Ref. No. : Ex/ME(M2)/PE/H/T/416C/2025

B.E. MECHANICAL ENGINEERING FOURTH YEAR FIRST SEMESTER – 2025

CONTINUM MECHANICS (HONS.)

Full marks 100 **Duration 3 hr.**

Answer suitable values for any missing data. Symbols have their usual meanings.

1. CO1: Answer any six. [6 × 5=30]

   a. Show that in a rigid body motion, the velocity gradient $\boldsymbol{L} \in skw$.

   b. Show that $tr(\boldsymbol{L}) = \frac{1}{J}\frac{dJ}{dt}$.

   c. Derive the relationship between the area elements in deformed and undeformed coordinates.

   d. Show that Green's strain and Almansi strain are true strain measures and small strain is not.

   e. Show that $\frac{d\boldsymbol{A}}{dt} = \boldsymbol{D} - \boldsymbol{L}^T\boldsymbol{A} - \boldsymbol{A}\boldsymbol{L}.\ \boldsymbol{A} = Almansi\ strain$

   f. Perform the polar decomposition ($\boldsymbol{F} = \boldsymbol{R}\boldsymbol{U}$) for simple shear described below. [Hint: General expression for a planar rotation tensor $\boldsymbol{Q} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$]

      i. $x_1 = X_1 + \gamma X_2$

      ii. $x_2 = X_2$

   g. Show that $\int_{V(t)}[2\boldsymbol{W}\boldsymbol{v} + (\nabla.\boldsymbol{v})\boldsymbol{v}]\, dV = \int_{S(t)}\left[(\boldsymbol{v}.\boldsymbol{n})\boldsymbol{v} - \frac{\boldsymbol{v}.\boldsymbol{v}}{2}\boldsymbol{n}\right] dS$

2. CO2: Answer any eight. [8 × 5 =40]

   a. State and prove the transport theorem I (TTI) for a scalar function.

   b. State and prove the transport theorem II (TTII) for a vector function.

   c. Show that the incompressibility condition can be written as $\nabla.\boldsymbol{v} = 0$ in Eulerian form.

   d. Show that traction at any given location depend on the normal to the plane of action as $\boldsymbol{t} = \boldsymbol{\sigma}\boldsymbol{n}$.

   e. Show that the above expression leads to the equation of motion given by $\rho\frac{d\boldsymbol{v}}{dt} = \nabla.\boldsymbol{\sigma} + \rho\boldsymbol{b}$.

   f. Show that the Eigen values of the Cauchy stress ($\boldsymbol{\sigma}$), and for that matter any symmetric tensor, are all real numbers.

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g. Show that the Eigen vectors of the Cauchy stress ($\boldsymbol{\sigma}$), and for that matter any symmetric tensor, can always be chosen to form an orthonormal base.

h. Show that the Eigen vectors (principal directions) represent the normals to planes where the normal component of the traction is extremum.

i. If the stress tensor at a given location in the continuum has the form $\begin{bmatrix} \sigma_{xx} & 0 \\ 0 & \sigma_{yy} \end{bmatrix}$, then determine the normal to the plane in which the shear component (i.e., perpendicular to the normal) component of the traction is maximum.

3. CO3: Answer all the question. [5 × 3=15]

   a. What is the difference between the following? Explain with expressions.

      i. $\boldsymbol{DE}(\boldsymbol{F})[\Delta \boldsymbol{F}]$

      ii. $\boldsymbol{DE}(\boldsymbol{I})[\boldsymbol{\nabla u}]$

   b. Derive the linearized expressions of the following quantities $\boldsymbol{A}, J, \boldsymbol{U}, \boldsymbol{R}, \boldsymbol{T}$.

   c. For a homogenous circular bar made of isotropic material under torsion

      i. $u_r = 0$

      ii. $u_\theta = \alpha r z$

      iii. $u_z = 0$

   Determine α for an applied torque T, satisfying all the boundary conditions. Also determine the stress components and the traction in the cross-section and the cylindrical surface. [Hint: $\varepsilon_{\theta\theta} = \frac{1}{r}\frac{\partial u_\theta}{\partial \theta}$, $2\varepsilon_{r\theta} = \frac{\partial u_\theta}{\partial r} - \frac{u_\theta}{r} + \frac{1}{r}\frac{\partial u_r}{\partial \theta}$, $2\varepsilon_{z\theta} = \frac{\partial u_\theta}{\partial z} + \frac{1}{r}\frac{\partial z}{\partial \theta}$]

4. CO4: Answer all the questions. [5 × 3=15]

   a. Show that $\boldsymbol{DT}(\boldsymbol{F})[\boldsymbol{WF}] = \boldsymbol{WT}(\boldsymbol{F})$. [Hint: Second P-K stress is a direct function of right Cauchy-Green tensor.]

   b. Show that if $\boldsymbol{S} = \frac{\partial W}{\partial \boldsymbol{E}}$, then $\boldsymbol{T} = \frac{\partial W}{\partial \boldsymbol{F}}$. [Hint: $DW(\boldsymbol{F})[\Delta \boldsymbol{F}] := DW(\boldsymbol{E})[\boldsymbol{DE}(\boldsymbol{F})[\Delta \boldsymbol{F}]]$].

   c. From the above result, derive the expression for Cauchy stress for compressible Neo-Hookean model given below.

$$W = C_1(tr(\boldsymbol{C}) - 3 - 2\ln J) + D_1(J-1)^2$$

   Also determine the expression for the same for infinitesimal strain. [Hint: for infinitesimal strain $2\boldsymbol{\varepsilon} \approx \boldsymbol{B} - \boldsymbol{I}$ and $J \rightarrow 1$]