4. Obtain the equation of the cylinder, whose generators intersect the plane curve $ax^2 + by^2 = 1$, $z = 0$ and are parallel to the straight line $\frac{x}{l} = \frac{y}{m} = \frac{z}{n}$.

5. Find the radical axis of the circles $x^2 + y^2 + 2ax + c^2 = 0$ and $x^2 + y^2 + 2by + c^2 = 0$ and hence deduce that the circles touch each other, if $\frac{1}{a^2} + \frac{1}{b^2} = \frac{1}{c^2}$.

★ ★ ★

Ex/SC/MATH/UG/CORE/TH/01/2024(OLD)

Bachelor of Science Examination, 2024

(First Year, First Semester)

Mathematics (Honours)

(Old)

Paper : CORE - 01

(Algebra, Geometry and Calculus)

Time : 2 *Hours* *Full Marks* : 40

Use Separate Answer scripts for each Part.

PART—I

(24 Marks)

1. Answer ***any three*** questions : 4×3=12

   (a) If a and b are two roots of the equation $x^2 - 2x + 2 = 0$, then using De Moivre's theorem show that

   $$a^n + b^n = 2^{\frac{n}{2}+1} \cos\frac{n\pi}{4} \, . \qquad 4$$

   (b) Solve the equation by Cardan's method :

   $$x^3 - 15x^2 - 33x + 847 = 0 \, . \qquad 4$$

(c) Determine the values of α and β for which the following equations

$$x + y + z = 6$$
$$x + 2y + 3z = 10$$
$$x + 2y + \alpha z = \beta$$

have

(i) no solution;

(ii) a unique solution;

(iii) an infinite number of solutions. 4

(d) If x, y and z are positive real numbers and $x + y + z = 1$, prove that

$$8xyz \le (1-x)(1-y)(1-z) \le \frac{8}{27}$$ 4

(e) State the fundamental theorem of Arithmetic. Prove that the product of any three consecutive integers is divisible by 6. (1+3)

2. Answer ***any three*** questions : 4×3=12

(a) Find the radius of curvature of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ at (a cos ϕ, b cos ϕ). 4

(b) If $y = \sin ax + \cos bx$, prove that

$$y_n = a^n[1 + (-1)^n \sin 2ax]^{\frac{1}{2}}\ \cdot$$ 4

(c) Determine the rectilinear asymptotes of the curve

$$x^3 - 2y^3 + xy(2x - y) + y(x - y) + 1 = 0$$ 4

(d) Draw the curve $xy^2 + (x + a)^2(x + 2a) = 0$ and find the area of the loop. 4

(e) If $J_n = \int \sec^n x\, dx$, then show that

$$(n-1)J_n = \tan x \sec^{(n-2)} x + (n-2)J_{n-2}\cdot$$

Hence find a reduction formula for $\int_0^{\frac{\pi}{4}} \sec^n x\, dx$. 4

PART—II

(16 Marks)

1. PSP' is a focal chord of the conic. Prove that the angle between tangents at P and P' is $\tan^{-1}\left(\frac{2e\sin\alpha}{1-e^2}\right)$, where α is the angle between the chord and the major axis.

2. A sphere of radius k passes through the origin and meets the axes in A, B, C respectively. Prove that the locus of the centroid of the triangle ABC is the sphere $9(x^2 + y^2 + z^2) = 4k^2$.

3. Show that the equation of the cone whose generators pass through the point (α, β, γ) and have their direction cosines satisfying the relation $al^2 + bm^2 + cn^2 = 0$ is $a(x-\alpha)^2 + b(y-\beta)^2 + c(z-\gamma)^2 = 0$.