4. *(a)* Patients arrive at a clinic according to a Poisson distribution at the rate of 30 patients per hour. The waiting room does not accommodate more than 14 patients. Examination time per patient is exponential with mean rate 20 per hour.

   *(i)* What is the probability that an arriving patient will not wait?

   *(ii)* What is the expected waiting time until a patient is discharged from the clinic?

   *(b)* In a car manufacturing plant, a loading crane takes exactly 10 minutes to load a car into a wagon and again comes back to the position to load another car. If the arrival of cars is a Poisson stream at an average rate of one after every 20 minutes, calculate the average time of a car in a stationary state. 5+5

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**MASTER OF SCIENCE EXAMINATION, 2024**

**(2nd Year, 1st Semester)**

**MATHEMATICS**

**PAPER : DSE - 04 - A26**

**( Probability, Games and Queuing Theory )**

*Time* : *Two Hours* *Full Marks* : 40

*Use a separate Answer-Script for each Group.*

*Notations / Symbols have their usual meanings.*

## GROUP—A (20 Marks)

Attempt ***any two*** questions. Each question carries **ten** marks : 10×2=20

1. *(a)* Prove the first Borel-Cantelli Lemma :

   If $\sum_n P(A_n)$ converges, prove that $P(\limsup A_n) = 0$.

   *(b)* Define a Tail Event. Prove that : If $\{A_n\}$ is an independent sequence of events, then every tail-event generated by the sequence $\{A_n\}$ has probability either zero or one. What is the name of this result?

2. Stating all the results you are using, find the characteristic function of a Cauchy Random Variable.

3. Prove that :

(a) A class of sets that is both a $\pi$-system and a $\lambda$-system, is a $\sigma$-field.

(b) If a $\lambda$-system contains $A$ and $A \cap B$, then it also contains $A - B$.

**GROUP—B / (20 Marks)**

Answer ***any two*** questions.

1. (a) State minimax-maximin principle.

Let $f(x, y)$ be a real valued function of $x$ and $y$ such that $f(x, y)$ is a real number for $x \in A$ and $y \in B$ ; $A, B$ being two sets. If both $\max_{x\in A} \min_{y\in B} f(x,y)$ and $\min_{y\in B}$ $\max_{x\in A} f(x,y)$ exist, then show that

$$\max_{x\in A} \min_{y\in B} f(x,y) \le \min_{y\in B} \max_{x\in A} f(x,y)$$

(b) $A$ and $B$ play a game in which each has three coins, a 5P, a 10P and a 20P. Each selects a coin without the knowledge of the others choice. If the sum of the coins is an odd amount, $A$ wins $B$'s coin; if the sum is even $B$ wins $A$'s coin. Find the best strategy for each player and the value of the game. 5+5

2. (a) If mixed strategies are allowed, then show that there always exists a value of the game i.e. $\overline{v} = \underline{v} = v$.





(b) Solve the following game graphically :

| | | Player B | | | |
|---|---|---|---|---|---|
| | | $B_1$ | $B_2$ | $B_3$ | $B_4$ |
| **Player A** | $A_1$ | 2 | 1 | 0 | –2 |
| | $A_2$ | 1 | 0 | 3 | 3 |

5+5

3. (a) Obtain the steady state equations for the model $(M/M/1) : (N/FCFS)$.

(b) The rate of arrival of customers at a public telephone booth follows Poisson distribution, with an average time of 10 minutes between one customer and the next. The duration of a phone call is assumed to follow exponential distribution with a mean of 3 minutes.

(i) What is the probability that a person arriving at the booth will have to wait?

(ii) What is the average length of the non-empty queues that form from time to time?

(iii) The management will install a second booth when it is convinced that the customers would expect waiting for at least 3 minutes for their turn to make a call. By how much time should the flow of customers increase in order to justify a second booth? 6+4