

being filled with liquid. The impulsive forces are so applied that the inner sphere starts moving with velocity u and the outer sphere with velocity v in the same direction. Determine the velocity potential of the liquid motion. 8

- b) A source S and a sink T of equal strengths m are situated within the space bounded by a circle whose centre is O . If S and T are at equal distances from O on opposite sides of it and on the same diameter AOB , show that the velocity of the liquid at any point P is

$$2m \frac{OS^2 + OA^2}{OS} \cdot \frac{PA \cdot PB}{PS \cdot PS' \cdot PT \cdot PT'},$$

where S' and T' are the inverse points of S and T with respect to the circle. 8

4. a) Define vortex rows. Show that the motion due to a set of line vortices of strength K at points $z = \pm na$ ($n = 0, 1, 2, 3, \dots$) is given by the relation

$$w = \frac{iK}{2\pi} \log \left\{ \sin \left(\frac{\pi z}{a} \right) \right\}. \quad 8$$

- b) Prove that the velocity of propagation of surface wave of length λ of a canal of depth h is given by

$$c^2 = \left(\frac{g\lambda}{2\pi} \right) \tanh \left(\frac{2\pi h}{\lambda} \right). \quad 8$$

M. SC. MATHEMATICS EXAMINATION, 2024

(2nd Year, 1st Semester, Special Supplementary)

MATHEMATICS

PAPER – DSE-03 A5

[FLUID MECHANICS - I]

Time : 2 hours

Full Marks : 40

Notations / Symbols have their usual meanings.

Answer **Q. No. 1** and **any two** from the rest.

1. Derive the Euler's equation of motion of an inviscid fluid in the form $\frac{D\vec{q}}{Dt} = \vec{F} - \frac{1}{\rho} \text{grad } p.$ 8

Or

Derive the equation of continuity using Lagrangian method in the form

$$\rho J = \rho_0 \text{ (a constant).} \quad 8$$

2. a) State and prove the Milne-Thomson circle theorem. 8

- b) Show that $\frac{x^2}{a^2} \tan^2 t + \frac{y^2}{b^2} \cot^2 t = 1$ is a possible form of the boundary surface of a liquid, where a and b are constants. Find the normal velocity to this surface. 8

3. a) Let a sphere of radius a surrounded by a concentric sphere of radius b ($a < b$), the intervening space

[Turn over