Ref. No.: Ex/ET/PC/B/T/216/2025

B.E. ELECTRONICS AND TELE-COMMUNICATION ENGINEERING

SECOND YEAR FIRST SEMESTER EXAM 2025

Subject: MATHEMATICS III

Time: Three Hours **Full Marks: 100**

Answer Question No. 1 and any FIVE questions from the rest. Q.1 carries 20 marks and the rest are of 16 marks each. Answer in brief with proper justification. All the symbols used carry the standard meanings.

Q. No.		Marks
1.	***Answer with proper reasons.*** (a) Find the partial differential equation obtained by eliminating arbitrary constants a & b from $z = ax^2 - by^2$. (b) Average time required to repair a machine is 0.5 hours. What is the probability that the next repair will take more than 2 hours? (c) A random variable X has the following pdf: $f(x) = \begin{cases} k, -2 < x < 2 \\ 0, \ elsewhere \end{cases}$. Find the value of the constant k. (d) Evaluate $\lim_{z \to -1+i} \frac{z^2+2z+2}{z^2+2i}$. (e) Determine the constant a so that the vector: $V = (x+3y)\hat{\imath} + (y-2z)\hat{\jmath} + (x+az)\hat{k}$ is solenoidal. (f) A random variable X has Poisson distribution such that $P(1) = P(2)$. Calculate the standard deviation of X. (g) Three different numbers are selected at random from the set $A = \{1,2,3, \dots, 11,12\}$. Find the probability that the product of two of the numbers is equal to the third. (h) Mention two properties of mathematical expectation. (i) Comment on the limitations of correlation coefficient. (j) Give an example of Linear and Non-linear partial differential equations.	$(2 \times 10) = 20$
2.	(a) From the fundamental concept of Divergence and the basic equations of Electrostatics establish Poisson's Equation and hence Laplace's equation. (b) Determine the Divergence and Curl of the following vector field: $\vec{\phi} = yz\hat{a}_x + 4xy\hat{a}_y + y\hat{a}_z$ (c) Show that $\boldsymbol{F} = (2xy + z^3)\hat{\imath} + x^2\hat{\jmath} + 3xz^2\hat{k}$ is a conservative field. Find the scalar potential. Find the work done in moving an object in this field from (1, -2, 1) to (3, 1, 4).	8 + 4 + 4
3.	(a) Prove that $\lim_{z \to 0} \frac{Re(z)}{\lvert z \rvert}$ does not exist. (b) Using the complex variable technique prove that $\int_0^\infty \frac{\sin mx}{x} dx = \frac{\pi}{2}$ (c) Evaluate $\int_0^\infty \frac{dx}{(1+x^2)^2}$ using Cauchy's integral. (d) Expand $\sin^{-1} z$ in powers of z.	4 + 4 + 4 + 4
4.	(a) What are the different types of random variables. Mention the context of choosing a random variable based on a particular situation. (b) How is mathematical expectation expressed for the different random variables? (c) Define the terms probability mass function and probability density function. (d) Compare between Continuous and Discrete Probability Distributions.	3 + 2 + 3 + 8

[Turn over

Ref. No.: Ex/ET/PC/B/T/216/2025

B.E. ELECTRONICS AND TELE-COMMUNICATION ENGINEERING

SECOND YEAR FIRST SEMESTER EXAM 2025

Subject: MATHEMATICS III

5.	(a) There are 3 multiple choice questions in a MCQ test. Each MCQ consists of four possible choices and only one of them is correct. If an examinee answers those MCQ randomly (without knowing the correct answers) a. What is the probability that exactly any two of the answers will be correct? b. What is the probability that at least two of the answers will be correct? c. What is the probability that at most two of the answers will be correct? d. What will be the average or expected number of correct answers? e. Find the standard deviation of number of correct answers. (b) The waiting time (in minutes) for train is uniform (10, 50). Find a. The probability that you have to wait at least 20 minutes. b. Average waiting time. c. Standard deviation of waiting time. (c) Prove that for normal distribution, the mean deviation from the mean equals nearly $\frac{4}{5}$ times of the standard deviation.	$6+5+5$
6.	(a) A string is stretched and fastened to two points l units apart. Motion is started by displacing the string in the form $y = a\sin\frac{\pi x}{l}$ from which it is released at $t = 0$. Prove that the displacement of any point at a distance x from one end at time t is given by $y(x,t) = a\sin\frac{\pi x}{l}\cos\frac{\pi ct}{l}$. (b) Solve the PDE: $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}\,(0 < x < \lambda, t > 0)$ with initial condition $u(x,0) = 3\sin\frac{n\pi x}{\lambda}$ (n is a positive integer) and boundary conditions $u(0,t) = u(\lambda,t) = 0$.	$10+6$
7.	(a) Let $X_1, X_2, \dots X_n$ be a sequence of independent random variables, each having mean m and variance σ^2. Show that $\frac{1}{n}\sum_{i=1}^{n} X_i \to m$ in mean square as $n \to \infty$. (b) With the number of trials increased indefinitely and the probability of success in a single trial being very small, show that the probability mass function (p.m.f.) of Poisson variate can be obtained as a limiting case of the p.m.f. of Binomial variate. (c) Solve $\frac{\partial y}{\partial x} = 2\frac{\partial y}{\partial t} + y,\ y(x,0) = 6e^{-3x}$, which is bounded for $x > 0,\ t > 0$.	$5+7+4$
8.	(a) Explain the significance of coefficient of correlation and coefficient of determination. (b) How do we differentiate between correlation and regression? (c) What are the common errors involving correlation? (d) What are the limitations of linear regression? (e) Mean demand of an oil is $1000l$ per month with SD of $250l$. If $1200l$ are stocked, what is the satisfaction level? For an assurance of 95%, what stock must be kept?	$3+3+2+3$ $+5$
9.	(a) Let $f(z) = u(x,y) + iv(x,y)$ be an analytic function. If $u = 3x + 2xy$, then find v and express $f(z)$ in terms of z. (b) Considering a vector mathematically show how can Divergence Theorem and Stokes' theorem form the basis of conversion of volume integral to surface integral and then to line integral and vice versa. (c) For a vector field $\bar{A}$, show explicitly that $\nabla . \nabla \times \bar{A} = 0$. What is the significance of such a field?	$5+8+3$