Master of Science Examination, 2024

(2nd Year, 1st Semester)

Mathematics

Paper : DSE 04 – A28

(Einstein's General Theory of Relativity)

Time : 2 *Hours* *Full Marks* : 40

Use a separate Answer-Script for each Part.

PART—I (20 Marks)

***All** questions carry equal marks.*

Symbols/Notations have their usual meanings.

Answer ***any two*** questions : 2×10

1. *(a)* Derive Einstein field equations from action principle.

(b) Write down spherically symmetric vacuum solution for Einstein gravity. Interpret the constant parameter of it. 6+(2+2)

2. Determine radial ingoing and outgoing null geodesics in Schwarzschild space-time. Show that the null geodesics become parallel to the time axis as the radial co-ordinate approaches toward the event horizon. Determine the time taken by a time-like particle to reach the event horizon from outside in proper time. 6+2+2

3. "Due to strong gravitational force the path of light rays bend"—prove this conjecture in Einstein gravity, clearly describing by diagram. 8+2

4. *(a)* Deduce the Oppenheimer–Volkov equation for static spherically symmetric space-time.

(b) Define blackhole from the point of view of space-time geometry. What do you mean by geodesically complete manifold? 6+(2+2)

PART—II (20 Marks)

Answer ***any four*** questions : 4×5=20

1. For the following Schwarzschild line element

$$ds^2 = \left(1-\frac{16}{r}\right)dt^2 - \frac{dr^2}{\left(1-\frac{16}{r}\right)} - r^2\left(d\theta^2 + \sin^2\theta\, d\phi^2\right),$$

find the proper time for light takes to reach from horizon to $r = 0$ (origin).

2. If $p \in D^+(S) - H^+(S)$, then show that each past inextendible causal curve from p meets $I^-(S)$.

($D^+(S)$ and $H^+(S)$ are future domain of dependence and future Cauchy horizon, respectively.)

3. Show that chronological future is an open set but not closed.

4. Define with example Cauchy surface. Give an example of a space time which is not globally hyperbolic.

5. Show that future Cauchy horizon, $H^+(S)$ is achronal and closed.

6. Define stable causality and strong causality. Show that stable causality implies strong causality.

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