B.E. MECHANICAL ENGINEERING THIRD YEAR SECOND SEMESTER EXAM – 2025

Subject: MECHANICAL VIBRATION ANALYSIS **Time: Three Hours** **Full Marks: 100**

The value of the acceleration due to gravity (g) can be taken as 10 m/s^2, if it is not specified.

Any missing information may be suitably assumed with appropriate justification.

Q1. Answer any one question from this group.

Q1(a). A bullet of mass m = **10 kg** travelling with the velocity v = **50 m/s** strikes and becomes embedded in a massless board supported by a spring of stiffness k = **6.4 × 10^4 N/m** in parallel with a dashpot with the coefficient of viscous damping c = **400 N·s/m** as shown in **Fig. Q1a**. **Determine the time required** by the board to reach the maximum displacement and the **value of the maximum displacement**. **[10]**

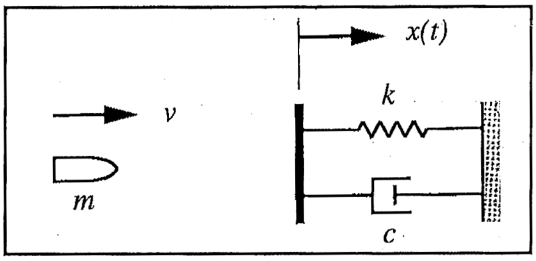


Fig. Q1a

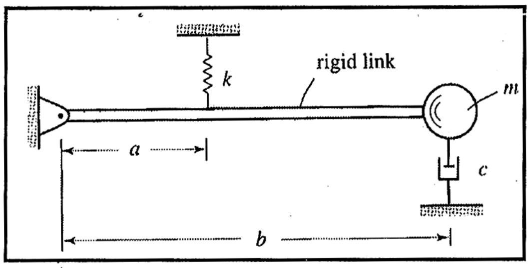
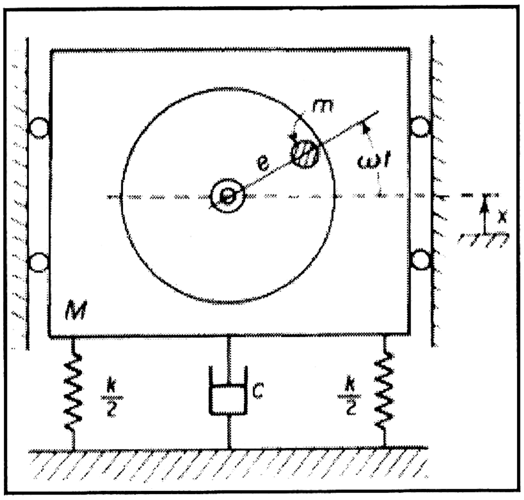


Fig. Q1b

Q1(b). Derive the differential equation of motion for the **small amplitude angular oscillation** of the system shown in **Fig. Q1b** and **determine the natural frequency of damped oscillation,** ω_d and **the critical damping coefficient,** c_C. Left end of the **light** horizontal rigid link is hinged with the wall. Consider m = **1750 kg,** c = **3500 N·s/m,** k = **7 × 10^5 N/m,** a = **1.25 m** and b = **2.5 m. Also find the expression of displacement of the mass m as a function of time** if it is initially released from rest after being displaced by **1 mm** in the downward direction. **[10]**

Q2. Answer any two questions from this group.

Q2(a). Derive the equation motion for the vibration of a spring-mass-dashpot system constrained to move in the vertical direction and excited by a rotating unbalanced mass m, as shown in **Fig. Q2a**. Consider M is the total mass of rotating and non-rotating system. **Obtain an expression of the steady-state vibration response amplitude X,** of the non-rotating mass in non-dimensional form **(MX/me)** **and the corresponding phase angle between the excitation and the steady-state response. Show the approximate plot** between (**MX/me**) and the frequency ratio (ω/ω_n) for various values of damping ratio, ζ, where ω_n is the undamped natural frequency of the system. **Find the frequency ratio** as a function of ζ, **at which (**MX/me**) will be maximum** for that particular ζ. **[15]**



