

B.E in INFORMATION TECHNOLOGY

2ND YEAR, 1ST SEMESTER EXAMINATION, 2025

MATHEMATICS FOR IT-1

Time: 3 hours

Full Marks: 100

*Note: Use separate answer scripts for each part*PART –A
[50 MARKS]

CO1 [25 MARKS]	1. a) If vector $\mathbf{A} = \mathbf{a}_x + 3\mathbf{a}_z$ and vector $\mathbf{B} = 5\mathbf{a}_x + 2\mathbf{a}_y - 6\mathbf{a}_z$, find the angle θ_{AB} between $\mathbf{A}$ and $\mathbf{B}$. Or, Let $\mathbf{G} = x^2\mathbf{a}_x - y\mathbf{a}_y + 2z\mathbf{a}_z$ and $\mathbf{H} = yz\mathbf{a}_x + 3\mathbf{a}_y + xz\mathbf{a}_z$, find the angle between $\mathbf{G}$ and $\mathbf{H}$ at point (1,-2,3). Where $\mathbf{a}_x$, $\mathbf{a}_y$ and $\mathbf{a}_z$ are unit vectors along x, y and z axis respectively. b) If vector $\mathbf{P} = 2\mathbf{a}_x - \mathbf{a}_z$, vector $\mathbf{Q} = 2\mathbf{a}_x - \mathbf{a}_y + 2\mathbf{a}_z$, vector $\mathbf{R} = 2\mathbf{a}_x - 3\mathbf{a}_y + \mathbf{a}_z$ Determine: i) $(\mathbf{P} + \mathbf{Q}) \times (\mathbf{P} - \mathbf{Q})$ ii) $\mathbf{Q} \cdot \mathbf{R} \times \mathbf{P}$ iii) $\sin \theta_{AB}$ iv) $\mathbf{P} \times (\mathbf{Q} \times \mathbf{R})$ v) The component of $\mathbf{P}$ along $\mathbf{Q}$ vi) A unit vector perpendicular to both $\mathbf{Q}$ and $\mathbf{R}$ c) If vector $\mathbf{A} = x^2y\mathbf{a}_x + x\mathbf{a}_y + 2yz\mathbf{a}_z$, find divergence ($\nabla \cdot \mathbf{A}$) at the point (-3, 4, 2). Or, Given the vector field $\mathbf{G} = (16xy - z)\mathbf{a}_x + 8x^2\mathbf{a}_y - x\mathbf{a}_z$, check whether $\mathbf{G}$ is irrotational or not. Where $\mathbf{a}_x$, $\mathbf{a}_y$ and $\mathbf{a}_z$ are unit vectors along x, y and z axis respectively. d) Let $V = xy^2z^3$, evaluate ∇V and $\nabla^2 V$ at point P(1,2,3). Or, State Stoke's theorem. Hence prove it. [4+(2x6)+4+5=25]
CO2 [25 MARKS]	2. Solve the following differential equations. [Any 5] i) $(1 + 3e^{y/x})dy + 3e^{y/x}(1 - y/x)dx = 0$ ii) $(4D^2 - 20D + 25)y = 0$, where D stands for d/dx iii) $\sin 2x \frac{dy}{dx} - y = \tan x$ iv) $x(x-1) \frac{dy}{dx} - y = x^2(x-1)^2$ v) $(1 + y + x^2y)dx + (x + x^3)dy = 0$ vi) $(1 + x^2) \frac{dy}{dx} + y = e^{\tan^{-1}x}$ vii) $(x + y \cos \frac{y}{x}) dx = x \cos \frac{y}{x} dy$ viii) $\frac{dy}{dx} = \frac{3x + 4y + 1}{-4x + 2y - 3}$ [5x5=25]

[Turn over

Part-B
[50 MARKS]

(Note: Answers of all parts/subparts of a question should be written together)

