Ref. No.: Ex/ME(M2)/PE/B/T/316A/2025

B.E. MECHANICAL ENGINEERING
THIRD YEAR
FIRST SEMESTER EXAM 2025

INTRODUCTION TO FINITE ELEMENT METHOD

Time : Three hours | Full Marks : 100

Answer any five questions

All questions carry equal marks

Assume appropriate value for missing data

Question 1

a) Derive the stiffness matrix of a 2-node bar element starting from linear shape functions and using minimization of potential energy principle.

b) Write down the transformation matrix of a two-dimensional bar element and show the process of determining the stiffness matrix in global coordinates.

c) What transformation matrix should you use for a three-dimensional bar element?

[Turn over

Question 2

a. Start from

$$N_1 = 1 - \frac{3x^2}{l^2} + \frac{2x^3}{l^3}$$

$$N_2 = x - \frac{2x^2}{l} + \frac{x^3}{l^2}$$

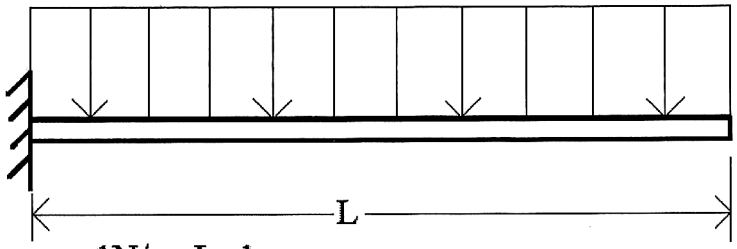
$$N_3 = \frac{3x^2}{l^2} - \frac{2x^3}{l^3}$$

$$N_4 = -\frac{x^2}{l} + \frac{x^3}{l^2}$$

Using minimization of potential energy to show that

$k_{12} = \frac{6EI}{l^2}$

b. Shown below is a steel cantilever beam with uniformly distributed load (Figure Q2). Find out the deflection at the free end. Take $E = 200GPa$
Use single element
(The expression for stiffness matrix is given below :-)



w=1N/m, L=1m
b=24mm, h=20mm

Figure Q2

$$[K^e] = \begin{bmatrix} \frac{12EI}{l^3} & \frac{6EI}{l^2} & -\frac{12EI}{l^3} & \frac{6EI}{l^2} \\ & \frac{4EI}{l} & -\frac{6EI}{l^2} & \frac{2EI}{l} \\ sym & & \frac{12EI}{l^3} & -\frac{6EI}{l^2} \\ & & & \frac{4EI}{l} \end{bmatrix}$$

Question 3

For a constant strain triangle (CST) start from the following relation (Figure Q3):-

$$N_i = \frac{1}{2\Delta}(a_i + b_i x + c_i y)$$

Where, $a_1 = x_2 y_3 - x_3 y_2$ $\quad b_1 = y_2 - y_3$ $\quad c_1 = x_3 - x_2$

a) Find out the strain displacement matrix $[B]$
b) Consider **plane stress** and material as **steel**. Find out the stress-strain matrix $[D]$
c) Find out the surface force vector ($1N/m$ on the slant edge)
d) Find out the body force vector due to self-weight

