B. Sc. MATHEMATICS EXAMINATION, 2024

(2nd Year, 1st Semester)

MATHEMATICS—II

PAPER : GE-03

Time : *Two Hours* *Full Marks* : 40

Use separate Answer-Scripts for each Part.
Symbols / Notations have their usual meanings.

PART— I (12 Marks)

Answer ***any three*** questions :

1. If $x = \cos\alpha + i\sin\alpha,\quad y = \cos\beta + i\sin\beta\,,\quad z = \cos\gamma + i\sin\gamma$ and $x + y + z = xyz,$ then show that

$$\cos(\beta-\gamma) + \cos(\gamma-\alpha) + \cos(\alpha-\beta) = -1\,.$$ 4

2. Solve by Cardon's method : $x^3 - 27x - 54 = 0$. 4

3. If $A = \begin{pmatrix} 1 & 0 \\ -1 & 1 \end{pmatrix}$, then show that $A^2 - 2A + I_2 = O_2$. Hence find A^{15}. 4

4. Show that the product of all values of $(\sqrt{3}+i)^{3/5}$ is $8i$. 4

5. *(a)* Apply Descartes' rule of signs to find the nature of the roots of the equation $x^6 + x^2 = 0$.

(b) The roots of the equation $x^3 + px^2 + qx + r = 0$ $(r \neq 0)$ are α, β, γ.

Find the equation whose roots are

$$\alpha - \frac{\beta\gamma}{\alpha}, \beta - \frac{\gamma\alpha}{\beta}, \gamma - \frac{\alpha\beta}{\gamma}.$$ 2+2

PART—II (16 Marks)

Answer ***any four*** questions : 4×4=16

1. Solve : $(x^2y - 2xy^2)\,dx - (x^3 - 3x^2y)\,dy = 0$.

2. Solve the equation $\frac{xdx + ydy + (xdy - ydx)}{(x^2 + y^2)} = 0$, given $y = 1$ when $x = 1$.

3. Solve : $\frac{dy}{dx} + x\sin 2y = x^3\cos^2 y$.

4. Examine the equation $y = 2px + p^2$ for singular solution. Then find the general solution, where $p = \frac{dy}{dx}$.

5. Solve : $(1 + y^2) + \left(x - \exp(\tan^{-1} y)\right)\frac{dy}{dx} = 0$.

6. Solve : $(2D^2 - 5DD' + 2D'^2)z = 24(2y - x)$, where $D \equiv \frac{\partial}{\partial x}$ and $D' \equiv \frac{\partial}{\partial y}$.

PART—III (12 Marks)

Answer ***any three*** questions :

1. Find the real root of the equation $x^3 - 9x + 1 = 0$ which lies between 2 and 3, by iteration method, correct to four significant digits. (4)

2. Find the real root of the equation $3x^2 + 2x = 9$ by Newton-Raphson method, correct to three significant digits. (4)

3. Use Simpson's one-third rule and Trapezoidal rule to evaluate $\int_0^6 \frac{dx}{(1 + x^4)}$, taking 6 equal sub-intervals, correct to three decimal places. (4)

4. Solve the following system of equations $x + 3y + 3z = 1$, $x + 4y + 3z = 0$, $x + 3y + 4z = 2$ by Gauss-Jordan method. (4)

5. Solve numerically the differential equation $\frac{dy}{dx} = x + y^2$, given that $y = 1$ at $x = 0$ and find $y(0.5)$ (4)

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