Ref. No.: Ex/ME(M2)/PE/H/T/414A/2025

**B.E. MECHANICAL ENGINEERING FOURTH YEAR FIRST SEMESTER EXAM.– 2025**

**Subject: FINITE ELEMENT METHOD FOR NON-STRUCTURAL APPLICATIONS (HONS)**

**Time: 3 Hours** **Full Marks: 100**

***Use separate answer-scripts for different parts.***

## PART 1 (Marks 50)

***Answer all the questions.***

**Q1.** Consider the differential expression $L(\phi)=\frac{d}{dx}\left(x\frac{d\phi}{dx}\right)$ within the domain $0 \le x \le l$. Determine if the operator is self-adjoint for the boundary conditions $\phi(0)=\phi(l)=0$. **[5]**

**Q2**. A steel rod of diameter $D$= **80 mm** and, length $l$= **150 mm**, thermal conductivity $K$= **50 W/m·K** is subject to steady state heat conduction with a heat influx of **200 W** specified at one end ($x$ = **0**). The other end of the rod ($x$ = **150 mm**) is kept at an ambient temperature of **20°C**. The convective heat transfer coefficient between the rod and the surroundings is $h$= **64 W/m²·K**.

(a) Obtain the governing differential equation of one-dimensional steady-state heat transfer through the rod along with the necessary boundary conditions. **[5]**

(b) Using Mikhilin's theorem, obtain the functional of the problem whose minimization will lead to the above governing differential equation. **[10]**

(c) Using three finite elements of equal length and with linear interpolation function obtain the finite element equations for the one-dimensional steady-state heat transfer through the rod highlighting following features:

(i) Element level equations in matrix form for each of the three elements.

(ii) Assembly of the element level equations.

(iii) Modified finite element equations for the entire rod in matrix form after invoking appropriate essential boundary conditions.

**[15]**

**Q3.** Discuss, in short, the method to obtain finite element equations for solving two-dimensional Poisson's equation with respect to electrostatic problems using variational technique. Use one triangular finite element. **[15]**

[ Turn over

**PART II (Marks 50)**

***Answer any two questions.***

**Q4.** Consider first order equations of the following form:-

$$[K_t]\{\dot{\phi}\} + [K]\{\phi\} = \{R(t)\}$$

For a finite difference solution, **consider $\boldsymbol{t = t_n + \theta\Delta t}$**

a. Depending on the value of $\theta$ **classify** the finite difference schemes
b. What is an **explicit scheme**? State its advantage and disadvantage
c. When is such a finite difference scheme **conditionally and unconditionally stable**? Discuss with the help of a **suitable graph**.

**[25]**

**Q5.**

**a.** For **velocity potential formulation** in 2-dimensional flow

$$u = \frac{\partial\phi}{\partial x}, v = \frac{\partial\phi}{\partial y}$$

Find out the governing partial differential equation from continuity equation.

Now consider computation of flow around a cylinder. Take boundary sufficiently away from the cylinder. Specify the boundary conditions. Refer Figure Q5.

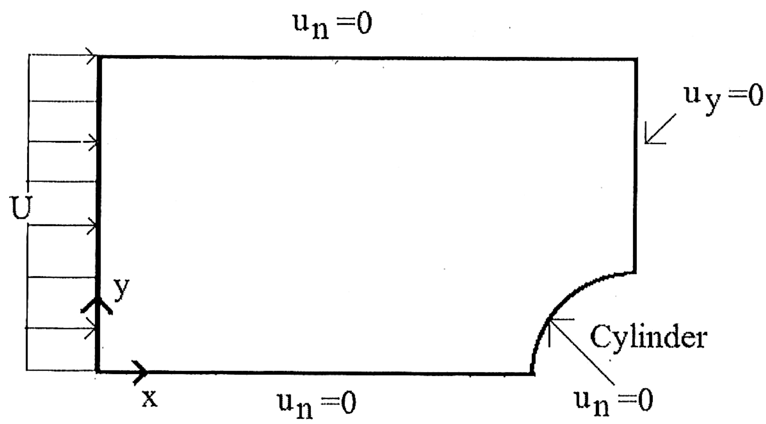


**Figure Q5**

**b.** Consider the one-dimensional equation of the following form:-

$$\frac{T}{m}\frac{d^2\phi}{dx^2} + \lambda\phi = 0, \qquad 0 < x < l$$

$$\phi(0) = \phi(l) = 0$$

$$T = 100N, l = 1.5m, m = 0.1kg/m$$

Write down the **variational principle**. Take 3 equal elements. Formulate the matrix eigenvalues problem from **variational principle.**

**[25]**

**Q6.** Consider a problem of the following form:-

$$\frac{\partial\phi}{\partial t} = \frac{\partial^2\phi}{\partial t^2} + e^{-t}, \qquad 0 < x < l$$

$$\phi(0) = \phi(l) = 0$$

$$\phi = 10\frac{x(l-x)}{2} \quad at \ t = 0$$

**Take $l = 1.5$ and a 3 element discretization**

Use mode superposition method to find out the **2 uncoupled** equations

What are the **initial conditions** in the **modal space**?

How do you get the **solution in physical space**?

**[25]**