Master of Science Examination, 2025

(1st Year, 1st Semester)

Mathematics

Paper : CORE – 04

(GENERAL MECHANICS)

Time : 2 *Hours* | *Full Marks* : 40

Symbols/Notations have their usual meanings.

Use separate Answer Script for each Part.

PART—I (20 Marks)

Answer *any **two*** questions.

1. *(a)* A mass m, initially at rest, is released from a height h above the earth's surface, where h is small relative to the earth's radius. Given that the earth rotates about its axis with a constant angular speed, ω, demonstrate that after a duration t, the falling object experiences an eastward displacement from the vertical. Furthermore, derive an expression for the magnitude of this deflection.

(b) Define Euler's angles θ, ϕ and ψ.

(c) A particle of mass, *m,* is constrained to move on the surface of a cylinder. The particle is subjected to a force directed towards the origin and proportional to the distance of the particle from the origin. Construct the Hamiltonian and Hamilton's equations of motion.

5+2+3

2. *(a)* Define Poisson bracket of two dynamical variables. Show that for any three dynamical variables u, v, w the following identity is satisfied

$$[u,[v,w]]+[v,[w,u]]+[w,[u,v]]=0.$$

(b) If a co-ordinate is cyclic then will it be present in the Hamiltonian or not? Justify your answer.

(c) Find the horizontal component of the Coriolis force acting on a body of mass 1·5 kg moving nothward with a horizontal velocity of 100 m/sec., at 30º N latitude on earth. (1+4)+3+2

3. *(a)* Define action of a mechanical system. State and prove the principle of stationary action.

(b) A bead slides on a wire in the shape of a cycloid described by the equations

$$x = a(\theta - \sin\theta)$$
$$y = a(1+\cos\theta)$$

where $0 \leq \theta \leq 2\pi$. Find —

(i) the Lagrangian function;

(ii) the equation motion.

Neglect friction between the bead and wire.

(2+4)+(2+2)

PART—II (20 Marks)

Answer ***any four*** questions. 5×4=20

1. Find the relation between Poisson bracket and Lagrange bracket.

2. If the transformation $Q_i(q_i, p_i, t)$ and $P_i\left(q_{i,} p_i, t\right)$ is a canonical transformation, then show that the Jacobian of the transformation is ± 1.

3. Show that the Hamiltonian is invariant under infinitesimal canonical transformation.

4. Show that the transformation

$Q = \ln\left(\frac{\sin p}{q}\right)$ and $P = q\cot p$ is canonical. Find the generating function.

5. Find the values of α and β so that the equations

$$Q = q^{\alpha}\cos\beta p,\ P = q^{\alpha}\sin\beta p$$

represent a canonical transformation.

6. Use Hamilton-Jacobi method and obtain the equation of motion for one dimensional Harmonic oscillator.

★ ★ ★