Master of E. & Tel. E. Examination, 2025

Ex/PG/ETC/PE/T/113D/2025

Specialization CSN

1st Year (1st Semester)

STATISTICAL COMMUNICATION THEORY

Time: **Three hours** Full Marks: **100**

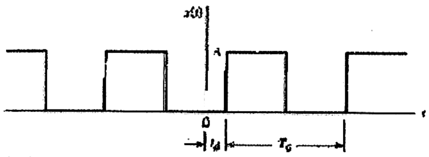
Module I: CO1 Answer any 2 questions Marks : 40

Q1. a) Thermal noise in a communication system is described by a zero-mean *Gaussian* random process n(t). Write an expression for Gaussian probability density function and discuss its significance. Write power spectral density and autocorrelation function of this process. What is the implication of AWGN channel? 08

b) If X and Y are random variables what would be the covariance of X and Y and Cov (aX, bY)? a and b are constants.
Find the characteristic function of a random variable X with parameter 'a' and its pdf given by $f(x) = a/\pi(x^2+a^2)$? $-\infty<x<\infty$ 05

c) Write expressions for mean, autocorrelation and covariance of a strictly stationary Random Process X(t). Prove and explain the property of autocorrelation function $|R_x(\tau)|\le R_x(0)$.
When are two random variables called independent and uncorrelated? 07

Q2. In the random binary wave X(t) generated in the below picture t_d is the random variable described



by the probability density function

$$f_{T_d}(t_d) = \begin{cases} \dfrac{1}{T_0}, & -\dfrac{1}{2}T_0 \le t_d \le \dfrac{1}{2}T_0 \\ 0, & \text{otherwise} \end{cases}$$

(a) Determine the probability density function of the random variable $X(t_k)$ obtained by observing the random process X(t) at time t_k.
(b) Determine the mean and autocorrelation function of X(t) using ensemble -averaging.
(c) Determine the mean and autocorrelation function of X(t) using time -averaging.
(d) Establish whether or not X(t) is stationary. In what sense is it ergodic? 12
(e) Establish the Rician distribution function as given for narrowband noise represented in terms of envelop and phase in presence of sinusoidal signal with amplitude A. From which find the condition for Rayleigh distribution. Point out the significance of these two distributions in communication. 08

Q3. (a) Given the random process $X(t)=10\cos(100t+\Theta)$ where Θ is uniformly distributed random variable in the interval $(0, \pi)$, is the process WSS? Establish the reason. 05

(b) Let x(n) be a stationary random process with zero mean and auto correlation $r_x(k)$. We form the process y(n) as, $y(n) = x(n) + f(n)$, where f(n) is a known deterministic sequence. Find the mean $m_y(n)$ and the autocorrelation $r_y(k,l)$ of the process y(n). 05

(c) Define Discrete Chain Markov Process with its states and transition probability function. 05

(d) Check whether the given matrix is from absorbing Markov Chain. Show the state transition diagram. 05

	S1	S2	S3	S4
S1	1	0	0	0
S2	½	0	0	½
S3	¼	¼	¼	¼
S4	0	0	0	1

[Turn over

Module II: CO2 Answer Q4 Marks : 20

Q4. (a) Consider an ARMA process with p poles and q zeros, write the expression for linear shift causal filter response for this ARMA (p, q) process and it's power spectral density. 04

(b) Consider a first order AR process that is generated by the difference equation

$$y(n) = a\,y(n-1) + w(n)$$

Where $|a|<1$ and w(n) is a zero mean white noise random process with variance σ_w^2,
Find the unit sample response of the filter that generates y(n) from w(n) and find the auto-correlation function of y(n). 06

(c) If $r_x(0)=1$, $r_x(1)=0.5$, and $r_x(2)=0.75$, then find the value of a(1), a(2) and b(0) in the following AR(2) model for x(n),

$$x(n) + a(1)x(n-1) + a(2)x(n-2) = b(0)\,w(n)$$

Where, w(n) is unit variance white noise. 10

Module III: CO3 Answer any one from Q5 and Q6 Marks : 20

Q5. (a) A discrete-time random process x(n) is generated as follows:

$$x(n) = \sum_{k=1}^{p} a(k)\,x(n-k) + w(n)$$

Where w(n) is a white noise process with variance σ_w^2. Another process z(n) is formed by adding noise to x(n),

$$z(n) = x(n) + v(n)$$

Where v(n) is a white noise with variance σ_v^2 and is uncorrelated with w(n).

(i) Find the power spectrum of x(n)
(ii) Find the power spectrum of z(n) 08

(b) The power spectrum of a wide sense stationary process x(n) is

$$Px(e^{jw}) = (5 - 4\cos w) / (6 - 5\cos w)$$

Find the whitening filter H(z) that produces unit variance white noise when input is x(n). 08

(c) Plot the autocorrelation function given the power spectral densities

$$Px(e^{jw}) = 1/(5 + 3\cos w)$$ 04

Q6. (a) Let d(n) be an AR(1) process with autocorrelation sequence $r_d(k) = \alpha^{|k|}$ with $0<\alpha<1$. If d(n) is observed in the presence of uncorrelated white noise v(n), that has a variance of σ_v^2. $X(n) = d(n) + v(n)$
Design a first order FIR Wiener Filter to reduce the noise in x(n) with $W(z) = w(0) + w(1)z^{-1}$

(i) Write Wiener Hopf equation.
(ii) Determine power spectrum of d(n), minimum mean square error and SNR at the output of Wiener Filter by considering $\alpha = 0.4$ and 0.8. Comments on the results. 20

Module IV: CO4 and CO5 Answer any one from Q7 and Q8 Marks :20

Q7. (a) When is Neyman-Pearson test applicable in detection problem? Outline the process of this test. 06

b) What is the receiver Operating Characteristic (ROC)? How does SNR influence the ROC? 06

c) In a binary hypothesis-testing problem, the received signals under the two hypotheses are

$$H_1 : Z = X_1^2 + X_2^2$$
$$H_2 : Z = X_1^2$$

Where X_1 and X_2 are independent, identically Gaussian distributed with zero mean and unity variance. Obtain the optimum decision rule for the Bay's criterion. Assume that $P_0 = P_1 = 1/2$, $C_{00} = C_{11} = 0$, $C_{10} = C_{01} = 1$. 08

Q8. Write short notes on 10+10

(a) Spectrum Estimation by Periodogram method
(b) Binary Hypothesis testing and Minimax Test