Ref. No. : Ex/IT/BS/B/T/222/2025

B.E. INFORMATION TECHNOLOGY SECOND YEAR SECOND SEMESTER - 2025

MATHEMATICS FOR IT-II

Time: 3 Hrs. **(Please use separate answer scripts for each part.)** Full Marks: 100

Part I

Marks : 60

CO1: Define and illustrate the concept of probability and random variables.

1. Answer any *three* questions. [3 × 5 = 15]
   a. In a bolt factory machine M_1, M_2 and M_3 manufacture respectively 25%, 35% and 40% of the total of their output with 5%, 4%, and 2% respectively are defective bolts. One bolt is chosen at random from the product and it is found defective. What is probability that it was manufactured in machine M_1, M_2, and M_3?
   b. A random variable X has the pdf: $f(x) = cx^2$ for $0 \leq x \leq 1$ and 0 otherwise. Find the value of c, and $P(0 \leq X \leq 1/2)$, $P(1/4 < X < 3/4)$. Also find the cdf of the given pdf.
   c. i) For what value of "a" will the function $f(x) = ax$; $x = 1, 2, 3, \ldots, n$ be the pmf of a discrete random variable X. Find the variance of X.
   ii) Obtain the expectation of the number of tails preceding the first head in an indefinite series of the tosses of the same coin.
   d. An unbiased coin is thrown three times. If the random variable X denotes the number of heads obtained, find the cumulative distribution function of X. Also, find the mode of X and the probability that at least two heads are obtained.

CO2: Discuss the concept of the continuous and discrete distribution along with its mean and variance and illustrate with different distributions.

2. Answer any *four* questions (where, F or φ carries their usual meaning). [4 × 5 = 20]
   a. Derive the expressions of mean and variance of Binomial distribution with given parameters.
   b. i) A die is thrown 1200 times. Find the probability that the number of sixes lies between 190 and 200. (Given that the area of standard normal curve between 0 and 0.78 is 0.2823 and between 0 and 0.81 is 0.2910.)
   ii) A discrete random variable X follows Poisson distribution such that $P(X = 0) = P(X = 1)$. Find the variance of the distribution and $P(X > 0)$.
   c. i) In a sample of 120 workers in a factory, the mean and s.d. of wages were INR 11.35 and INR 3.03 respectively. Find the number of workers getting wages between INR 9 – INR 17 in the whole factory, assuming that the wages are normally distributed. (Given: area under standard normal curve from 0 - 0.78 is 0.2823 and 0 - 1.86 is 0.4686).
   ii) Show that the probability that the number of heads in 400 tosses of a fair coin lies between 180 to 220 is approximately $2\varphi(2) - 1$.
   d. i) 2% of the items made by a machine are defective. Find the probability that 3 or more items are defective in a sample of 100 items (Given $e^{-3} = 0.0498$).
   ii) If X and Y are independent Poisson variates, find the conditional distribution of X, given X + Y.
   e. i) In an examination, marks obtained by the student in C, JAVA and Python are independently and normally distributed with mean 50, 52, 48 and s.ds. 15, 12, 16 respectively. Find the probability that total marks of a student are between 90 and 180 (Given $\varphi(1.2) = 0.8849$ and $\varphi(2.4) = 0.9918$).
   ii) A variable X follows Poisson distribution with mean 16. Find $P(X \geq 20)$. Given $F(0.875) = 0.8092$.

CO3: Explain the concept of random sampling and measure central tendency, correlation, and regression.

3. Answer any *three* from {a, b, c, d} and *one* from {e, f}. [3 × 4 + 3 = 15]
   a. Show that the mean and standard error of sample mean ($\bar{x}$) from simple samples of size n are $E(\bar{x}) = \mu$ and S.E. $(\bar{x}) = \sigma/\sqrt{n}$.
   b. Two lines of regression are given by $x + 2y = 5$ and $2x + 3y = 8$ and $\sigma_x^2 = 12$. Calculate mean of x, σ_y, and r.
   c. If m_r and α_r denote the r^{th} central moments of x and of the standard variable $z = (x - \bar{x})/\sigma$ respectively, where σ is the s.d. of x, then show that $\alpha_r = m_r/\sigma^r$ and justify "if all standardized moments α_r for r >2 are 0, then x must follow a normal distribution".
   d. The regression equation calculated from a given set of observations are $x = -0.2y + 4.2$ and $y = -0.8x + 8.4$. Calculate r and estimated value of y when x = 4.

[Turn over

e. If m_2' and m_2 are respectively the second moment about an arbitrary origin "a" and that about mean of x, then show that $m_2' = m_2 + d^2$, where $d = \bar{x} - a$.

f. The mean of a certain normal distribution is equal to the standard error of a sample of 25 from the distribution. Find the probability that the mean of a sample of 49 from the distribution will be negative. (Given that area under standard normal curve to the left of the ordinate at 1.4 is 0.9192.)

CO4: Explain and illustrate Estimation of Parameters and curve fitting.

