MASTER OF SCIENCE EXAMINATION, 2024

(Second Year, First Semester)

MATHEMATICS

PAPER : CORE - 11

(Integral Equation, Integral Transform and Calculus of Variation)

Full Marks : 40 *Time* : 2 *Hours*

Symbols/Notations have their usual meanings

Use separate Answer-Scripts for each Group

GROUP—A

(24 Marks)

1. Answer ***any one*** : 8×1=8

(a) Find the extremals of the functional

$$J[y] = \int_0^1 (1 + y''^2)\, dx$$

subject to the boundary conditions $y(0) = 0$, $y'(0) = 1$, $y(1) = 1$, $y'(1) = 1$.

(b) Find the curves for which the functional

$$J[y(x)] = \int_0^{x_1} \frac{(1 + y'^2)^{1/2}}{y}\, dx$$

can have extrema if $y(0) = 0$ and the point (x_1, y_1) can vary along the line $y = x - 5$.

2. Answer ***any four*** : 4×4=16

(a) Reduce the following boundary value problem to a Fredholm integral equation

$$\frac{d^2y}{dx^2}+y=0\,,\ 0\le x\le 1\quad y(0)=0,\ y'(1)+3y(1)=1.$$

(b) Prove that the eigenfunctions of a symmetric kernel corresponding to different eigenvalues are orthogonal.

(c) Use the method of successive approximations to solve

$$u(x)=1-\int_0^x u(t)\,dt,\ 0\le x\le 1\,.$$

(d) Find the nth iterated kernal of the following integral equation

$$\phi(x)=x+\int_0^{\frac{1}{2}}\phi(t)\,dt,\ 0\le x\le \frac{1}{2}\,.$$

(e) Find the non trivial solution of following Fredholm integral equation of second kind

$$\phi(x)=\lambda\int_0^1 4x\phi(t)\,dt,\ 0\le x\le 1\,.$$

(f) For what values of λ does the unique solution of

$$\phi(x)=x+\lambda\int_0^1(1+x+t)\phi(t)\,dt,\ 0\le x\le 1$$

exist? Hence find the unique solution.

GROUP—B

(Marks : 16)

(Integral Transforms)

Answer ***any two*** questions.

1. Find $\mathcal{F}^{-1}\left\{\dfrac{1}{s^2+c^2}\right\}$ by using the complex integration formula. 8

2. *(a)* Using zero order Hankel transform to solve PDE

$$\frac{\partial^2\phi}{\partial r^2}+\frac{1}{r}\frac{\partial\phi}{\partial r}+\frac{\partial^2\phi}{\partial z^2}=0\,,\ 0<r<\infty\,,\ z\ge 0$$

with the condition $\phi(r,z)\,,\ \dfrac{\partial\phi}{\partial r}\to 0$ as $(r^2+z^2)^{\frac{1}{2}}\to\infty$, $z>0$ and $\phi(r,0)=f(r)\,,\ r>0\,,\ f(r)$ is prescribed function and its Hankel transform exists.

(b) Find $\mathcal{L}^{-1}\left\{\dfrac{p}{(p^2+a^2)^2}\right\}$. 6+2

3. *(a)* If $\mathcal{L}\{f(t)\}=F(p)$, then show that $\lim_{|p|\to\infty}|F(p)|=0$.

(b) State and prove convolution theorem for Fourier transform.

(c) Find $\mathcal{L}\{te^{-at}\cos bt\}$. 2+3+3

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