B. Sc. (Mathematics Hons) Examination, 2025

(1st Year, 1st Semester)

DISCRETE MATHEMATICS

Paper : MDC/TH/11/101B

Time : *Two Hours* *Full Marks* : 40

The figure in the margin indicate full marks.

Special credit will be given for precise answer.

(Unexplained Symbols/Notations have their usual meanings.)

PART—I

(MARKS : 24)

(Q1, Q2, Q3 are from CO1, Q4, Q5, Q6 are from CO2**)**

Answer *any* **four** questions taking **two** from Q1, Q2, Q3 and **two** from Q4, Q5, Q6. 6×4=24

1. *(a)* Let P be a poset and $a, b \in P$ with $a \leq b$. Prove that $\sup\{a, b\}$ and $\inf\{a, b\}$ exist. Find them. Hence prove that every chain is a lattice. 1

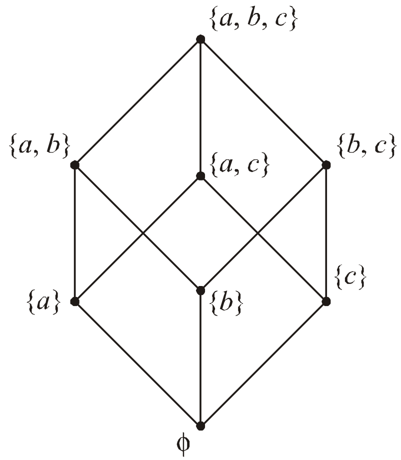
(b) Suppose C is a finite lattice. Determine the elements of C which have complements. 2

(c) Prove that the set $\mathbb{N}_0$ of nonnegative integers is a lattice with respect to divisibility relation. 3

2. Consider the vector space $V = F^2$ where the field $F = \mathbb{Z}_2$ and draw the Hasse diagram of the poset sub(V) of all subspaces of V with set inclusion as the partial order. Is this lattice bounded, complemented, distributive and modular? Answer with reason. 2+4

3. *(a)* Suppose V is a vector space over a field F. Let $P(V)$ denote the power set of V and sub(V) denote the set of all subspaces of V. It is known (no proof is required) that both $P(V)$ and sub(V) are lattices with respect to the same partial order relation 'set inclusion'. Prove that sub(V) is not a sublattice of $P(V)$. 3

(b) Prove that the lattice represented by the following Hasse diagram is a Boolean Algebra. 3



4. Using generating function solve the recurrence relation $a_n = a_{n-1} + 2a_{n-2}, a_0 = 2, a_1 = 0$. 4

Also solve the above recurrence relation as a difference equation. 2

CO 4 : Answer *any* **one** question.

7. Let G be a simple graph.

(a) Define cliques and independent sets of G. 1+1

(b) Let $\omega(G)$ be the largest clique size and $\alpha(G)$ be the size of the largest independent set in G. Prove that $\omega(G) = \alpha(\overline{G})$, where $\overline{G}$ is the complement of G. 2

8. Let G be the complete bipartite graph $K_{m,n}$.

(a) Exhibit a plane drawing of G when $m = 2$. 1

(b) Prove that G is not planar if and only if $m, n \geq 3$. 3

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5. *(a)* Find the coefficient of x^5 in $(1-2x)^{-7}$. 2

(b) Obtain a recurrence relation (do not solve) for the value of D_n and verify that $D_1 = 1, D_2 = 3$, where

$$D_n = \begin{vmatrix} 2 & 1 & 0 & 0 & 0 & \ldots & 0 & 0 & 0 & 0 \\ 1 & 2 & 1 & 0 & 0 & \ldots & 0 & 0 & 0 & 0 \\ 0 & 1 & 2 & 1 & 0 & \ldots & 0 & 0 & 0 & 0 \\ & \ldots & & & & \ldots & \ldots & & & \\ 0 & 0 & 0 & 0 & 0 & \ldots & 1 & 2 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & \ldots & 0 & 1 & 2 & 1 \\ 0 & 0 & 0 & 0 & 0 & \ldots & 0 & 0 & 1 & 2 \end{vmatrix}$$ 4

6. Suppose m is an odd positive integer. Using Pigeonhole Principle prove that there exists a positive integer n such that m divides $2^n - 1$. 6

PART—II

(MARKS : 16)

CO 1 : Answer *any* **one** question.

1. *(a)* Prove that the sum of degrees of all vertices of a graph is twice the number of edges of the graph. 2

(b) Let G be a graph with n vertices and $n-1$ edges. Show that G has either an isolated vertex or a vertex of degree 1. 2

2. *(a)* Let G be a graph and u, v be two distinct vertices in G. If there is a walk from u to v, then prove that there is a path form u to v. 2

(b) Prove that every closed walk of length 3 is a cycle. 2

CO 2 : Answer *any* **one** question.

3. Let G be the Petersen graph.

(a) Show that G is a regular graph. 2

(b) Prove that any two non-adjacent vertices of G have a unique common neighbor in G. 2

4. *(a)* Define an even graph. 1

(b) Show that every non-null even graph contains a cycle. 3

CO 3 : Answer *any* **one** question.

5. *(a)* Prove that a tree with more than one vertex is a bipartite graph. 2

(b) What is a Hamiltonian cycle? Let G be a connected simple graph with 10 vertices and 40 edges. Does G contain a Hamiltonian cycle? 1+1

6. *(a)* Suppose G is an acyclic graph with 16 vertices and 15 edges. Is G a connected graph? 1

(b) Let G be a planar graph with n vertices, e edges, f faces and k components. Then show that $n - e + f - k = 1$. 3