Ref. No. : EX/SC/MATH/PG/DSE/TH/02A/2024(S)

Master of Science Examination - 2024 (Supplementary)

(Second Year, First Semester)

Mathematics

DSE - 02

Numerical Analysis - I (Theory)

Full Marks : 24 | **Time : One hour fifteen minutes**

Symbols / Notations have their usual meaning.

Use separate answer script for each part.

Group – A

(16 Marks)

Answer **any two** questions

1. Deduce Thomas algorithm for the solution of tri-diagonal system of linear equations. When this algorithm is stable? Is this algorithm be applicable to the following system of equations? Justify.

$$\begin{aligned} x_1 + x_2 \quad\quad &= 3, \\ -x_1 + 2x_2 + x_3 &= 6, \\ 3x_2 + 2x_3 &= 12. \end{aligned}$$

$5 + 1 + 2$

2. Suppose $AX = b$ and $\tilde{A}\tilde{X} = \tilde{b}$ are two linear systems, where $X(t)$ be the solution of the system $A(t)X(t) = b(t)$. Then prove that

$$\frac{||X - \tilde{X}||}{||X||}\left(1 - \kappa(A)\frac{||A - \tilde{A}||}{||A||}\right) \leq \kappa(A)\left(\frac{||b - \tilde{b}||}{||b||} + \frac{||A - \tilde{A}||}{||A||}\right),$$

where $\kappa(A) = ||A||\cdot||A^{-1}||$. 8

3. Define cubic spline interpolation and hence construct a natural cubic spline that passes through the points $(1, 3)$, $(2, 4)$ and $(3, 2)$.

$3 + 5$

[Turn over

Group – B

(8 Marks)

Answer **any one** question

4. Let $A = (a_{ij})$ be a square matrix of order n whose eigenvalues are λ_i, $(i = 1, 2, \cdots, n)$. If P_k be the sum of the moduli of the elements along the kth row excluding the diagonal element a_{kk}, then prove that every eigenvalue of A lies inside or on the boundary of at least one of the circle

$$|\lambda_i - a_{kk}| = P_k, \qquad k = 1, 2, \cdots, n.$$

8

5. For the matrix

$$A = \begin{pmatrix} 1 & 2 & -2 \\ 1 & 1 & 1 \\ 1 & 3 & -1 \end{pmatrix}$$

(i) find all the eigenvalues and the corresponding eigenvectors,

(ii) Verify that $S^{-1}AS$ is a diagonal matrix, where S is the matrix of eigenvectors.

$4 + 4$