

B.E COMPUTER SCIENCE AND ENGINEERING 3rd YEAR 1st SEMESTER EXAMINATION 2025**Formal Languages and Automata Theory**

Time: 3 hours

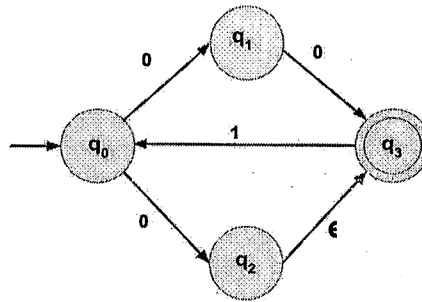
Full Marks: 100

Answer from Groups A, B, C, D**Group A**i) Which one of the strings below matches the regular expression $a(a+b)^*b+b(a+b)^*a$?

- (A) aba (B) bab (C) abba (D) aabb

ii) Which binary strings does the regular expression $(0+10)^*(\epsilon+1)$ represent ?

iii) Draw the transition diagram of an equivalent NFA for the following:



iv) Let A be an NFA with n states. Which of the following is necessarily true?

- (A) The shortest word in $L(A)$ has length at most $n-1$.
 (B) The shortest word in $L(A)$ has length at least n .
 (C) The shortest word not in $L(A)$ has length at most $n-1$.
 (D) The shortest word not in $L(A)$ has length at least n .

v) Let L be the language over $\{a,b\}$ that contains the same number of occurrences of a and b. Which of the following languages is regular?

- (A) $L \cap a^*b^*$ (B) $(L \cap a^*b^*) \cup a^*b^*$ (C) $L \cup a^*b^*$ (D) $(L \cap a^*b^*) \cup b^*a^*$

vi) Which of the following languages over the alphabet $\{0,1\}$ are **not** recognized by deterministic finite state automata (DFA) with three states?

- (A) Strings not containing 11 (B) Binary representations of multiples of three
 (C) Strings with 11 as a suffix (D) Strings not containing 101

[Turn over

vii) Which of the following statements about regular languages is NOT true?

- (A) Every language has a regular superset.
- (B) Every language has a regular subset.
- (C) Every subset of a regular language is regular.
- (D) Every subset of a finite language is regular.

viii) Consider the following languages over the alphabet $\{a,b,c,d\}$

$$L_1 = \{a^n b^n c^m d^m \mid n, m \geq 0\}, L_2 = \{a^n b^m c^n d^m \mid n, m \geq 0\}, L_3 = \{a^n b^m c^m d^n \mid n, m \geq 0\}.$$

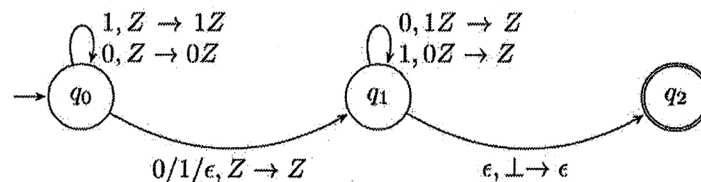
Which of these languages is/are context-free?

- (A) Only L_1 and L_2
- (B) Only L_1 and L_3
- (C) All of them
- (D) None of them

ix) The language which is generated by the grammar $S \rightarrow aSa \mid bSb \mid a \mid b$ over the alphabet of $\{a,b\}$ is the set of

- (A) Strings that begin and end with the same symbol
- (B) All odd length palindromes
- (C) All odd and even length palindromes
- (D) All even length palindromes

x) Consider the transition diagram of the following Non Deterministic Pushdown Automaton (NPDA). In this diagram z represents a string of stack symbols, currently stored on the associated stack of the NPDA, and the stack symbol $\perp$ the initial stack symbol.



Which one of the following sequences must follow the string 101100 so that the overall string is accepted by the automaton?

- (A) 10110
- (B) 10010
- (C) 01010
- (D) 01001

xi) Let $L_1 = \{a^n b^m \mid m, n \geq 0 \text{ and } m \geq n\}$ and $L_2 = \{a^n b^m \mid m, n \geq 0 \text{ and } m < n\}$. The language $L_1 \cup L_2$ is:

- (A) regular, but not context-free
- (B) context-free, but not regular
- (C) Both regular and context-free
- (D) neither regular nor context-free

xii) Given the productions $S \rightarrow aSb$, $S \rightarrow \epsilon$, $S \rightarrow A$, $A \rightarrow aA$, $B \rightarrow C$, $C \rightarrow D$, what will be the ratio of number of useless variables to number of useless productions?

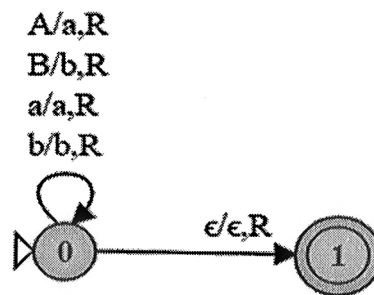
- (A) 1
- (B) $\frac{3}{4}$
- (C) $\frac{2}{3}$
- (D) 0

xiii) Let $\langle M \rangle$ be the encoding of a Turing machine as a string over $\Sigma=\{0,1\}$.

