Ex/CSE/BS/B/Math/T/221/2025

Bachelors of Computer Science and Engineering 2025

(2nd Year, 2nd Semester)

Mathematics IV

Time: Three hours

Full Marks: 100

USE SEPARATE ANSWER SCRIPTS FOR GROUP A AND GROUP B

Group A (FULL MARKS: 50)

Answer question 1 any SIX from the rest:

1. Let A, B, C be three subsets of a set X. If $A \Delta C = B \Delta C$, then prove that $A = B$. 2

2. Given $m \in \mathbb{N}$ integers $a_1, a_2, ...a_m$, show that there exist integers k, s with $0 \leq k < s \leq m$ such that $a_{k+1} + a_{k+2} + ... + a_s$ is divisible by m. 8

3. Prove that a mapping $f : A \to B$ is surjective if and only if f has a right inverse. Hence show that $f : \mathbb{N} \to \mathbb{N}$ defined by $f(x) = 2x + 1$ has no right inverse. 6+2

4. Find the number of different reflexive relations and anti-symmetric relations on a set containing n elements. 4+4

5. Define a countable set. Prove that every countably infinite set is cardinally equivalent to a proper subset of itself. 2+6

6. Prove that $|A| < |\mathscr{P}(A)|$ for any set A, where $\mathscr{P}(A)$ is the power set of A. 8

7. Find all upper bounds of $\{2, 9\}$ and all lower bounds of $\{60, 72\}$ for the poset $\{2, 4, 6, 9, 12, 18, 27, 36, 48, 60, 72\}$ with the divisibility relation. Is this poset a lattice? Justify your answer. 6+2

8. Define a *well-ordered set*. Show that the set of all natural numbers is well-ordered. 2+6

9. Find the truth table of $[(p \to q) \vee (q \to r)] \wedge (p \to r)$. 8

[Turn over

Group – B (Marks: 50)

Attempt *any Five* questions.

All questions carry equal marks.

1. Let A and B be two events in a probability space. Let $P(A) \neq 0$, $P(B) \neq 0$.

   Prove that

   a) A and B are independent $\Rightarrow$ (i) A and B^C are so.

   (ii) A^C and B^C are so.

   b) A and B are independent $\Rightarrow A \cap B \neq \phi$.

   c) $A \cap B = \phi \Rightarrow$ A and B cannot be independent.

2. Let (x, y) be uniformly distributed in A where $A = \{(x, y) \varepsilon \mathbb{R}^2 \| x| + |y| \leq 1\}$.

   a) Find the joint probability density function (pdf) of (x, y).

   b) Find the marginal pdf of x and that of y.

   c) Find the correlation coefficient of x and y.

3. Two points are chosen at random from a line with unit length. Let the points divide the line in three parts of lengths a unit, b unit and c unit of length respectively. Find the probability that a, b and c from the three sides of a triangle.

4. Let X_1, X_2 and X_3 be three independent random variables each with pdf $f(x) = \begin{cases} \lambda e^{-\lambda x} & \text{if } x > 0 \\ 0 & \text{otherwise} \end{cases}$.

   Find the pdf of their sum.

5. If $A_1, A_2, \ldots, A_n$ are n events in a sample space, use the axioms of Probability to prove that

$$1 + P\left(\bigcap_{i=1}^{n} A_i\right) \geq \sum_{i=1}^{n} P(A_1) - n$$

6. Suppose

$$P = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{array} \begin{array}{c} \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 \\ \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \\ 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 \\ \frac{1}{2} & 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

   Let P be the one-step transition matrix of a Markov chain $\{X_n : n = 1, 2, \ldots\}$, with state-space $\varphi = \{1, 2, 3, 4, 5, 6\}$. Find which states are recurrent and which states are transient.