# Master of Science Examination, 2024

**(1st Year, 1st Semester)**

**Mathematics**

**Paper : Core - 03**

**( Topology )**

*Time* : 2 *Hours* *Full Marks* : 40

*Answer **any five** questions.*

*Notations and Symbols have their usual meanings.*

**1.** Let $\tau$ be the class of subsets of $\mathbb{N}$ consisting of $\phi$ and all subsets of $\mathbb{N}$ of the form $E_n = \{n, n+1, n+2, \ldots\}$ with $n \in \mathbb{N}$.

*(i)* Show that $\tau$ is a topology of $\mathbb{N}$; 3

*(ii)* List the open sets containing the positive integer 6; 2

*(iii)* Find the accumulation points of the set $S = \{4, 13, 28, 37\}$. 3

**2.** *(i)* Consider the discrete topology $\mathcal{D}$ on $X = \{a, b, c, d, e\}$. Find a subbase $S$ for $\mathcal{D}$ which does not contain any singleton sets. 3

*(ii)* Show that if $S$ is a subbase for the topologies $\tau_1$ and $\tau_2$ on $X$, then $\tau_1 = \tau_2$. 3

*(iii)* Show by counterexample that separability is not a hereditary property. 2

**3.** *(i)* Define homeomorphic spaces. Show that the relation in any collection of topological spaces defined by "$X$ is homeomorphic to $Y$" is an equivalence relation. 3

*(ii)* Let $\tau_c$ be a cofinite topology on the real line $\mathbb{R}$. Show that $(\mathbb{R}, \tau_c)$ does not satisfy the first axiom of countability. 3

*(iii)* Show that every subspace of a second countable space is second countable. 2

**4.** *(i)* Let $X$ be a $T_1$-space containing at least two elements. Show that if $\mathcal{B}$ is a base for $X$, then $\mathcal{B} \setminus \{X\}$ is also a base for $X$. 3

*(ii)* Prove/Disprove : Every countably compact set is sequentially compact. 3

*(iii)* Let $(X, \tau)$ be compact and let $(X, \tau^*)$ be Hausdorff. Show that if $\tau^* \subset \tau$, then $\tau^* = \tau$. 2

**5.** *(i)* Prove/Disprove : Continuous image of a countably compact set is countably compact. 2





*(ii)* Prove/Disprove : Every path connected space is connected. 2

*(iii)* Prove/Disprove : Every locally connected set is connected. 2

*(iv)* Prove/Disprove : Every connected set is locally connected. 2

**6.** *(i)* Prove/Disprove : The subset

$A = \{(x, y) \in \mathbb{R}^2 : x^2 - y^2 \geq 4\}$ is connected. 2

*(ii)* Prove that a topological space $(X, \tau)$ is a $T_1$ space if, and only if, every singleton subset of $X$ is closed. 3

*(iii)* Prove that a function $f : X \to Y$ is a continuous if, and only if, inverse image of every closed subset of $Y$ is closed in $X$. 3

**7.** *(i)* Show that being a Cauchy sequence is not a topological property. 3

*(ii)* Show that every point $p$ in a discrete space $X$ has a finite local base. 3

*(iii)* List all topologies on $X = \{a, b, c\}$ which consists of exactly four members. 2

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