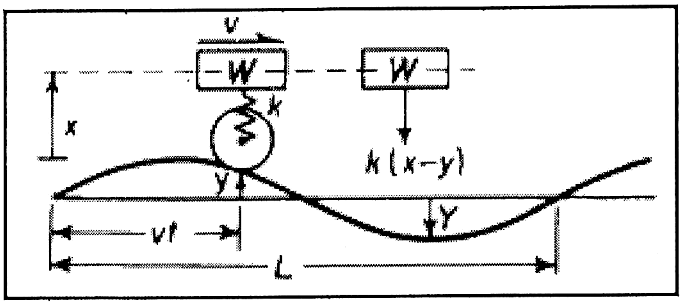


Fig. Q2b

Fig. Q2a

Q2(b). Figure Q2b represents a simplified diagram of a spring-supported vehicle of weight W traveling over a rough road with a profile approximated by a sine wave of amplitude Y and wavelength of L. **Derive the equation for the amplitude** for total displacement of the vehicle as a function of the speed of the vehicle, and **determine the most unfavorable speed**. If the supporting spring under the vehicle is compressed by **120 mm** under its own weight **find the value of most unfavorable speed** when the trailer is traveling over the road with Y = **80 mm** and L = **18 m**. What will be the amplitude of total vibration response of the vehicle when it is moving at a speed of **60 km/h**? **[15]**

Q2(c). Consider a single-degree-of-freedom undamped vibratory system having a mass m and stiffness k. Using convolution integral and principle of superposition **obtain the expression of the transient vibration response** of the system when it is acted on by a force as shown in **Fig. Q2c**. Express the result in dimensionless form as $\left(\frac{kx}{F_0}\right)$ for $t < t_0$ and $t > t_0$. **[15]**

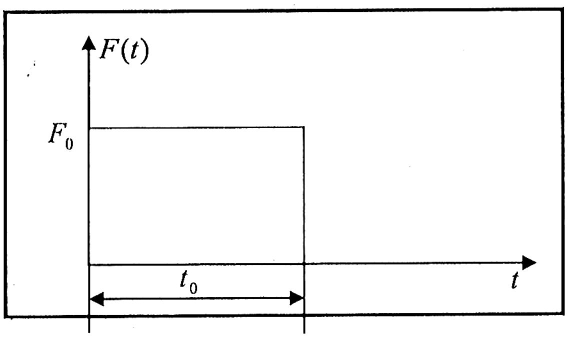


Fig. Q2c

Q3. Answer any three questions from this group.

Q3(a). A vehicle as shown in **Fig. Q3a** has a mass of **1200 kg**. The distance between its front wheel axle and the centre of gravity (C.G.) is l_1 = **1.2 m** and that between the rear wheel axle and the C.G. is l_2 = **1.5 m**. The radius of gyration of the vehicle about an axis perpendicular to the plane of the vehicle and passing through its C.G. is **0.9 m**. The equivalent stiffness of the front and rear suspension and wheels are k_f = **20 kN/m** and k_r = **24 kN/m** respectively.

This system can be modeled as a rigid bar of length $l = l_1 + l_2$, supported on two springs k_f and k_r.

(i) **Obtain the differential equations of free vibration** of the vehicle for its pitching and bouncing motion (represented by x and θ respectively) in the vertical plane.

(ii) **Obtain the corresponding natural frequencies** of the system.

(iii) **Obtain the ratios of amplitudes** of linear motion (x) and angular motion (θ) of the vehicle during the free vibration in each of the normal modes corresponding to the respective natural frequencies. **[15]**

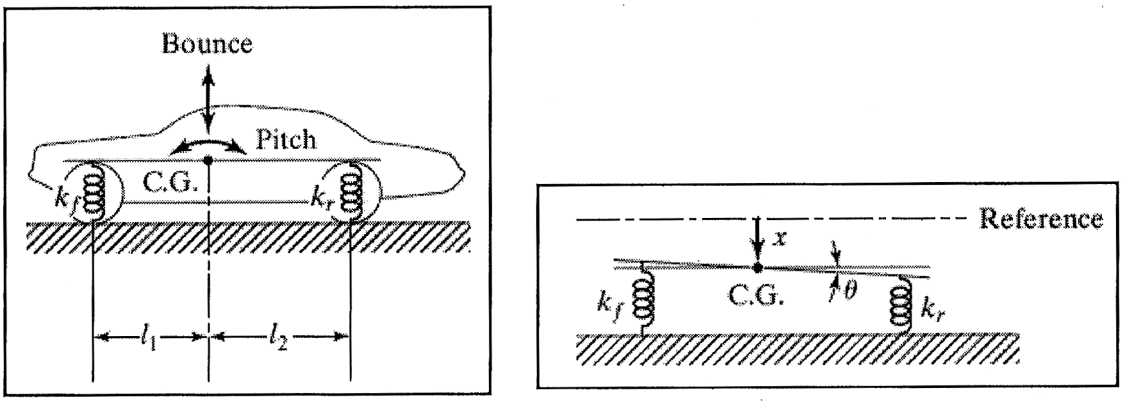


Fig. Q3a

Q3(b). Consider the following equations of motion for free vibration of a two-degree-of-freedom vibratory system:

