

Bachelor of Science Examination, 2024

(3rd Year, 2nd Semester, Special Supplementary)

Mathematical Modelling

Paper - DSE-3B

Time : Two Hours

Full Marks : 40

The figures in the margin indicate full marks

Symbols / Notations have their usual meanings

*Answer any **FIVE** questions*

1. Assuming that the emotion is measurable, propose a simple model for the love affair of Romeo and Juliet. Under what parametric conditions, the two lovers are said to be exactly opposite? Determine the stability condition of the equilibrium love levels under this situation. [CO1][CO3] [3 + 5]

2. Suppose two nations, M and N , are engaged in an arms race. Assuming $m(t)$ and $n(t)$ as the armament levels of the nations M and N at time t , describe the rate equations of their armament levels mentioning the basic assortment of psychological and strategic motivations. What is the equilibrium defence level? Under what parametric conditions the two nations will be able to avoid instability in the arms race?

[CO1][CO3] [3 + 5]

3. (a) What do you mean by dimensional homogeneity?
(b) Consider the system

$$\begin{aligned}\frac{dB}{dt} &= rB\left(1 - \frac{B}{K}\right) - \frac{\alpha BP}{a + B}, \\ \frac{dP}{dt} &= \frac{\theta \alpha BP}{a + B} - gP,\end{aligned}$$

where B and P are two species. The parameters r, K, α, a, g are all positive, and θ is dimensionless. Nondimensionalize the above system with suitable transformations.

[CO2] [2 + 6]

[Turn over

4. (a) What do you mean by the half-life of a drug? If a drug's elimination rate follows first-order pharmacokinetics, determine its half-life.
- (b) A 60 kg patient was given a single dose of an antibacterial drug at a dose of 5 mg/kg. Blood samples were taken at various time intervals, and the concentration of the drug (C_p) in plasma was determined as given below:

Time (t) in hrs	C_p ($\mu\text{g/ml}$)
0.0	8.40
0.25	8.21
0.50	7.87
1.0	7.23
3.0	5.15
4.0	4.20
6.0	3.09
12.0	1.11
18.0	0.40

What are the values of this drug's elimination rate constant and half-life? This antibacterial agent is ineffective at a plasma concentration of less than 3 $\mu\text{g/ml}$. When should the next dose of this drug be given? [CO4] [4 + 4]

5. Suppose the growth of a population, N , is determined by the equation

$$\frac{dN}{dt} = rN - bN^2,$$

where r , b are positive constants and $N(0) = N_0$.

- (a) Determine the size of the population at any time t .
- (b) What would be the doubling time of this population? [CO3] [5 + 3]
6. What do you mean by optimum lot size of an inventory? Derive the optimum lot size and minimum total inventory cost in the usual notations when the lead time is zero, demand is uniform, production is instantaneous and there is no shortage. [CO1][CO4] [1 + 7]
