Master Of Science Examination, 2025

(2nd Year, 1st Semester)

MATHEMATICS

Paper : DSE-03/A8

(Solid Mechanics – I)

Time : *Two Hours* *Full Marks* : 40

The figures in the martin indicate full marks.

(*Symbols/Notations have their usual meanings*)

Answer *any* **four** questions : 10×4=40

1. Deduce Clapeyron formula in the form

$$U = \frac{1}{2}\tau_{ij}\, e_{ij}\ (i,\ j = 1, 2, 3)$$

where U is strain energy density function, τ_{ij}, e_{ij} are stress and strain components respectively.

2. A beam of length l is supported in a suitable manner at the point $(0, 0, l)$ on its upper end, the xy-plane of the coordinate system being chosen to coincide with its lower end before deformation takes place. If the force of gravity directed downwards, is the only external force acting throughout the beam, show that, when the point $(0, 0, l)$ of the beam is prevented from being displaced or being rotated about z-axis, then the displacement components are given by

$$u = -\frac{\sigma\rho g}{E} zx, \quad v = -\frac{\sigma\rho g}{E} zy \quad \text{and}$$

$$w = \frac{\rho g}{2E}\left[z^2 + \sigma x^2 + \sigma y^2 - l^2\right]$$

3. A bar of length l and of rectangular cross-section is bent by two equal and opposite couples M. If there is no rigid body motion of translation or rotation about the origin, then show that displacement components along the coordinate axes are given by

$$u = \frac{Mxz}{EI_y}, \quad v = -\frac{\sigma Myz}{EI_y} \quad \text{and} \quad w = -\frac{M(x^2 - \sigma y^2 + \sigma z^2)}{2EI_y}$$

4. *(a)* Write Beltrami-Michell compatibility equations for stress components, hence show that when the components of the body force $\overline{F}$ are constants, then the stress and strain invariants Θ and θ are harmonic functions and the stress components τ_{ij} and strain components e_{ij} are biharmonic functions.

(b) Evaluate strain energy function U for the stress field (for an isotropic solid) $\tau_{11} = \tau_{22} = \tau_{33} = \tau_{12} = 0$ and $\tau_{13} = -\mu\alpha x_2$, $\tau_{23} = \mu\alpha x_1$, α is a constant and μ is the Lame's constant.

5. A disk of uniform thickness is rotating about an axis through the centre with angular velocity ω. Show that the stresses are greatest at the centre of the disk. Also, show that by making a small circular hole at the centre of the disk, the maximum tangential stress approaches a value twice as great as that for a solid disk (assuming that there is no external forces applied at the boundaries).

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