M.Sc. Mathematics Examination, 2024 (S)

(2nd year, 2nd semester Special Supplementary)

Paper- Core 12

Control Theory and Optimization

Use separate answer script for each part

Part – I (24 marks)

Answer *any three* questions from the following:

1. a) Define **controllability matrix**. [CO2]

b) Prove that for a controllability matrix G, rank G = n iff $0 \in C^0$. [CO2] (2+6)

2. a) State **Mangasarian's Theorem** with proper validations. [CO1]

b) Solve the control problem, Max $\int_0^T [1 - \text{tx (t)} - u(t)^2]\, dt$, $\dot{x} = \text{u (t)}$; x (0) $= x_0$; x (T) = free, where x_0 and T are positive constants.

c) Solve: Max $\int_0^T (x - u^2)\, dt$, $\dot{x} = \text{x} + \text{u}$; x (0) =0; x (T) free, $\text{u} \in R$. [CO1]

(2+3+3)

3. (a) Describe briefly the **Moon-lander problem**. [CO1]

(b) Explain how the Pontryagin Maximum Principle can be applied to this problem to land the moon-lander safely while maximizing the remaining fuel m (τ) and equivalently, minimizing the total applied thrust before landing. [CO1]

(2+6)

4. Consider the differential equation of the system $\boldsymbol{M\ddot{x} + F\dot{x} + K = 0}$ with $\boldsymbol{x}(\mathbf{0}) = \mathbf{1}$ and $\boldsymbol{\dot{x}}(\mathbf{0}) = \mathbf{1}$. Find the condition for which the system will be asymptotically stable & hence, state the Lyapunov's first theorem from your analytical solution. [$\boldsymbol{M}$, $\boldsymbol{F}$and$\boldsymbol{K}$ as their usual notation] [CO3]

(4+4)

EX/SC/MATH/PG/CORE/TH/12/2024(S)

Master of Science Examination, 2024(S)

(2rd Year, 2nd Semester)

MATHEMATICS

Core 12

(Optimization and Control Theory)

Full Marks:40 Time: Two hours

Use separate answer script for each group.

Part - II (16 Marks)

Answer **any four** *questions. All questions carry equal marks.*

1. What is the difference between Fibonacci and Golden section methods?
Maximize the following unimodal function

$$f(x) = \begin{cases} 2x/3, & x \leq 3 \\ 5 - x, & x > 3 \end{cases}$$

in the interval [1, 4] by Fibonacci method using $n = 4$. [CO4]

2. Minimize $f(x, y) = 2(y - x^2)^2 + (1 - x)^2$
by Nelder-Mead method. Take the initial simplex as $v_1 = (0, 0)$, $v_2 = (1, 0)$ and $v_3 = (0, 1)$ and complete two iterations. [CO4]

3. Minimize $f(x_1, x_2) = x_1 - x_2 + 2x_1^2 + 2x_1x_2 + x_2^2$
by Hooke and Jeeves pattern search method. Complete two iterations. [CO4]

4. Minimize $f = 4x_1^2 + 3x_2^2 - 5x_1x_2 - 8x_1$ staring from the point (0, 0) using Powell's method. Complete two iterations. [CO4]

5. Minimize $f = 2x_1^2 + x_2^2$ by Steepest Descent method. Take the starting point as $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$ and perform two iterations only. [CO4]

6. What is the difference between interior and exterior penalty function methods?
Minimize $f(x_1, x_2) = \frac{1}{3}(x_1 + 1)^3 + x_2$ subject to $x_1 \geq 1$, $x_2 \geq 0$,
using interior penalty function method with the calculus method of unconstrained minimization. [CO4]