Master Of Science Examination, 2025

(2nd Year, 1st Semester)

MATHEMATICS

Paper : DSE-03/A5

(Fluid Mechanics – I)

Time : *Two Hours* *Full Marks* : 40

Symbols/Notations have their usual meanings.

Answer **Question No. 1** and *any* **two** from the rest.

1. *(a)* *(i)* Define stream lines and show that the equations of stream lines are given by

$\frac{dx}{u} = \frac{dy}{v} = \frac{dz}{w}$, where $\vec{q} = u\hat{i} + v\hat{j} + w\hat{k}$.

(ii) Show that

$\frac{x^2}{a^2}\tan^2 t + \frac{y^2}{b^2}\cot^2 t = 1$

is a possible form of the boundary surface of a liquid. 3+5=8

(OR)

(b) Stating the assumption clearly, derive the Bernoulli's equation in its most general form. Hence obtain the Bernoulli's equation for the steady motion of a homogeneous liquid. 8

2. *(a)* State and prove the Kelvin's minimum energy theorem. 7

(b) A source S and a sink T of equal strengths m are situated within the space bounded by a circle whose centre is O.

If S and T are at equal distances from O on opposite sides of it and on the same diameter AOB, show that the velocity of the liquid at any point P is

$$2m\,\frac{OS^2 + OA^2}{OS}\cdot\frac{PA\cdot PB}{PS\cdot PS'\cdot PT\cdot PT'}$$

where S' and T' are the inverse points of S and T with respect to the circle. 9

3. *(a)* Find the velocity potential φ, the stream function ψ and the complex potential ω for the motion of a circular cylinder moving in an infinite mass of liquid at rest at infinity with velocity v in the direction of x-axis. 8

(b) A circular cylinder is fixed across a stream of velocity u with circulation k round the cylinder. Show that the maximum velocity in the liquid is $2u + \frac{k}{2\pi a}$, where a is the radius of the cylinder. 8

4. *(a)* Define vortex rows. Show that the motion due to a set of line vortices of strength k at points $z = \pm na\ (n = 0, 1, 2, 3, ...)$ is given by

$$\omega = \frac{ik}{i\pi}\log\left\{\sin\frac{\pi z}{a}\right\}.$$ 8

(b) Show that the velocity of propagation of surface wave of length λ in a canal of depth h is given by

$$c^2 = \left(\frac{g\lambda}{2\pi}\right)\tanh\left(\frac{2\pi h}{\lambda}\right).$$ 8

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