Ref. No.: EX/SC/MATH/UG/DSE/TH/01/A/2024(S)

BACHELOR OF SCIENCE EXAMINATION, 2024

(3rd Year, 1st Semester, Special Supplementary)

MATHEMATICS

Linear Programming and Game Theory

Time: Two Hours | Full Marks: 40

USE SEPARATE ANSWER SCRPITS FOR EACH PART

PART - 1 **(20 Marks)**

The figures in the margin indicate full marks

(Symbols/Notations have their usual meanings)

Answer any two questions

1. (a) Find all basic feasible solutions of the following system :

$$-x_1 + 2x_2 + x_3 + x_4 - 2x_5 = 5$$
$$x_1 - 2x_2 + 2x_4 - x_5 = 6.$$ 5

(b) Solve the linear programming by two - phase method:

$$\text{Minimize } z = 4x_1 + 2x_2$$
$$\text{Subject to } 3x_1 + x_2 \geq 27,$$
$$x_1 + x_2 \geq 21,$$
$$x_1 + 2x_2 \geq 30,$$
$$x_1,\ x_2 \geq 0.$$ 5

2. (a) Reduce the following linear programming problem to standard form:

$$\text{Minimize } z = 2x_1 - 3x_2 + 4x_3$$
$$\text{Subject to } |x_1 - 3x_2 - 2x_3| \leq 4,$$
$$3x_1 + x_2 - 2x_3 \geq 5,$$
$$5x_1 + 3x_2 - x_3 = 6,$$
$$x_1,\ x_2 \geq 0, x_3 \text{ is an unrestricted in sign.}$$ 4

(b) Solve the linear programming by Charnes Big-M method:

$$\text{Maximize } z = 2x_1 + x_2 + x_3$$

$$\text{Subject to } \quad 4x_1 + 6x_2 + 3x_3 \leq 8,$$

$$3x_1 - 6x_2 - 4x_3 \leq 1,$$

$$2x_1 + 3x_2 - 5x_3 \geq 4,$$

$$x_1, x_2, x_3 \geq 0.$$ 6

3. (a) Find the dual of the following linear programming problem:

$$\text{Minimize } z = 3x_1 + 2x_2 - x_3$$

$$\text{Subject to } \quad 2x_1 - x_2 + 2x_3 = 6,$$

$$x_1 - x_2 + x_3 \leq 5,$$

$$2x_2 - x_3 \geq -2,$$

$x_1, x_2 \geq 0, x_3$ is an unrestricted in sign. 4

(b) Use duality to solve the following linear programming problem:

$$\text{Maximize } z = 3x_1 + 2x_2$$

$$\text{Subject to } \quad 7x_1 + 2x_2 \geq 30,$$

$$5x_1 + 4x_2 \geq 20,$$

$$2x_1 + 8x_2 \geq 16,$$

$$x_1, x_2, \geq 0.$$ 6

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BACHELOR OF SCIENCE EXAMINATION, 2024(S)

(3RD Year, 1st Semester)

MATHEMATICS (Honours)

Paper: DSE 1(A)

(Linear Programming and Game Theory)

Time: Two Hours **Full Marks: 40**

USE SEPARATE ANSWER SCRIPTS FOR EACH PART

The figures in the margin indicate full marks

(Symbols/Notations have their usual meanings)

Part I (20 Marks)

Answer *any two* questions

4 (a) A manufacturer wants to ship 8 loads of his product as shown in the following table. The matrix gives the mileage from origin O to destination D. Shipping costs are Rs. 10 per load per mile. What shipping schedule should be used by the manufacturer?

	D_1	D_2	D_3	Available
O_1	50	30	220	1
O_2	90	45	170	3
O_3	250	200	50	4
Required	4	2	2	

(b) Solve the following game problem graphically:

Player B

Player A		B_1	B_2	B_3	B_4
	A_1	1	3	0	2
	A_2	3	0	1	-1

5+5

5(a) A team of 4 horses and 4 riders has entered in a jumping show contest. The number of penalty points to be expected when each rider rides any horse is shown below:

Riders

Horses	R_1	R_2	R_3	R_4
H_1	2	3	7	6
H_2	4	1	5	2
H_3	6	8	1	2
H_4	4	2	5	7

How should the horses be allotted to the riders so as to minimize the expected loss of the team?

(b) Is

	1	2	3	4
1			50	20
2	55			
3	30	35		25

an optimal solution of the following transportation problem? If not, modify it to obtain a better feasible solution.

	1	2	3	4	a_i
1	6	1	9	3	70
2	11	5	2	8	55
3	10	12	4	7	90
b_j	85	35	50	45	

5+5

6 (a) Two companies A and B are competing for the same product. Their different strategies are given in the following pay-off matrix:

Player A	Player B: B_1	B_2
A_1	2	-3
A_2	-2	5
A_3	3	-1

Use linear programming to determine the best strategies for both the players.

(b) Solve the following travelling salesman problem:

		To			
		A	B	C	D
From	A	∞	15	30	4
	B	6	∞	4	1
	C	10	15	∞	16
	D	7	18	13	∞

5+5