Ref. No. Ex/PG/ETC/PE/T/113F/2025

M.ETCE First Year First Semester Examination, 2025

Full Marks: 100 **Time: 3 Hours**

Nonlinear Dynamics, Control and Chaos Theory

Answer any FOUR Questions.

1. A first-order system is represented by a differential equation

$$\dot{x}(t) = x(t) + u(t).$$

Given the optimizing criterion

$$J = \frac{1}{4}x^2(T) + \int_0^T \frac{1}{4}u^2(t)dt,$$

where T is final time.

Determine the optimal control law and hence determine the optimized system dynamics. You may use the well-known Hamilton-Jacobi-Bellam equation given below, the parameters of which are standard.

$$J_t^*(x(t),t) + H(x(t),u^*(x(t),J_x^*,t),J_x^*,t) = 0$$ [25]

2. Given a discrete system dynamics:-

$$\vec{x}(k+1)=A\vec{x}(k)+B\vec{u}(k)$$

Where, $A = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.2 \end{bmatrix}$
and $B^T = \begin{bmatrix} 1 & 1 \end{bmatrix}$.

Determine the control law u^0(k) so that Q=I, V(x)=$\vec{x}^T(k)P\vec{x}(k)$ and the solution of P is obtained from:

$$-Q = A^T PA - P$$

a) Find the positive definite matrix P.
b) Compute optimal feedback $G = (B^T PB)^{-1} B^T PA$ and hence the optimal control signal $u^0(k)$.
c) Prove the optimal state-feedback equation. [7+8+10]

3. Given matrix Ricatti Equation:

$$\dot{P}(t) + Q(t) - P(t)BR^{-1}B^T P(t) + P(t)A + A^T P(t) = 0$$

with boundary condition: $P(t_f) = 0$, where the parameters used have usual meaning.
Use the Ricatti equation to determine the optimal control law for the system:

$$\dot{\vec{x}}(t) = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} \vec{x}(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t)$$

[Turn over

in order to optimize: $J = \int_0^\infty (\vec{x}^T \vec{x}(t) + u^2(t))dt$. [25]

4. Consider Fig. 1 comprising one non-linear element $u=g(e)$. Given $r=0$ and $c=-e$. Assuming $x_1=e$, $x_2 = \dot{x_1}$, find the Jacobian matrix J. Assuming P to be positive definite matrix, determine Q = J^TP+PJ.
   Determine the condition for asymptotic stability of the dynamics by testing positive definiteness of -Q. Hence determine the region of asymptotic stability of $x_1(t)$.

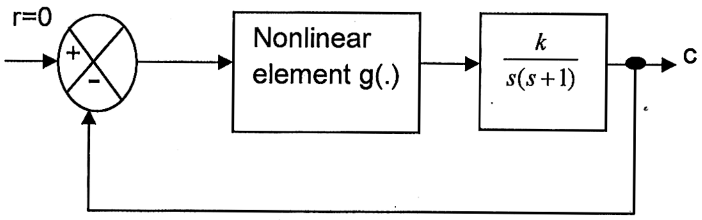

Fig. 1 [10+15]

5. a) Explain the essence of Sliding Mode Control.

   b) Draw a schematic block diagram of a Quadrotor control using Sliding Mode technology. What is the importance of ESO and LFDC in the system?

   c) How will you replace the ESO in the above block diagram using Fuzzy Logic?

   d) Show that the fuzzy controller is unconditionally stable. [4+8+7+6]

6. a) What is Duffing oscillator? Write down the dynamics of a non-linear spring.

   b) What is a Lyapunov exponent? What is the condition of chaos in a dynamical system?

   c) Draw one illustrative phase portrait of a stable, unstable, chaotic and limit cycle dynamic system.

   d) Also draw the corresponding time-response of the system.

   e) Show that linear dynamics can never have chaos. [5+5+5+5+5]