MASTER OF SCIENCE EXAMINATION - 2024

(2nd year, 1st Semester)

Mathematics

(Solid Mechanics-1)

Full Marks: 40 **Time: Two Hours**

The figures in the margin indicate full marks.

(Symbols/Notations have their usual meanings)

Answer any four questions

1. Deduce generalized Hook's law in the form

$$\tau_{ij} = c_{ijkl}e_{kl}, \quad i, j, k, l = 1,2,3$$

Further show that due to symmetry of stress and strain tensors and existence of strain energy density function the number of coefficients in the generalized Hook's law reduces from 81 to 21. 10

[CO1+CO2]

2. Deduce compatibility equations in the form

$$e_{ij,kl} + e_{kl,ij} - e_{ik,jl} - e_{jl,ik} = 0$$

Hence deduce the Beltrami –Michell compatibility equations in the form

$$\nabla^2\tau_{ij} + \frac{1}{1+\sigma}\,\Theta_{,ij} = -\frac{\sigma}{1+\sigma}\,\delta_{ij}\,div\,\bar{F} - (F_{i,j} + F_{j,i})$$

Where $\Theta = \tau_{ii}$ and F_i is the components of body force.

If field of force $\vec{F} = \vec{\nabla}\varphi$, then show that the above equation can be written as

[CO1+CO3]

$$\nabla^2\tau_{ij} + \frac{1}{1+\sigma}\,\Theta_{,ij} = -\frac{\sigma}{1+\sigma}\,\delta_{ij}\,\nabla^2 - 2\varphi_{,ij}$$ 10

3a. Show that principal directions of strain at each point of a linearly elastic isotropic body are coincide with the principal directions of stress. 5

b. State and prove Clapeyron's theorem. 5

[CO2+CO4]

4. Prove that the problem of torsion of a beam by the terminal couples can be reduced either to a Neumann problem or to a Dirichlet problem.Hence find the resultant force and

resultant moment of the external Forces applied to the end of the beam. Also show that the torsional rigidity depends on the modulus of rigidity and shape of the cross section.

[CO3+CO4] 10

5. Prove that the components for plane problem in cylindrical polar co-ordinates can be expressed in the form

$$e_{rr} = \frac{\partial u_r}{\partial r}, \quad e_{\theta\theta} = \frac{u_r}{r} + \frac{1}{r}\frac{\partial u_\theta}{\partial \theta}, \qquad e_{r\theta} = \frac{1}{r}\frac{\partial u_r}{\partial \theta} + \frac{\partial u_\theta}{\partial r} - \frac{u_\theta}{r}.$$

10.

[CO2+CO4]

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