Master of Science Examination, 2024(S)

(2nd Year, 2nd Semester)

MATHEMATICS

Paper - DSE 06 (B7)

(Production Planning and Inventory Control)

Time : Two hours **Full Marks : 40**

The figures in the margin indicate full marks.

(Notations/Symbols have their usual meanings)

Answer ***any five*** questions.

1. Derive the expression for Economic Production Quantity (EPQ) with uniform rate of replenishment and no shortages. Find also the minimum average cost of the production-inventory system. [CO1] 1+7

2. Show that, for a stochastic inventory model with instantaneous demand and no set up cost, the optimum stock level S* in discrete units can be obtained by satisfying the following inequality:

$$\sum_{x=0}^{s-1} p(x) < \frac{c_2}{c_1 + c_2} < \sum_{x=0}^{s} p(x)$$

where the symbols have their usual meanings. [CO2] 8

3. Consider a shop which produces and stocks three items. The management desires never to have an investment in inventory of more Rs. 15000. The items are produced in lots. The demand rate for each item is constant and can be assumed to be deterministic. No back orders are to be allowed. The pertinent data for the items are given in the following table.

Item	**1**	**2**	**3**
Demand rate (units/year)	1,000	500	2,000
Variable cost (Rs./unit)	20	100	50
Set up cost per lot (Rs.)	50	75	100

The carrying charge on each item is 20% of average inventory valuation per annum. Determine the optimal lot size for each item. [CO4] 8

4. Suppose that an inventory system operates over a prescribed time horizon H and, during H, there exists a total demand for D units. Assume that there is no shortage in inventory. If n be the number of equal replenishments made during the period H then show that the optimal value of n i.e., n^* can be obtained by satisfying the relation

$$f(n^*-1) \leq \frac{c_1 DH}{c_3} \leq f(n^*)$$

where $f(n) = \dfrac{n(n+1)}{(n+1)\,h(n) - n\,h(n+1)}$ and $h(n) = \dfrac{2}{3}n - \dfrac{\sqrt{1}+\sqrt{2}+\sqrt{3}+\cdots+\sqrt{(n-1)}}{n}$

[CO1] 8

5. Derive the expected cost function for a stochastic order-level inventory system with uniform demand and no set up cost, when the stock level S and demand x are restricted to discrete units. Also show that the optimal value of S can be obtained by satisfying the inequality:

$$\sum_{x=0}^{S-1} p(x) + \left(S - \frac{1}{2}\right)\sum_{x=S}^{\infty}\frac{p(x)}{x} < \frac{c_2}{c_1+c_2} < \sum_{x=0}^{S} p(x) + \left(S + \frac{1}{2}\right)\sum_{x=S+1}^{\infty}\frac{p(x)}{x}$$

where $p(x)$ = p.m.f. of x, c_1 = holding cost per unit per unit time, c_2 = shortage cost per unit per unit time. [CO2] 8

6. Find the optimum order quantity for a product for which the price breaks are as follows:

Quantity	**Unit cost(Rs.)**
$1 \leq q_1 < 500$	25.0
$500 \leq q_2 < 1500$	24.8
$1500 \leq q_3 < 3000$	24.6
$3000 \leq q_4$	24.4

Given that the annual demand for a product is 500 units. The cost of storage per unit per year is 10% of the unit cost. The ordering cost is Rs. 180 for each order. (CO4) 8

7. Suppose that an item is ordered per week. The demand for the item during the week is uniform and with the probabilities P(0) = 0.04, P(5) = 0.20, P(10) = 0.37, P(15) = 0.30, P(20) = 0.09. The carrying cost is Rs. 2/unit/week. The shortage cost is Rs. 24/unit/week. What should the optimal order level be at the beginning of each week? (CO3) 8

8. Show that the probabilistic order-level inventory systems with instantaneous and uniform demands are equivalent. (CO2) 8