Ex/PG/MEC/PC/T/115A/2025

M.E. MECHANICAL ENGINEERING FIRST YEAR FIRST SEMESTER EXAM – 2025

Subject: PRINCIPLES AND APPLICACATIONS OF LINEAR CONTROL THEORY

Time: Three Hours **Full Marks: 100**

Attempt All Questions

Any missing information may be suitably assumed with appropriate justification.

Q1. Simplifying the block diagram shown in **Fig. Q1**, obtain the expression of the overall transfer function $\frac{C(s)}{R(s)}$ of the corresponding system. **[10]**

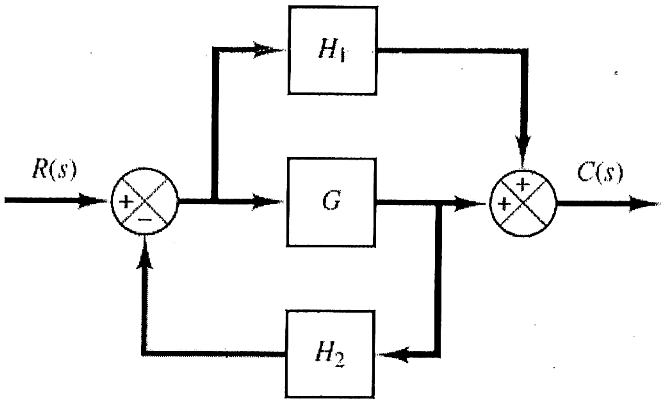

Fig. Q1

Q2. With a neat sketch obtain the transfer function of the angle of rotation ($\boldsymbol{\theta}$) to the armature input voltage ($\boldsymbol{V_a}$) for an armature-controlled D.C. motor. Use appropriate symbols for different variables and constants. Write the necessary equations. Mention the relevant assumptions (if any). **[15]**

Or,

With neat sketches and necessary block diagrams describe the operating principle of a hydraulic proportional controller using a zero-lapped pilot valve and two-port power cylinder. Mention relevant assumptions. **[15]**

Q3. Consider the closed loop system schematically shown in **Fig. Q3**.

(i) Obtain an expression of closed loop transfer function of the system. **[5]**

(ii) Determine the values of the damping ratio and the natural frequency of the closed loop system for $\boldsymbol{K} = \mathbf{8}$ and $\boldsymbol{a} = \mathbf{0}$. **[5]**

(iii) With $\boldsymbol{K} = \mathbf{8}$ and $\boldsymbol{a} = \mathbf{0}$, using final value theorem, determine of the steady-state error of the closed loop system due to unit ramp input. **[5]**

(iv) Determine the value of $\boldsymbol{a}$ so that damping ratio of the system will become **0.7**. With this value of the damping ratio what will be the steady-state error of the closed loop system due to unit ramp input?

[10]

[Turn over

(v) What will be the value of K for achieving the same value of steady-state error due to unit ramp input as obtained in part (iii) of the question, maintaining the damping ratio of the closed loop system at **0.7**?

[5]

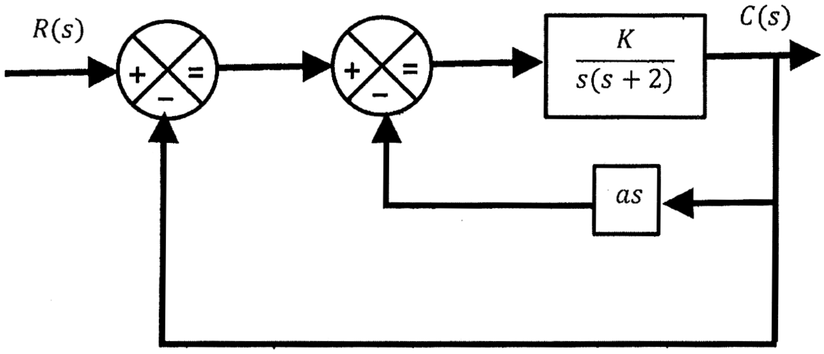

Fig. Q3

Q4. Consider the following closed loop system with unity feedback arrangement shown in **Fig Q4.** Obtain the characteristic equation of the closed loop system. Using Routh's stability criteria obtain the range of K, for which the closed loop system will remain stable. **[15]**

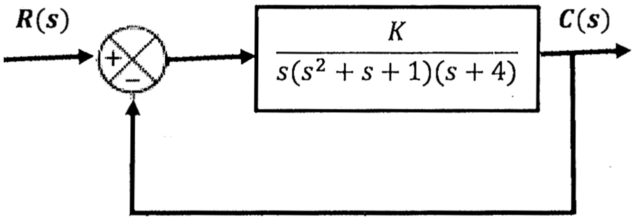

Fig. Q4

Q5. Consider the closed loop system shown in **Fig. Q5.**

(i) How many branches of root-loci will be possible for this system? **[1]**

(ii) Using the angle condition obtain the analytical equations of the root loci for the system. Using free-hand sketch draw the tentative plots of the root loci, indicating originating, terminating and break-away points. **[10]**