$$[M]\{\ddot{X}(t)\} + [K]\{X(t)\} = \{0\}$$

where $[\mathbf{M}] = \begin{bmatrix} 10 & 0 \\ 0 & 6 \end{bmatrix}$ and $[\mathbf{K}] = \begin{bmatrix} 120 & -30 \\ -30 & 120 \end{bmatrix}$, expressed in appropriate units.

(i) **Determine the values of the natural frequencies** of the system.

(ii) **Determine mode shape vectors of the system** when they are vibrating in normal modes corresponding to two natural frequencies respectively. **Normalize the mode shape vector with respect to mass matrix.**

(iii) **Determine the expressions of free vibration response** of the system with initial conditions: $x_1(0) = x_2(0) = \mathbf{1}$ unit and $\dot{x}_1(0) = \dot{x}_2(0) = \mathbf{0}$. **[15]**

Q3(c). Consider a multi-degrees of freedom undamped linear vibratory system whose equations of motion during free vibration is given by $[M]\{\ddot{X}(t)\}+[K]\{X(t)\}=\{0\}$, where $\{X(t)\}$ is the $n\times 1$ vector for the system degrees of freedom, $[M]$ is the $n\times n$ mass matrix and $[K]$ is the $n\times n$ stiffness matrix of the system.

Discuss the mathematical procedure to obtain the eigenvalues and eigenvectors of the above equations. Explain how the eigenvalues and eigenvectors of the above equations are related, respectively, to the natural frequencies and mode shapes of the vibratory system.

Also **show that the mode shapes** corresponding to any two distinct natural frequencies (ω_i and ω_j) of the system **are mutually orthogonal to each other.** [15]

Q3(d). With neat sketches **derive the governing differential equation** for the free torsional vibration of a uniform prismatic elastic shaft with circular cross-section. When one end of such a shaft is fixed and the other end is free, **show that the n^{th} normal mode and n^{th} natural frequency are given by respectively,**

$$U_n(x)=\sin\left\{\left(\frac{2n+1}{2}\right)\frac{\pi x}{l}\right\} \qquad \text{and} \qquad \omega_n=\left(\frac{2n+1}{2}\right)\frac{\pi}{l}\sqrt{\frac{G}{\rho}}$$

where l is the total length of the bar and x is the coordinate measured along its length, G is the modulus of rigidity and ρ is the mass per unit volume. [15]

Q4. Answer any one question from this group.

Q4(a). Consider that the system shown in **Fig. Q4a** is subject to proportional damping: $c_i=0.1k_i$, for i = 1, 2. **Derive the equations of motion for forced vibration** of the system in matrix form. **Obtain the frequency response function** (FRF) matrix for the system. **Obtain the steady-state response of the system** due to the harmonic excitation F_2, which is applied on mass m_2, with the following data: m_1 **= 9 kg**, m_2 **= 1 kg**, k_1 **= 24 N/m** and k_2 **= 3 N/m**.

[15]

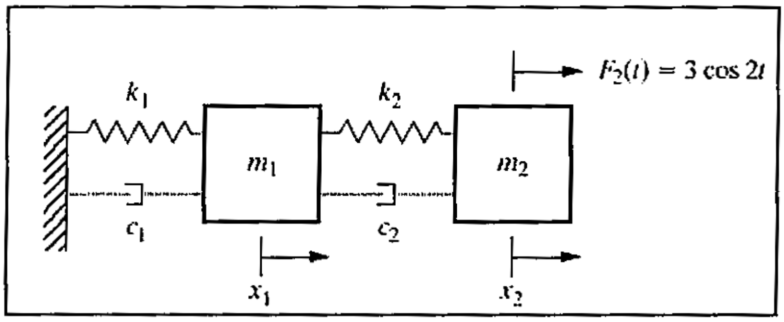


Fig. Q4a

Q4(b). Explain, with neat sketches and necessary mathematical derivations, **the operating principle of an undamped vibration absorber or tuned mass damper.** [15]