Ex/PG/ETC/PE/T/113A/2025

M.E. ELECTRONICS AND TELE-COMMUNICATION ENGINEERING
FIRST YEAR FIRST SEMESTER EXAM - 2025

Advanced Algorithms **Time: 3 hours** **Full Marks: 100**

Answer All Questions. Please answer all parts of the same question in the same place.

MODULE 1 (Based on CO 1)

Q.1 (a) Prove that for any two functions $f(n)$ and $g(n)$, we have $f(n) = \Theta(g(n))$ iff $f(n) = O(g(n))$ and $f(n) = \Omega(g(n))$. 5

(b) Prove or disprove $f(n) = O(g(n))$ implies $\log(f(n)) = O(\log(g(n)))$, where $\log(g(n)) \geq 1, f(n) \geq 1$ for all sufficiently large n. Assume $f(n)$ and $g(n)$ are asymptotically positive functions. 5

(c) Show that the solution to the recurrence relation $T(n) = T\left(\lfloor \frac{n}{2} \rfloor\right) + 1$ is $O(log(n))$. Use substitution method. 10

OR

(a) Define the three asymptotic notations O, Θ, and Ω, which are used in the analysis of algorithmic complexity. 6

(b) Find the solution to the recurrence relation $T(n) = 3T\left(\frac{n}{2}\right) + n$ using the recursion tree approach. 10

(c) Apply Master Theorem to solve the recurrence relation in (b). 4

MODULE 2 (Based on CO 2)

Q.2 (a) Define a heap with an example. 2

(b) Write an algorithm for building a max-heap, and analyze its complexity 6+2

(c) Using the algorithm in (b), build a step-by-step max-heap for the following data: 10
4, 3, 12, 16, 9, 10, 10, 14

OR

(a) Define a binary search tree with an example. 2

(b) Build a step-by-step binary search tree for the following data: 5
4, 3, 12, 16, 9, 10, 10, 14

(c) Write an algorithm for sorting the binary search tree-based data, and analyze its complexity. 5+2

(d) Apply the algorithm in (c) to sort the data in (b). 6

MODULE 3 (Based on CO 3)

Q.3 (a) What is an optimal prefix code? Give examples. 2

(b) Write an algorithm for construction of a Huffman tree to represent optimal prefix codes. 5

(c) Show a step-by-step Huffman tree for the following table using the algorithm in (b): 6

Alphabets	a	b	c	d	e	f
Frequency (in thousands)	35	12	13	11	20	9

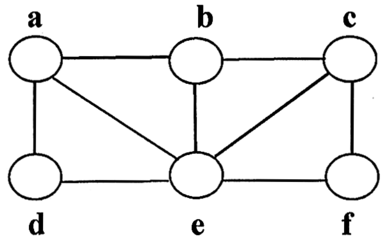
(d) State and prove the lemma validating the greedy choice used in the algorithm (b). 2+5

MODULE 4 (Based on CO 4)

Q.4 (a) Define Minimum Spanning Tree (MST). Why obtaining a MST can be useful? 2+1

(b) Write Prim's algorithm for obtaining MST, and analyze its complexity. 5+2

(c) Apply the algorithm in (b) to obtain a MST in the following weighted graph. Show your steps. 10



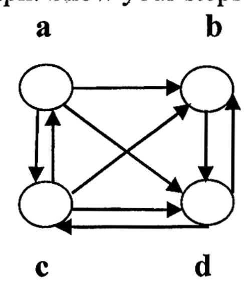
Consider the node '**a**' as the root vertex. The edge weights are as follows:
$w(e_{ab}) = 4$, $w(e_{ad}) = 3$, $w(e_{ae}) = 6$, $w(e_{bc}) = 5$, $w(e_{be}) = 3$, $w(e_{ce}) = 6$, $w(e_{cf}) = 4$, $w(e_{de}) = 8$, $w(e_{ef}) = 7$.

OR

(a) Explain the Single Source Shortest Path problem in a graph. Why solving such a problem can be useful? 2+1

(b) Write Dijkstra's algorithm for solving the problem in (a), and analyze its complexity. 5+2

(c) Apply the algorithm in (b) to obtain the shortest paths from a single source in the following weighted directed graph. Show your steps. 10



Consider the node '**a**' as the source. The edge weights are as follows:
$w(e_{ab}) = 3$, $w(e_{ac}) = 2$, $w(e_{ad}) = 6$, $w(e_{bd}) = 2$, $w(e_{ca}) = 4$, $w(e_{cb}) = 6$, $w(e_{cd}) = 3$, $w(e_{db}) = 3$, $w(e_{dc}) = 4$.

MODULE 5 (Based on CO 5)

Q.5 (a) Differentiate between P and NP class of problems. Mention two algorithms each from the P and NP classes. 2+4

(b) Define the Travelling-Salesman Problem (TSP). Prove that TSP is NP-Complete. 1+8

(c) Write a 2-approximation algorithm for TSP. State its complexity. 4+1