Ref. No.: Ex/PG/MEC/PC/T/111A/2025

M. E. MECHANICAL ENGINEERING FIRST YEAR FIRST SEMESTER EXAMINATION - 2025

SUBJECT: THEORY OF ELASTICITY

Time: 3 Hours **Full Marks: 100**

Notations/Symbols carry its usual meaning
Any missing data/information may be assumed with suitable justification

ALL QUESTIONS CARRY EQUAL MARKS

ANSWER ANY FOUR QUESTIONS

Q1. **[10+10+5]**

(a) Derive Cauchy's equations for stress.

(b) Show that the octahedral normal stress (σ_{oct}) and octahedral shear stress (τ_{oct}) in terms of stress invariants are respectively given by: $\sigma_{oct} = \frac{I_1}{3}$ and $\tau_{oct} = \frac{\sqrt{2}}{3}\left(I_1^2 - 3I_2\right)^{1/2}$.

(c) For the following state of stress (in kPa), find the normal and shear stress components on a plane given by the direction cosines $(1/2, \sqrt{3}/2, 1/2)$:

$$\begin{bmatrix} 1 & 2 & 1 \\ 2 & -2 & -3 \\ 1 & -3 & 4 \end{bmatrix}.$$

Q2. **[10+15]**

(a) Write down the differential equations of equilibrium in cylindrical coordinates. Derive any one of them.

(b) Show that the normal strain (ε) at any point in any given direction (direction cosines: l, m, n) is given by, $\varepsilon = \varepsilon_{xx}l^2 + \varepsilon_{yy}m^2 + \varepsilon_{zz}n^2 + \gamma_{xy}lm + \gamma_{yz}mn + \gamma_{xz}ln$.

Q3. **[10+10+5]**

(a) For constant body forces, show that the differential equations of equilibrium in rectangular Cartesian coordinates are given by: $\nabla^2(e) = 0$, where, e is the volumetric strain.

(b) Assuming a second degree polynomial for Airy's stress function, determine the stress fields. Explain the nature of the derived stress fields and show it on the boundary of a thin rectangular plate.

(c) Explain plane strain problems of elasticity.

[Turn over

Q4. **[12+13]**

(a) Derive the components of shear stress induced in a uniform elliptical bar under torsional loading.

(b) Derive the stress fields for a thin rotating circular disk. Using them, find the stress fields for a rotating solid disk.

Q5. Write short notes on the following: **[5 × 5]**

(a) State of stress.

(b) Stress invariants.

(c) Compatibility relations.

(d) Generalized Hooke's law.

(e) Field equations of elasticity.

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