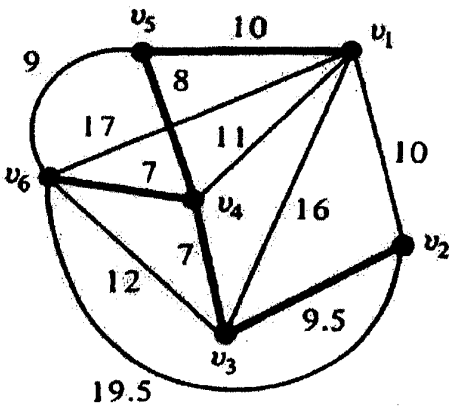


Bachelor Of Engineering In Information Technology
2nd Year 2nd Semester, Semester Examination, 2025
Subject Name –Graph Theory & Combinatorics (IT/PC/B/T/224)

Time: 3Hrs.

Full Marks=100

CO1 [10]	Q1. (i) Consider a graph with four vertices labeled A, B, C, and D. The graph contains the following undirected edges: A is connected to B, A is connected to C, B is connected to C, and C is connected to D. Based on this graph, draw the corresponding structure. Find its adjacency matrix and incidence matrix. Additionally, compute the path matrix of length 2. (ii) Prove that “If a graph has exactly two vertices of odd degrees, there must be a path joining these two vertices”. (iii) Find the maximum and minimum number of edges in a 3-component graph having 10 vertices. OR In a group of twelve (12) persons, is it possible that each person has a friendship with exactly five (5) others in the group? Justify this. [3+4+3=10]
CO2 [20]	Q2. (i) Prove that every tree has either one or two centers. (ii) Prove that any subgraph g of a connected graph G is contained in some spanning tree of G if and only if g contains no circuit. (iii) In an electric network, there are 5000 cables connecting 300 nodes. The network is currently fully connected, and each cable represents an edge in the graph. Calculate the minimum number of cables that must be removed to form a spanning tree in the network, ensuring the network remains connected but without any cycles. (iv) Using Prim's Algorithm, find the minimum spanning tree (MST) of the given graph (Figure: 1). Show all the steps involved and calculate the total weight of the MST.  Figure: 1 [4+5+5+6=20]
CO3 [20]	Q3. (i) Consider the following two undirected graphs: G₁: Vertices = {A, B, C, F}, Edges = {AB, AC, BC, CF} G₂: Vertices = {C, D, E, G}, Edges = {CD, DE, EG, GC}

	(a) Perform the union operation on G_1 and G_2. (b) From the union graph, delete vertex D. (c) Identify at least one cut vertex and one cut edge in the union graph, and justify how their removal affects the structure and connectivity of the graph. (d) On the union graph, perform a fusion operation by merging vertices A and D, and update the graph accordingly. (ii) "The vertex connectivity of any graph G can never exceed the edge connectivity of G". Justify this. [(3+3+4+4)+6=20]
CO4 [20]	Q4. (i) "A graph is 2-colorable if it is bipartite and every cycle has an even length." Justify this with a suitable example. (ii) Consider a graph $G=(V, E)$ where $V=\{A,B,C,D,E,F\}$ and $E=\{(A,B),(A,C),(A,D),(B,C),(C,D),(D,E),(E,F),(C,F)\}$. Find the chromatic number of the graph. (Mention all steps properly) (iii) Consider the graph $G=(V,E)$ where $V=\{A,B,C,D,E,F,G,H\}$ and $E=\{(A,B),(B,C),(C,D),(E,F),(F,G),(G,H)\}$ (a) Draw the graph G. (b) Determine the maximum independent set of G. (c) What is the size of this maximum independent set? Justify your answer with a proper explanation. (iv) Given the graph G with the following details: • Vertices = $\{A, B, C, D, E\}$ • Edges = $\{AB, AC, AD, BC, BD, CD, DE, CE\}$ Determine whether the graph is planar or non-planar. Justify your answer using Kuratowski's Theorem. (v) Find out how many vertices can be matched using maximum matching in the bipartite graph algorithm of the following graph given below (Figure: 2). Figure: 2 [3+4+(2+3+2)+3+3=20]
CO5 [20]	Q5. (i) How many license plates can be made using either three uppercase English letters followed by three digits or four uppercase English letters followed by two digits? (ii) How many cards must be selected from a standard deck of 52 cards to guarantee that at least three cards of the same suit are chosen?

	(iii) A club has 25 members. How many ways are there to choose four members of the club to serve on an executive committee? (iv) There are six different candidates for governor of a state. In how many different orders can the names of the candidates be printed on a ballot? [4+5+6+5=20]
CO6 [10]	Q6. (i) Find the generating functions of the following sequences in closed form. $\langle 1, a, a^2, a^3, \dots \rangle$ (ii) Find out the solution of the recurrence relation given below with $a_0 = 2$ and $a_1 = 7$. $a_n = a_{n-1} + 2a_{n-2}$ [5+5=10]

CO1: Explain and discuss the concept of different types of Graphs with fundamental properties and **express** different types of matrix representation. (K2)

CO2: Illustrate different types of trees such as (i) rooted tree (ii) spanning tree etc, and **explain** their properties. (K3)

CO3: Apply operations like Union, Deletion, and decomposition of graphs and **illustrate** Cut vertex and Cut edge and their properties. (K3)

CO4: Illustrate planar graph and their properties and Graph Coloring and Matching. (K3)

CO5: Apply and evaluate basic counting rules, pigeon-hole principle and principle of inclusion-exclusion. (K3)

CO6: Apply and Solve problems using Generating Function and Recurrence Relations. (K3)