M.E. ELECTRONICS AND TELE-COMMUNICATION ENGINEERING
FIRST YEAR, SECOND SEMESTER EXAMINATION, 2025

QUANTUM COMPUTING

Time: Three Hours Full Marks: 100

Answer ***all*** questions.

1. a) Show that the following two states form an orthonormal basis:
$|\mathbf{a}\rangle = \sqrt{3}/2\ |0\rangle + i/2\ |1\rangle$ and $|\mathbf{b}\rangle = i/2\ |0\rangle + \sqrt{3}/2\ |1\rangle$.
Consider a qubit in the state $1/2\ |0\rangle - \sqrt{3}/2\ |1\rangle$. Write the **qubit state** in terms of $|\mathbf{a}\rangle$ and $|\mathbf{b}\rangle$. If you measure the qubit in the basis $\{|\mathbf{a}\rangle, |\mathbf{b}\rangle\}$, what states can you get and with what probabilities? 12

b) Consider the two-qubit state : $1/4\ |00\rangle + 1/2\ |01\rangle + 1/\sqrt{2}\ |10\rangle + \sqrt{3}/4\ |11\rangle$.
If you measure (i) only the left qubit, (ii) only the right qubit, (iii) both the qubits simultaneously, what are the resulting states, and with what probabilities? 8

2. a) A qubit is in the state: $((1 - i) / 2\sqrt{2})\ |0\rangle + \sqrt{3}/2\ |1\rangle$.
Where on the **Bloch sphere** is this state? Give your answer in Cartesian (x, y, z) coordinates by the method of computing the "expected value of an observable". Sketch the point on the Bloch sphere. 10

b) Show that the following two states are located on the opposite points of the Bloch sphere: $|\mathbf{a}\rangle = \sqrt{3}/2\ |0\rangle + i/2\ |1\rangle$ and $|\mathbf{b}\rangle = i/2\ |0\rangle + \sqrt{3}/2\ |1\rangle$. 10

3. a) Give a quantum circuit that will convert the **Bell state** $|B00\rangle$ to $|B01\rangle$. Also, give a quantum circuit to detect the Bell state $|B11\rangle$. Discuss how these circuits can be used in **superdense coding.** 10

b) What is **quantum entanglement**? Discuss with the help of a quantum circuit how the principle of quantum entanglement may be used in **quantum teleportation.** 10

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4. a) Discuss the "**two beam splitters** " interference experiment and give its quantum explanation. 10

b) Explain in detail the **Deutsch-Jozsa algorithm** along with the **quantum oracle** used in it. 10

5. a) Describe the action of the **Toffoli gate** on the basis states and hence obtain the matrix representation of the same. Express this matrix as a sum of **outer products** of basis states. Prove also the **unitarity property** of the gate. 10

b) Is the classical full adder circuit **reversible**? If not, turn it into a reversible circuit and hence obtain a quantum version of the same circuit. 10

5. (OR) Consider the quantum circuit shown below and answer the following.

a) Obtain the output through the linear map method.
b) Obtain the matrix equivalent of the complete circuit.
c) Compute the output using the matrix obtained in (b).
d) Show the corresponding reverse circuit.
e) Obtain the matrix equivalent of the reverse circuit.

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