4. Answer *all* questions. [5 + 5 = 10]

a. The profits y (Rs. in lakh) of a certain company in the x^{th} year of its life are given y: (2.18, 2.44, 2.78, 3.25, 3.83) for x: (1, 2, 3, 4, 5). Obtain the profit at 6^{th} year by fitting a second degree parabola to the given data.

b. Fit an exponential model of the form $v = ab^u$ to the following data. u: (2, 3, 4, 5, 6, 7) with v: (640, 512, 410, 328, 262, 210) and find the value v at u = 0.5 and u = 8.5.

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Part II

Use Separate Answer scripts for each Part

Marks : 40

CO5: **Understand** the basics of game theory and Inventory Control and some simple applications

5. Attempt (a or b), c and d [6 + 4 + 10 = 20]

   a. Which competitive situation is called a game? What are assumptions made in the theory of games? What is the maximin criterion of game?

   OR

   b. Find solution of the following game

$$\begin{array}{cc} & \text{Player II} \\ \text{Player I} & \begin{pmatrix} 6 & 8 & 7 & 5 \\ 8 & 9 & 5 & 7 \\ 7 & 4 & 6 & 3 \\ 6 & 6 & 8 & 7 \end{pmatrix} \end{array}$$

   c. Let $[a_{ij}]_{m\times n}$ be the payoff matrix and $(p_1, p_2, \ldots, p_m)$ and $(q_1, q_2, \ldots, q_n)$ be the probabilities with which players A and B select their strategies $(A_1, A_2, \ldots, A_m)$ and $(B_1, B_2, \ldots, B_n)$, respectively. If V is the value of the game, then express the loss of the player B.

   d. Consider the inventory model as defined below. Explain the model with a diagram and derive economic order quantity, optimal stock level and total variable inventory cost, optimum shortage level and total cycle time.

      i. One type of item in the inventory
      ii. Replenished instantaneously and single delivery
      iii. Shortage is allowed
      iv. Carrying cost per year is same
      v. Demand is known, constant
      vi. Lead time is constant and known
      vii. Purchase cost and reorder cost same, no discount

CO6: **Explain** and **illustrate** the M/M/1 queue and Project Management

6. Attempt any three (3) from {a, b, c, d, e} and one from {f , g} [3 × 4 + 8 = 20]

   a. For an activity (i, j) define total float time and free float time. State and prove the relation between total float time and free float time of an activity.

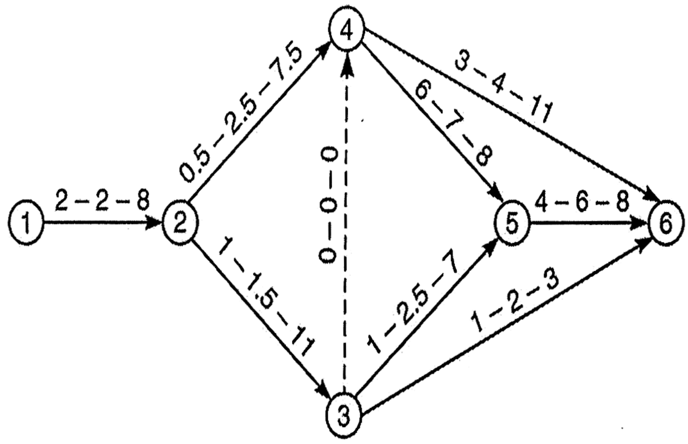
   b. For a critical activity (i, j), prove that $E_j = L_j$ and $E_i = L_i$ are necessary condition but not sufficient.

   c. For a project, if you have two critical paths, as a project manager which path do you follow, when the problem is solved by i) CPM; ii) PERT method. Justify your answer.

   d. What is an isolated event? Which distribution is used in the analysis of the isolated event? Write the expression of the distribution with explaining the meaning of the symbols.

   e. A queuing model is represented by (a/b/c) : (d/e/f). Define the symbols a – f.

f. Consider the network shown below. The three time estimates for the activities are given along the arrows. Determine the critical path. What is the probability that the project will be completed in 21 days? [Z score table is given below]



Standard Normal Distribution: Table Values Represent area to the left of the Z score										
Z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.7	.75804	.76115	.76424	.76730	.77035	.77337	.77637	.77935	.78230	.78524
0.8	.78814	.79103	.79389	.79673	.79955	.80234	.80511	.80785	.81057	.81327
0.9	.81594	.81859	.82121	.82381	.82639	.82894	.83147	.83398	.83646	.83891

g. In a queuing model, the steady state equation for this system is given below, where the symbols have usual meaning.

$$P_n'(t) = \lambda P_{n-1}(t) + \mu P_{n+1}(t) - (\lambda + \mu) P_n(t), \quad n \geq 1$$
$$P_0'(t) = \mu P_1(t) - \lambda P_0(t), \quad n = 0$$

Assuming above equation prove that $P_n = \left(1 - \frac{\lambda}{\mu}\right)\left(\frac{\lambda}{\mu}\right)^n$