Ref. No. : Ex/ME(M2)/PE/B/T/316F/2025

**B. E. MECH. ENGG. THIRD YEAR FIRST SEMESTER EXAM – 2025**

**Subject: ELEMENTS OF COMPUTATIONAL FLUID DYNAMICS Time: 3 hrs Full Marks: 100**

**Instructions: Answer any five (5) questions. Write all pertinent assumptions. Assume any missing data. Please be to the point. Verbose sentences and expressions will lead to the deduction in marks.**

1. (a) Derive the second-order accurate central difference finite difference approximations at the point $(i, j)$ of the following expression: $x^2 \frac{\partial^2 \theta}{\partial y^2} + y \frac{\partial^2 \theta}{\partial x^2} - xy\theta + 2$, where $\theta(x, y)$ is the dependent variable and, $(x, y)$ are the independent variables, respectively. **(10)**

(b) Using the Crank–Nicolson method, discretize the convection diffusion equation of a scalar field $\phi(x, y, t)$ as, $\frac{\partial \phi}{\partial t} + u\frac{\partial \phi}{\partial x} + v\frac{\partial \phi}{\partial y} = D\left(\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2}\right)$. Here, $u(x, y)$, $v(x, y)$ are velocity fields along $x$, $y$ directions, $D$ is the diffusion constant, and $t$ is the time. **(10)**

2. A copper plate of 20 cm thickness is initially at $550^0$C. It is suddenly plunged into cold water. As a consequence, the surface temperature drops to $70^0$C and remains at this value for the rest of the cooling process. Estimate the time elapsed for the middle plane temperature to cool down to $250^0$C by dividing the thickness of the plate into five equal divisions and applying FTCS finite difference method. Assume thermal conductivity, density, and specific heat of copper as: $k_{Cu}$=401 W/m.K, $\rho_{Cu}$=8850 kg/m$^3$, $Cp_{Cu}$=390 J/kg.K. **(20)**

3. Consider an one dimensional transient heat conduction equation with the initial (IC) and boundary (BC) conditions as,

$$\frac{\partial \theta}{\partial t} = \frac{\partial^2 \theta}{\partial x^2}$$

$$IC\text{: at } t = 0, \theta = 1 \text{ for all } x$$

$$\text{for } t > 0$$

$$BC1\text{: at } x = 0, \theta = 0$$

$$BC2\text{: at } x = 1, \frac{\partial \theta}{\partial x} = 0$$

Here, $\theta$ is the temperature, $t$ is the time, and $x$ is the spatial coordinate. Using von Neumann stability analysis, obtain the stability condition for explicit method. **(20)**

4. (a) What is the upwinding scheme? Starting with a steady, one-dimensional convection-diffusion problem for slug flow, demonstrate how solving the problem using the conventional finite difference method can lead to unphysical numerical solutions. Then, show how employing an upwinding scheme can resolve this issue. **(15)**

(b) Write a detailed note on the false transient method. **(5)**

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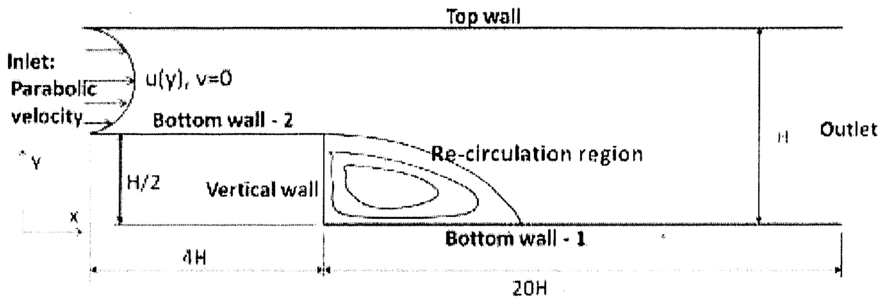
5. (a) What is pressure-velocity decoupling? How can pressure-velocity coupling be achieved? **(6)**

(b) Write the detailed numerical method for solving the Navier-Stokes equations using the SIMPLE algorithm. Also, provide a step-by-step numerical solution procedure. **(14)**

6. Flow over a backward-facing step is a very popular benchmark problem against which CFD codes are tested for comparison and accuracy. As shown in **Figure 1**, the fluid (assume air) is entering into the backward-facing step with a parabolic inlet velocity and at the outlet it is fully developed.

**Figure 1:**




Formulate the problem using the *fractional step* method so that the velocity and pressure field can be obtained. Assume the expression of the inlet parabolic velocity profile as: $u(y) = \frac{2y}{H}\left(1 - \frac{2y}{H}\right)$. (i) Write governing equations along with the boundary conditions. (ii) Formulate using the fractional step method. (iii) Discretize the formulated equations using the finite difference method. iv) Write the discretized form of the boundary conditions. (v) Write the step-by-step solution algorithm. **(20)**

7. Write short notes on: a) Root Mean Squared Error. b) Numerical methods for generating uniform and non-uniform meshes. c) Free-slip and fully developed boundary conditions. d) Steady state and Dynamic steady state conditions. **(5+5+5+5)**