CO3 (25)	Q.1 Answer (a) and any one from (b) and (c): a. Find the optimal solution of the problem i) graphically, and ii) using simplex method: i. Min $z = 20x_1 + 10x_2$, where x_1 and $x_2 \geq 0$ ii. Min $z = 4x_1 + 3x_2$, where x_1 and $x_2 \geq 0$ s.t. $x_1 + 2x_2 \leq 40$ s.t. $x_1 + 2x_2 \geq 8$ $3x_1 + x_2 \geq 30$ $3x_1 + 2x_2 \geq 12$ $4x_1 + 3x_2 \geq 60$ 4+8 b. i. Find the dual of the problem: Max $z = x_1 - x_2 + 3x_3 + 2x_4$, where x_1 and $x_4 \geq 0$, x_2 and x_3 are unrestricted in sign; subject to: $x_1 + x_2 \geq -1$, $x_1 - 3x_2 - x_3 \leq 7$ and $x_1 + x_3 - 3x_4 = -2$ ii. Show that the following L.P.P is degenerate. Hence find its optimal solution. Max $z = 3x_1 + 9x_2$, where x_1 and $x_4 \geq 0$, s.t. $x_1 + 4x_2 \leq 8$, $x_1 + 2x_2 \leq 4$ iii. Under which condition (s) an L.P.P. becomes degenerate while solving using simplex methods. 5+6+2 c. i. An agricultural firm has 180 tons of Nitrogen fertilizers, 250 tons of phosphate, and 220 tons of potash. It is able to sell 3:3:4 mixtures of these substances at a profit INR 15 per ton and 1:2:1 mixtures at a profit of INR 12 per ton respectively. Pose a linear programming problem on the given fact. ii. Give comments on the solutions of the following problems: Max $z = 3x_1 + x_2$, where x_1 and $x_2 \geq 0$, s.t. $2x_1 + 3x_2 \geq 2$, $x_1 + x_2 \geq 1$ Max $z = 3x_1 + 2x_2$, where x_1 and $x_2 \geq 0$, s.t. $x_1 - x_2 \leq 1$, $x_1 + x_2 \geq 3$ Max $z = 6x_1 + 10x_2$, where x_1 and $x_2 \geq 0$ s.t. $3x_1 + 5x_2 \leq 10$, $5x_1 + 3x_2 \leq 15$ 5+8																																																																		
CO4 (13)	Q.3 Answer any one: a. i. Find the initial basic feasible solution of the following transportation problem by using <i>three</i> well known methods (symbols carry their usual meaning). <table> <tr> <td></td><td>D₁</td><td>D₂</td><td>D₃</td><td>D₄</td><td>a_i</td></tr> <tr> <td>O₁</td><td>19</td><td>20</td><td>50</td><td>10</td><td>7</td></tr> <tr> <td>O₂</td><td>70</td><td>30</td><td>40</td><td>60</td><td>9</td></tr> <tr> <td>O₃</td><td>40</td><td>8</td><td>70</td><td>20</td><td>18</td></tr> <tr> <td>b_j</td><td>5</td><td>8</td><td>7</td><td>14</td><td></td></tr> </table> ii. What step (s) is/are taken if the number of occupied cells < m+n-1 w.r.t. a transportation problem? What step (s) is/are taken if some cells of an assignment problem are vacant and -ve? 9+2x2 b. i. Find the optimal assignment of the following assignment problem. <table> <tr> <td></td><td>M₁</td><td>M₂</td><td>M₃</td><td>M₄</td><td>M₅</td></tr> <tr> <td>J₁</td><td>3</td><td>5</td><td>10</td><td>15</td><td>18</td></tr> <tr> <td>J₂</td><td>4</td><td>7</td><td>15</td><td>18</td><td>8</td></tr> <tr> <td>J₃</td><td>8</td><td>12</td><td>20</td><td>20</td><td>12</td></tr> <tr> <td>J₄</td><td>5</td><td>5</td><td>8</td><td>10</td><td>6</td></tr> <tr> <td>J₅</td><td>10</td><td>10</td><td>15</td><td>25</td><td>10</td></tr> </table> ii. For a transportation problem what step (s) is/are taken if at least one cell-evaluation is -ve? Determine the conditions of non-degeneracy and finding basic feasible solution of it. 9+2x2		D ₁	D ₂	D ₃	D ₄	a _i	O ₁	19	20	50	10	7	O ₂	70	30	40	60	9	O ₃	40	8	70	20	18	b _j	5	8	7	14			M ₁	M ₂	M ₃	M ₄	M ₅	J ₁	3	5	10	15	18	J ₂	4	7	15	18	8	J ₃	8	12	20	20	12	J ₄	5	5	8	10	6	J ₅	10	10	15	25	10
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CO5
(12)

Q.2 Answer any one:

a.

i. Find the optimal solution of the following transportation problem:

	D ₁	D ₂	D ₃	D ₄	D ₅	a _i
O ₁	3	4	6	8	8	20
O ₂	2	10	0	5	8	30
O ₃	7	11	20	40	3	15
O ₄	1	0	9	14	16	13
b _j	40	6	8	18	6	

ii. What are the conditions of an assignment problem to be degenerate? Suggests some rules/techniques to avoid it.

9+3

b. How is enumerative method applied in an assignment problem for solving a travelling salesman problem (TSP)? Hence solve the following:

	D ₁	D ₂	D ₃	D ₄	D ₅	D ₆
S ₁	∞	20	23	27	29	34
S ₂	21	∞	19	26	31	24
S ₃	26	28	∞	15	36	26
S ₄	25	16	25	∞	23	18
S ₅	23	40	23	31	∞	10
S ₆	27	18	12	35	16	∞

3+10

Course Outcomes

CO1: Explain and illustrate sum and product of vectors with related applications.

CO2: Solve homogeneous, non-homogeneous linear ordinary differential equations of the 1st order and higher orders having constant and variable coefficients and system of linear differential equations.

CO3: Express a given real-world problem as a linear programming problem and use simplex method to solve it.

CO4: Solve transportation problem using suitable methods and test for optimality.

CO5: Solve assignment problem using suitable methods and examine for optimality.