B. Sc. Mathematics Examination, 2024

(2nd Year, 1st Semester)

Numerical Methods

Paper : CORE-07 (THEORY)

Time : *1 Hour 30 mins* *Full Marks* : 24

Use separate Answer scripts for each Part.

PART— I (12 Marks)

1. Answer ***any two*** questions :

(a) Find the number of significant digits in 0.5721, where the absolute error is 0.25×10^{-2}.

(b) Suppose that f is a continuous real-valued function with continuous second derivative f'', defined on the closed interval $I_\delta[\xi-\delta,\xi+\delta],\delta>0,$ such that $f(\xi)=0$ and $f''(\xi)\neq 0$. Suppose further that there exits a positive constant A such that

$$\frac{|f''(x)|}{|f'(y)|}\leq A \quad \forall x, y, \in I_\delta$$

If $|\xi-x_0|\leq h$, where h is the smaller of δ and $1/A$, then the sequence $\{x_k\}$ defined as $x_{k+1}=x_k-\frac{f(x_k)}{f'(x_k)}$, $k=0, 1, 2$ ... converges quadratically to ξ. 2+4

2. *(a)* Use secant method to find an approximate value of $\sqrt{12}$, correct to five decimal places.

(b) Show that for a system of linear equations $Ax = b$, if A is strictly diagonally dominant matrix, then the Gauss-Jacobi scheme converges for any initial starting vector. 3+3

3. *(a)* Calculate the rate of convergence of the iterative scheme $x_{k+1} = 10 - x_k^4 + 3x_k^2$ to find the root of the equations $x^4 - 3x^2 + x - 10 = 0$.

(b) What do you mean by partial and completed pivoting? Explain. 2+4

PART—II (12 Marks)

Answer ***any two*** questions :

1. The following data for the function $f(x) = x^4$ is given :

x	0.4	0.6	0.8
$f(x)$	0.0256	0.1296	0.4096

Find $f'(0.8)$ and $f''(0.8)$ using quadratic interpolation. Compare with the exact solution. Obtain the bound on the truncation errors. 3+1+2

2. Determine the coefficients w_0, w_1 and w_2 in the quadrative formula

$$\int_0^{2h} x^{-\frac{1}{2}} f(x)dx = (2h)^{\frac{1}{2}} \left[w_0 f(0) + w_1 f(h) + w_2 f(2h) \right]$$

which is exact for polynomial of highest possible degree. 6

3. Given the initial value problem

$$u' = 2tu^2, u(0) = 1$$

estimate $u(0.4)$ using modified Euler-Cauchy method, where $h = 0.2$, correct up to four decimal places. 6

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