BETCE EXAMINATION, 2025
(2nd Year 1st Semester)
Signals and Systems

Full Marks: 100 **Time: 3 hours**

Answer all parts & subparts of the questions under a CO serially in the same place

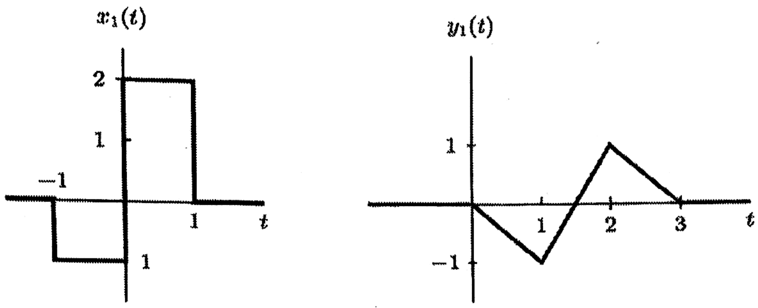
CO-1 (Marks: 25)

Q1. Consider an LTI system whose response $y_1(t)$ corresponding to the given input $x_1(t)$ is shown below in part (a). Another input signal $x_2(t)$ is also shown in part (b). [6+6+3]

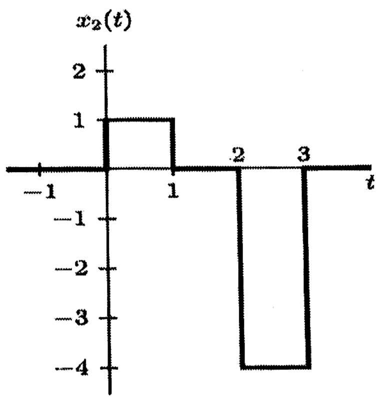
a) Express the input signal $x_2(t)$ in terms of a linear, time-shifted combination of the input $x_1(t)$ i.e. express $x_2(t)$ as the sum of $x_a(t)$ and $x_b(t)$. Also, sketch these components.

b) Let $y_a(t)$ and $y_b(t)$ be the outputs of the system corresponding to the inputs $x_a(t)$ and $x_b(t)$, respectively. From the given input-output pair and using LTI property, sketch $y_a(t)$ and $y_b(t)$.

c) From the diagrams obtained in part b), sketch the output response $y_2(t)$ due to input $x(t)$.



(a)

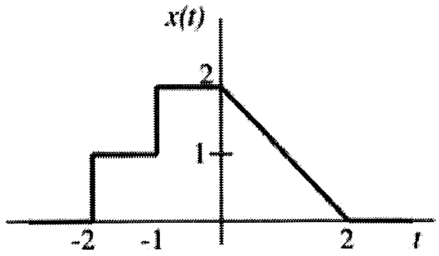


(b)

OR

Q1. a) Check whether the given signal $g(t) = (1 + e^{-5t})\, u(t)$ is energy signal or power signal or neither energy nor power signal. [4]

b) For the given signal x(t), as shown below, sketch x(2 - t/2). Show all necessary steps. [3]



c) Find the odd and even components of the signal, $x(t) = \sin(\omega_0 t + \pi/4)$. [3]

d) Sketch the signal $y(t) = r(t) - r(t - 1) - u(t - 1) + r(t - 2) - r(t - 3) - u(t - 3) + \ldots$
where, u(t) and r(t) represents unit step and unit ramp respectively. [5]

Q2. a) Compute the output response y(t) for a continuous time LTI system whose input x(t) and impulse response h(t) are given below. Use graphical method. [10]

$x(t) = u(t) - u(t - 3)$

$h(t) = u(t) - u(t - 2)$

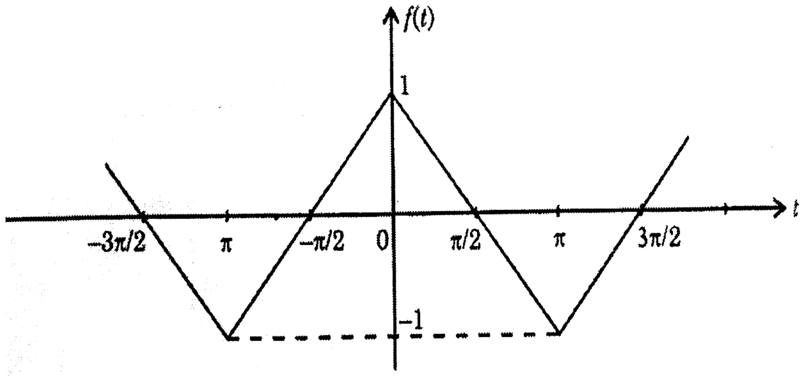
OR

Q2. a) State the significance of correlation coefficient C_n between two signals. Explain, how this concept is used for detection if signal in digital communication. [5]

