Ref. No.: **Ex/PG/CTS/PE/T/123A/2025**

M.E. ELECTRICAL ENGINEERING FIRST YEAR SECOND SEMESTER EXAM 2025
&
MASTER IN CONTROL SYSTEM ENGG FIRST YEAR SECOND SEMESTER EXAM 2025

OPTIMAL AND ROBUST CONTROL

Time: Three Hours; Full Marks: 100

Answer any ***four*** questions
All questions carry equal marks

1. a) Explain the terms 'supremum' and 'infimum' with the help of appropriate examples. 7+8+10

   b) Find the 2-norm and ∞-norm of the following signal:

$$u(t) = \begin{cases} 0, & if \quad t \le 0 \\ 4t^3, & if \quad 0 < t \le 1 \\ 0, & if \quad t > 1 \end{cases}$$

   c) For the system with transfer function $G(s) = \dfrac{2(1+s)}{(1+0.1s)(1+0.2s)}$, find $\|G\|_\infty$.

2. a) Distinguish between structured and unstructured uncertainty. 4+4+2+15

   b) With the help of diagrams, explain the terms additive and multiplicative unstructured uncertainty.

   c) State one limitation of Kharitonov's theorem.

   d) For a system whose characteristic polynomial is given by

$$s^3 + (a^2 - 1)s^2 + 2as + a = 0,$$

   where $a = 2$, check for the robust stability of the system for $\pm 10\%$ variation in each of the coefficients.

[Turn over

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3. a) A process plant given by $G_1(s)=\dfrac{10}{(s+1)(0.001s+1)}$ is modeled by using the transfer function $G_2(s)=\dfrac{10}{s+1}$. (8+7)+10

Analyze the suitability of using the above model for the given process plant on the basis of (i) the open loop unit step responses, (ii) the frequency responses (Bode plots) of the plant and its model.

b) For a system with plant transfer function P(s) and controller transfer function C(s), show that the largest value of complementary sensitivity is given by

$$M_t=\left\|\frac{PC}{1+PC}\right\|_\infty$$

c) Briefly explain how the method of M-circles may be used to calculate the permissible values of gain and phase variations for a system in the presence of uncertainty.

4. a) For the system shown in Fig. 1, with plant $P(s)$, controller $C(s)$, $F(s)=1$ and $r=0$, derive the expression for $\|H(P,C)\|_\infty$ when the input is chosen as $w=[-n,\ d]^T$ and the output is chosen as $z=[x,\ v]^T$. 10+15

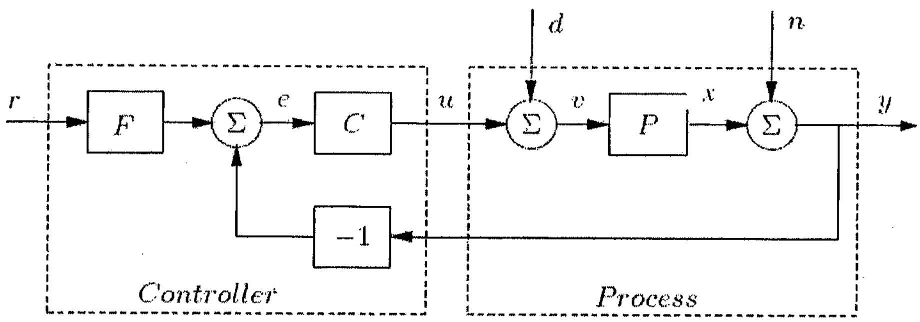

Fig. 1

b) For the closed loop system in Q 4 (a), given that the plant transfer function is $P(s)=\dfrac{1}{1+0.2s}$ and the controller transfer function is $C(s)=2$, find the value of $\|H(P,C)\|_\infty$.

5. a) Explain what you understand by the term *Quadratic performance index* with the help of an example. 5+(10+10)

b) A regulator contains a plant that is described by

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = -x_1 - 2x_2 + u$$
$$y = x_1$$

and has a performance index given by

$$J = \int_0^\infty \left\{ x_1^2 + x_2^2 + u^2 \right\} dt$$

(i) Find the elements of the Riccati matrix P in the steady state.

(ii) Design an optimum controller for the above system.

6. a) Write short notes on (i) the isoperimetric problem (ii) the geodesic problem. (5+5)+7+8

b) Derive an expression for the time of descent of a particle in a Brachistochrone problem.

c) Find the curve which is the solution to the above Brachistochrone problem, given initial position is $A(x_1, y_1)$ and final position is $B(x_2, y_2)$.

7 a) Given a functional $J = \int_a^b F\left(x; y; y'\right) dx$. For the fixed end-point problem, prove that the necessary condition to achieve an extremum of the functional is $F_y - \frac{d}{dx} F_{y'} = 0$. 7+(6+6+6)

b) Using the Euler equation find the extremal for the following functionals

(i) $J = \int_a^b \left(3x + \sqrt{y'}\right) dx$

(ii) $J = \int_a^b \left(xy' + y'^2\right) dx$

(iii) $J = \int_a^b \left(xy + 2y'^2\right) dx$