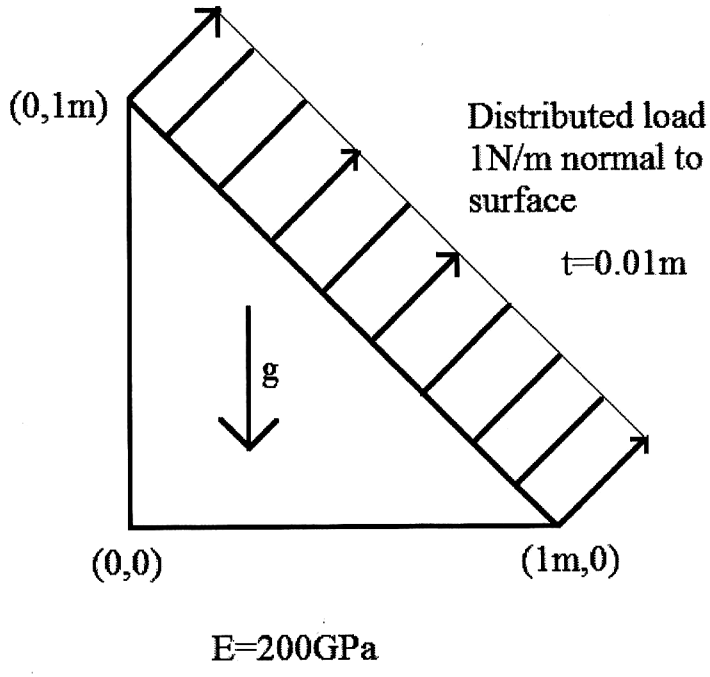


Figure Q3

Question 4

Consider an eight node isoparametric quadrilateral element

Draw and find out the shape functions

Using a flowchart describe how would you find the stiffness matrix for such an element

Question 5

In physical space a four node quadrilateral is mapped as shown (Figure Q5)

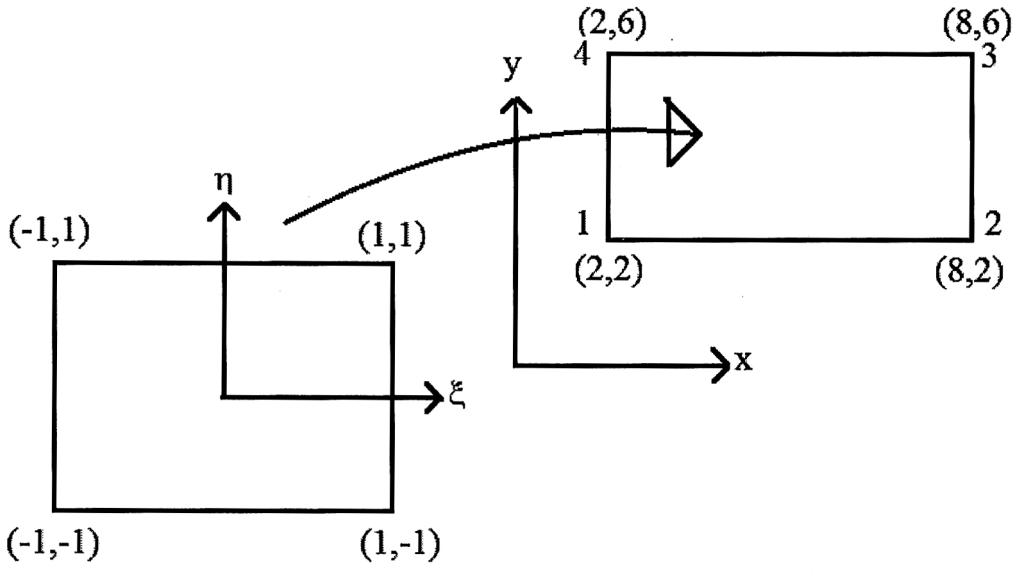


Figure Q5

a) Find out the Jacobian matrix and its determinant

b) Find out $\partial N_1/\partial x$ and $\partial N_1/\partial y$

c) Indicate what order of Gauss quadrature would integrate the stiffness matrix exactly

Question 6

a) Write down the shape functions of a nine-node isoparametric quadrilateral using Lagrange interpolation function

b) Evaluate the integral $\int_{-1}^{1}\int_{-1}^{1} r^4 s^4 \, drds$. Use 2 point and 3 point Gauss quadrature rule. Use the data given in Table 1

c) Are the results different? Why?

Table 1.Data for 2 point and 3 point Gauss quadrature rule

Number of points	Locations	Weights
2	$\pm 1/\sqrt{3}$	1
3	$\pm\sqrt{\frac{3}{5}}$ 0	5/9 8/9