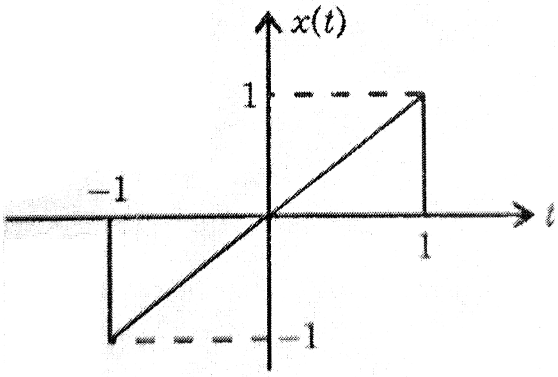
b) Consider an arbitrary signal $g(t)$ having an energy of E_g .The Fourier transform of this signal is represented by $G(f)$ or $G(\omega)$. Calculate E_g from $G(f)$ or $G(\omega)$ using Parseval's theorem. [5]

<u>CO-2 (Marks: 15)</u>

Q3. a) Obtain the Trigonometric Fourier Series representation for the periodic signal g(t). [8]



b) Find the Fourier transform of the given signal x(t). [7]



OR

Q3. a) Consider y(t) = x(t) * h(t) and g(t) = x(3t) * h(3t), where * implies convolution operation. Given, x(t) ↔ X(ω); h(t) ↔ H(ω); g(t) ↔ G(ω). [3+1+2]

i)Show that g(t) has the following form:
g(t) = A y(Bt), where, A and B are constants.

ii) Calculate the values of A and B.

iii) State the properties you have used in this context.

b) A causal LTI system has the frequency response H(ω) = (jω + 4) / (6 - ω^2 + 5jω). Determine the impulse response h(t). [4]

c) An arbitrary signal $g(t)$ is multiplied by a sinusoid $\cos \omega_o t$. State the significance of this phenomenon both in time and frequency domain with necessary diagrams. [5]

CO-3 (Marks: 10)

Q4.a) What do you mean by Aliasing effect? How will you mitigate this problem? Explain with proper diagrams. [8]

b) Find the Nyquist rate and Nyquist interval for the signal
$x(t) = 3\cos(50\pi t) + 5\sin(300\pi t) - \cos(200\pi t)$. [2]

CO-4 (Marks: 15)

Q5. a) Check whether the system described by $y(n) = x(n) + 3u(n + 1)$ is i) linear; ii) time invariant. Here, $x(n)$ and $y(n)$ represent the input and output of the system respectively and $u(n)$ represents unit step sequence. [2.5+2.5]

b) The input sequence $x(n)$ and impulse response $h(n)$ of an LTI system are given by
$x(n) = \delta(n) + 2\delta(n-1) - \delta(n-3)$ and $h(n) = 2\delta(n+1) + 2\delta(n-1)$.
Compute the convolution of the above sequences using graphical approach. Here $\delta(n)$ signifies unit impulse sequence. [10]

CO-5 (Marks: 10)

Q6. a) Given two zero mean unity variance random variables X and Y with a correlation coefficient $\rho_{XY} = 0.5$, form a new random variable, $U = (CX - 3Y)^2 - 2Y$, where C is a real valued constant. Given, $\rho_{XY} = E[XY] / \sigma_X \sigma_Y$, where symbols have their usual meanings. [4+1+1]

i) Find the value of C such that $E[U] = 9$.

ii) Find the value of C that minimizes $E[U]$.

iii) Can $E[U]$ be smaller than 27/4? Justify your answer.

b) A box contains 5 black balls and 10 white balls. One ball at a time is drawn at random, its colour is noted, and the ball is replaced in the box for the next draw. Find the probability that the first white ball is drawn on the third draw. [4]

CO-6 (Marks: 25)

Q7. a) Calculate 'Noise Temperature' of an amplifier block having power gain 'G' and noise factor 'F'. Also, discuss the significance of this term. [6]

b) How is Thermal noise generated by a resistor? Calculate the r.m.s. value of noise voltage generated by a resistor. [1+3]

OR

Q7. How will you quantify the degradation of Signal to Noise Ratio (SNR) by the receiver or amplifier in a communication system? Calculate the overall Noise Factor of two amplifiers in cascade having the input and output noise power N_i and N_O respectively. [3+7]

Given,
1st stage: Power gain: G_1; Noise Factor: F_1
2nd stage: Power gain: G_2; Noise Factor: F_2

Q8. a) Consider that noise is a random process and it can be represented as the superposition of noise spectral components. Also consider that the spectral component associated with the kth frequency interval is given by:
$n_k(t) = a_k \cos(2\pi k \Delta f t) + b_k \sin(2\pi k \Delta f t)$, where symbols have their usual meanings.
Discuss the statistical properties of a_k and b_k with necessary mathematical derivations. [10]

b) Explain the term 'Noise Bandwidth' with proper diagram. Hence, calculate the 'Noise bandwidth' of an RC low-pass filter. [5]

OR

Q8. a) In relation to the Narrow Band representation of noise, derive the mathematical expressions corresponding to the Quadrature components of noise. Also elaborate the significance of this representation. [5+3]

b) Define White Noise. Suppose, white noise is present at the input of an integrator placed before the demodulator to restrict the noise present in the communication system. If 'B' is the bandwidth of the ideal filter, calculate the noise power at the output of the filter. [2+5]