(c) A standard, fair die is thrown and the score X is recorded. Find skew (X). (2)

★ ★ ★

BACHELOR OF SCIENCE EXAMINATION, 2024

(First Year, First Semester)

MATHEMATICS

PAPER : Stat – I (Minor)

(Probability and Descriptive Statistics)

Full Marks : 32 *Time* : 1 *Hour 30 Minutes*

Use a separate Answer- Script for each part

PART—I

(Marks : 10)

Answer ***any two*** questions : 5×2=10

1. (a) Derive the linear regression equation of x on y. Show that it passes through a fixed point.

(b) Out of two regression lines $x+2y-5=0$ and $2x+3y-8=0$, which one is the regression line of x on y ? (3+2)

2. (a) Find the Spearman's rank correlation coefficient when there are no ties.

(b) Show that Spearman's rank correlation coefficient (with no ties) for perfect disagreement. (3+2)

3. If $u = x\cos\theta + y\sin\theta$, $v = y\cos\theta - x\sin\theta$, and u and v are uncorrelated for n pair of values $(x_i, y_i), \forall i = 1(1)n$, then prove that $\tan 2\theta = \dfrac{2r_{xy}s_x s_y}{s_x^2 - s_y^2}$, $s_x^2 =$ variance of x. (5)

PART—II

(Marks : 22)

Answer question number 1 and ***any four*** from the rest.

1. Suppose X is a real valued random variable with $E(X) = 5$ and $\text{var}(X) = 4$. Find the values of *(a)* $\text{var}(3X - 2)$ *(b)* $E(X^2)$ (2)

2. Suppose that the two-dimensional random variable (X, Y) has the probability density function f given by

$$f(x, y) = \begin{cases} 6x^2 y & \text{For } 0 \le x \le 1, 0 \le y \le 1 \\ 0 & \text{otherwise} \end{cases}$$

(a) Find the marginal probability density function of X (2)

(b) Find the marginal probability density function of Y (2)

(c) Are X and Y independent? (1)

3. If A and B are stochastically independent events such that $P(A) = 0{\cdot}37$, and $P(B) = 0{\cdot}44$, then find the values of

(a) $P(A')$ *(b)* $P(A \cup B)$ *(c)* $P(A \cap B)$ *(d)* $P(A \cap B')$

(e) $P(A' \cap B')$. (5)

4. A random variable X can assume the values 1, 2, 3, 4 with probabilities 1/4, 1/2, 1/8, 1/8 respectively. Determine the distribution function and draw the graph. (5)

5. If the distribution function of a random variable X is given by

$$F(x, y) = \begin{cases} 1 - (1 + x)e^{-x} & \text{for } x > 0 \\ 0 & \text{for } x \le 0 \end{cases}$$

find

(a) $P(X \le 2)$, $P(1 < X < 3)$ and $P(X > 4)$ (3)

(b) the density function of X (2)

6. *(a)* If X is a real valued random variable such that $\mu = E(X)$ and $\sigma^2 = \text{var}(X)$, then prove that

(i) skew $(X) = \dfrac{E(X^3) - 3\mu\sigma^2 - \mu 3}{\sigma^3}$ (1)

(ii) kurt $(X) = \dfrac{E(X^4) - 4\mu E(X^3) + 6\mu^2\sigma^2 + 3\mu^4}{\sigma^4}$

(2)