

Ex/SC/MATH/PG/DSE/TH/03/A10/2024

MASTER OF SCIENCE EXAMINATION, 2024

(Second Year, First Semester)

MATHEMATICS

PAPER : DSE - 03

(Advanced Differential Geometry)

Time : Two Hours

Full Marks : 40

Answer *any four* questions

10×4=40

1. (a) Compute $[X, Y]f$, where $X = \frac{\partial}{\partial x^1}$, $Y = \frac{\partial}{\partial x^2} + e^{x^1} \frac{\partial}{\partial x^3}$,
and $f = 5x^1 - 4(x^3)^2$ at $(2, -1, 3)$.
- (b) Define integral curve of a vector field. Find the integral
curve for the vector field $X = \frac{\partial}{\partial x^1} + (x^1)^2 \frac{\partial}{\partial x^2}$ on $\mathbb{R}^2$.
- (c) Prove that $(g \circ f)^* = f^* \circ g^*$, where the symbols have
their usual meaning. 3+4+3
2. (a) Prove that every vector field on a compact manifold is
complete.
- (b) Find the exterior differentiation of $x^2 y \, dy - xy^2 \, dx$. Is
it closed? Justify. 6+4

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[Turn Over]

(2)

3. (a) If w is a 1-form, then prove that

$$dw(X, Y) = \frac{1}{2} \{X(w(Y)) - Y(w(X)) - w([X, Y])\}$$

where X, Y are arbitrary vector fields on a differentiable manifold M .

- (b) Define curvature tensor R on a differentiable manifold M . Prove that $R(X, fY)Z = f R(X, Y)Z$ where $X, Y, Z \in \chi(M)$ and $f \in F(M)$.

- (c) Define Lie group. Is $GL(n, \mathbb{R})$ a Lie group? Justify.

3+4+3

4. Prove every Riemannian manifold (M, g) admits a unique Riemannian connection. 10

5. (a) Prove that a Riemannian manifold of constant curvature is an Einstein manifold. Is the converse true? If not up to what extent is it true? Justify.

- (b) Let $\bar{\nabla}$ be the Riemannian connection and ∇ be another linear connection such that $\nabla_X Y = \bar{\nabla}_X Y - w(X)Y$, where w is a 1-form, then show that ∇ is a semisymmetric non-metric connection. 6+4

6. (a) Define Kähler manifold with an example. Prove that a Kähler manifold of constant curvature is flat, provided $n > 2$.

(3)

- (b) Define k -contact Riemannian manifold. Prove that for a k -contact Riemannian manifold

$$g(\nabla_X \xi, Y) + g(\nabla_Y \xi, X) = 0 \quad 5+5$$

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