3. Evaluate

$$\int_E \int \frac{y}{e^{x+y}} dxdy,$$

where E is bounded by

$$x = 0, y = 0, x + y = 1.$$

4. Find the area of the loop $a^3y^2 = x^4(b + x)$.

5. Show that a necessary and sufficient condition for a vector function $\vec{F}(t)$ to have constant direction is $\vec{F}(t) \times \frac{d\vec{F}(t)}{dt} = \vec{0}$.

6. Find the angle between two surfaces $xy^2z = 3x + z^2$ and $3x^2 - y^2 + 2z = 1$ at $(1,-2,1)$.

7. Evaluate $\int_C \vec{F} \cdot d\vec{r}$, where $\vec{F} = \left(x^2 + y^2\right)\vec{i} - 2xy\,\vec{j}$ and the curve C is the rectangle in the xy plane bounded by $y = 0$, $x = a$, $y = b$, $x = 0$.

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BACHELOR OF SCIENCE EXAMINATION, 2024

(First Year, First Semester)

MATHEMATICS

PAPER : MINOR – 101A

Minor – 1 (Mathematics – 1)

Full Marks : 40 *Time* : 2 *Hours*

Use separate answer scripts for each Part.

Symbols/Notations have their usual meaning.

PART—I (20 Marks)

Answer ***any four*** questions : 4×5=20

1. *(a)* Discuss the nature of the conic given by the general equation of the second degree as below :

$$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$$

(b) Find the equation of the polar of the point (2, 3) with respect to the circle given by

$$x^2 + y^2 - 2x - 4y + 1 = 0$$ (3+2)

2. *(a)* Find the equation of the diameter of the ellipse $3x^2 + 4y^2 = 5$ conjugate to the diameter $y + 3x = 0$.

(b) Find the coordinates of the limiting points of the co-axial system given by the circles

$x^2 + y^2 + 5x + y + 4 = 0,\ x^2 + y^2 + 10x - 4y - 1 = 0.$

(2+3)

3. *(a)* Find the equation of the plane passing through the points (1, 1, 2) and (2, 4, 3) and perpendicular to the plane

$x - 3y\,7z + 5 = 0$

(b) Find the image of the point (–3, 8, 4) in the plane $6x - 3y - 2z + 1 = 0$. (3+2)

4. *(a)* Find the equation of the sphere which passes through the origin and touches the sphere $x^2 + y^2 + z^2 = 56$ at the point (2, –4, 6).

(b) Find the centre and radius of the circle $x^2 + y^2 + z^2 = 25$ and $x + 2y + 2z + 9 = 0$. (1+4)

5. *(a)* If $y = a\cos(\log x) + b\sin(\log x)$, then show that

$x^2 y_2 + x y_1 + y = 0$

(b) In the Mean Value Theorem $f(x+h) = f(x) + h f'(x+\theta h)$, show that if

$$f(x) = Ax^2 + Bx + C,\ A \neq 0,\ \text{then } \theta = \frac{1}{2}.$$

Give a geometrical interpretation of the result. (2+3)

6. *(a)* If $\lim_{x\to 0} (\sin 2x + a\sin x)/x^3$ be finite, find the value of 'a' and the limit.

(b) If $u = \tan^{-1}(x^3 + y^3)/(x - y)$, show that

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \sin 2u\,.$$

(3+2)

PART—II (20 Marks)

Answer ***any four*** questions : 5×4=20

1. Find the value of the integral :

(a) $\int_0^\infty \sqrt{x}\, e^{-x^3}\, dx$

(b) $\int_0^1 \frac{dx}{(1-x^6)^{\frac{1}{6}}}$

2. *(a)* Verify if the following improper integral is convergent or divergent :

$$\int_0^\infty \frac{x^{\frac{3}{2}} dx}{3x^2 + 5}$$

(b) Change the order of integration of $\int_0^1 dx \int_x^{\sqrt{x}} f dy$.