Let $L=\{\langle M \rangle \mid M \text{ is a Turing machine that accepts a string of length } 2014\}$.
Then L is:

- (A) decidable and recursively enumerable (B) undecidable but RE
(C) undecidable and not RE (D) decidable but not RE

xiv) What does the following Turing Machine compute?



xv) Assuming $P \neq NP$, which of the following is true ?

- (A) NP-complete=NP (B) NP-complete $\cap$ P = $\emptyset$
(C) NP-hard = NP (D) P = NP-complete (2x15=30)

Group B

Answer any three

2(a) Give the transition diagram of a Deterministic Finite Automaton (DFA) that accepts all binary strings containing neither '00' nor '11'.

(b) How would you decide whether one regular language is a subset of another regular language?

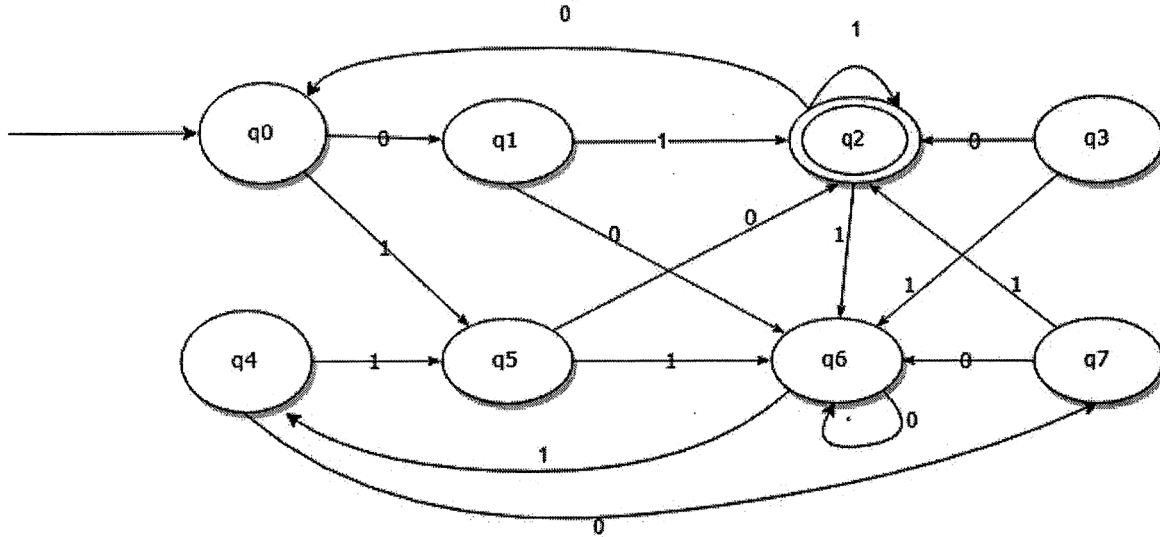
(8+4)

3(a) State the Pumping Lemma for regular languages.

(b) Prove that $L=\{0^p \mid p \text{ is a prime number}\}$ is not regular

(5+7)

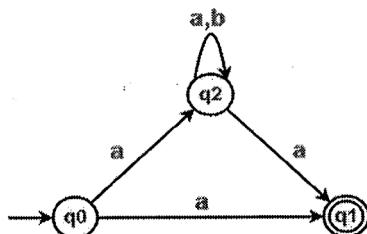
4(a) Consider the following Deterministic Finite Automaton (DFA). Form the table of distinguishability for this automaton and give the transition diagram of the minimum state equivalent DFA.



(b) Prove that no other equivalent DFA with lesser number of states than the one obtained by your method can exist.

(6+6)

5(a) Consider the following Non deterministic Finite Automaton (NFA). Give the transition diagram of an equivalent DFA.



(b) Let $L = \{0011001\}$ be a regular language on $\Sigma = \{0, 1\}$ and h a homomorphism defined as $h(a)=0$, $h(b)=1$, $h(c)=01$. Find $h^{-1}(L)$.

(c) Prove that regular languages are closed under complementation.

4+4+4

Group C

Answer any two

6(a) State the Pumping Lemma for context free languages.

(b) Let $L = \{0^p | p \geq 0\}$. Prove that L is not a context free language.

(4+6)

7(a) Prove that if there is a parse tree with root labeled A and yield w then $A \Rightarrow_{lm}^* w$

(b) Consider the language $L = \{0^i 1^j 2^k \mid i, j, k \geq 0 \text{ and } i+k=j\}$. Give a Context free grammar for the language L.

(7+3)

8) Prove that context free languages are not closed under intersection.

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Group D

Answer any two

9(a) State and prove the Halting problem for Turing machines.

(2+5)

10) Define P, NP, NP hard and NP complete classes of problems.

(1+(2x3))

11) Give the transition diagram of a Turing machine that decides if two binary positive integers, represented in unary, are equal.

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