Bachelor Of Mathematics Examination, 2025

(1^{st} Year, 1^{st} Semester)

MATHEMATICS (HONOURS)

(OLD)

Paper : CORE/01

(Algebra, Geometry and Calculus)

Time : *Two Hours* *Full Marks* : 40

Use separate Answer Scripts for each Part.

PART—I

1. Answer *any* **three** questions. 4×3=12

(a) (i) State the fundamental theorem of algebra.

(ii) Show that the equation $\tan\left(i\log\frac{x-iy}{x+iy}\right)=2$ represents the rectangular hyperbola $x^2-y^2=xy$. (1+3)

(b) Using row operations, compute the inverse of the matrix

$$A=\begin{bmatrix}3 & 1 & 1\\ 4 & 2 & -1\\ 7 & 3 & 1\end{bmatrix}$$ 4

(c) Solve the biquadratic equation by Ferrari's method

$$x^4+12x=5$$ 4

(d) *(i)* Define the rank of a matrix.

(ii) If α, β, γ be the roots of the equation $x^3 + px^2 + qx + r = 0$, then find the value of

$$\left(\frac{1}{\beta}+\frac{1}{\gamma}-\frac{1}{\alpha}\right)\left(\frac{1}{\gamma}+\frac{1}{\alpha}-\frac{1}{\beta}\right)\left(\frac{1}{\alpha}+\frac{1}{\beta}-\frac{1}{\gamma}\right)$$ (1+3)

(e) If x, y and z are positive real numbers and $x + y + z = 1$, prove that

$$8xyz \leq (1-x)(1-y)(1-z) \leq \frac{8}{27}$$ 4

2. Answer *any* **three** questions. 4×3=12

(a) The part of the parabola $y^2 = 4ax$ bounded by the latus rectum revolves about the tangent at the vertex. Find the volume and the area of the curved surface of the reel thus generated. 4

(b) *(i)* State the Leibnitz theorem on successive differentiation.

(ii) If $y = (\sin^{-1} x)^2$, show that

$$(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - x^2 y_n = 0$$ (1+3)

(c) Find the asymptotes of the curve $r = 2a\tan\theta$. 4

(d) Draw the curve $9ay^2 = (x-2a)(x-5a)^2$ and find the whole length of the loop of the curve. (1+3)

(e) What do you mean by envelope of a family of curves? Find the envelope of the family of straight lines $y = mx + \sqrt{a^2m^2 + b^2}$, m being the parameter. 4

GROUP—B

Answer *any* **four** questions. 4×4=16

1. Find the equation of the cone whose vertex is the origin and which passes through the curve of intersection of the plane $lx + my + nz = p$ and the surface $ax^2 + by^2 + cz^2 = 1$.

2. Show that the centres of spheres, which pass through the straight lines $y = mx, z = c$ and $y = -mx, z = -c$ lie on the surface $mxy + cz(1+m^2) = 0$.

3. Obtain the equation of the cylinder, whose generators intersect the plane curve $ax^2 + by^2 = 1, z = 0$ and are parallel to the straight line $\frac{x}{l} = \frac{y}{m} = \frac{z}{n}$.

4. Prove that the length of the common chord of the circles $(x-a)^2 + (y-b)^2 = c^2$ and $(x-b)^2 + (y-a)^2 = c^2$ is $\sqrt{4c^2 - 2(a-b)^2}$.

5. A conic touches the ellipse $\frac{l}{r} = 1 + e\cos\theta$ at the point $\theta = \alpha$. They have a common focus at the pole and have the same eccentricity. Show that the length of the latus rectum is

$$\frac{2l(1-e^2)}{e^2 + 2e\cos\alpha + 1}.$$

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