(iii) Obtain analytically the values of at break-away points of the root-loci branches. **[4]**

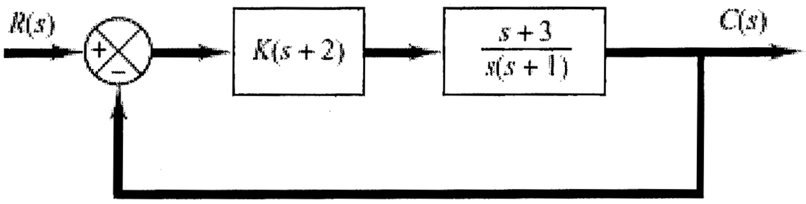

Fig. Q5

Q6. Consider the system represented by the transfer function $G(s) = \frac{200(s+1)}{(s+10)^2}$. Draw the asymptotic Bode-magnitude plot for the frequency response function of the system. Use **semi-log** graph paper for this purpose. Determine the corner frequencies and show them on the plot. **[15]**

---------------- **End of The Question Paper** --------------

Table of Laplace Transform Pairs

	$f(t)$	$F(s)$
1	Unit impulse $\delta(t)$	1
2	Unit step $1(t)$	$\frac{1}{s}$
3	t	$\frac{1}{s^2}$
4	$\frac{t^{n-1}}{(n-1)!} \quad (n = 1, 2, 3, \ldots)$	$\frac{1}{s^n}$
5	$t^n \quad (n = 1, 2, 3, \ldots)$	$\frac{n!}{s^{n+1}}$
6	e^{-at}	$\frac{1}{s+a}$
7	te^{-at}	$\frac{1}{(s+a)^2}$
8	$\frac{1}{(n-1)!}t^{n-1}e^{-at} \quad (n = 1, 2, 3, \ldots)$	$\frac{1}{(s+a)^n}$
9	$t^n e^{-at} \quad (n = 1, 2, 3, \ldots)$	$\frac{n!}{(s+a)^{n+1}}$
10	$\sin \omega t$	$\frac{\omega}{s^2+\omega^2}$
11	$\cos \omega t$	$\frac{s}{s^2+\omega^2}$
12	$\sinh \omega t$	$\frac{\omega}{s^2-\omega^2}$
13	$\cosh \omega t$	$\frac{s}{s^2-\omega^2}$
14	$\frac{1}{a}(1-e^{-at})$	$\frac{1}{s(s+a)}$
15	$\frac{1}{b-a}(e^{-at}-e^{-bt})$	$\frac{1}{(s+a)(s+b)}$
16	$\frac{1}{b-a}(be^{-bt}-ae^{-at})$	$\frac{s}{(s+a)(s+b)}$
17	$\frac{1}{ab}\left[1+\frac{1}{a-b}(be^{-at}-ae^{-bt})\right]$	$\frac{1}{s(s+a)(s+b)}$

18	$\frac{1}{a^2}(1 - e^{-at} - ate^{-at})$	$\frac{1}{s(s+a)^2}$
19	$\frac{1}{a^2}(at - 1 + e^{-at})$	$\frac{1}{s^2(s+a)}$
20	$e^{-at}\sin\omega t$	$\frac{\omega}{(s+a)^2+\omega^2}$
21	$e^{-at}\cos\omega t$	$\frac{s+a}{(s+a)^2+\omega^2}$
22	$\frac{\omega_n}{\sqrt{1-\zeta^2}}e^{-\zeta\omega_n t}\sin\omega_n\sqrt{1-\zeta^2}\,t \quad (0<\zeta<1)$	$\frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$
23	$-\frac{1}{\sqrt{1-\zeta^2}}e^{-\zeta\omega_n t}\sin(\omega_n\sqrt{1-\zeta^2}\,t-\phi)$ $\phi=\tan^{-1}\frac{\sqrt{1-\zeta^2}}{\zeta}$ $(0<\zeta<1,\ \ 0<\phi<\pi/2)$	$\frac{s}{s^2+2\zeta\omega_n s+\omega_n^2}$
24	$1-\frac{1}{\sqrt{1-\zeta^2}}e^{-\zeta\omega_n t}\sin(\omega_n\sqrt{1-\zeta^2}\,t+\phi)$ $\phi=\tan^{-1}\frac{\sqrt{1-\zeta^2}}{\zeta}$ $(0<\zeta<1,\ \ 0<\phi<\pi/2)$	$\frac{\omega_n^2}{s(s^2+2\zeta\omega_n s+\omega_n^2)}$
25	$1-\cos\omega t$	$\frac{\omega^2}{s(s^2+\omega^2)}$
26	$\omega t-\sin\omega t$	$\frac{\omega^3}{s^2(s^2+\omega^2)}$
27	$\sin\omega t-\omega t\cos\omega t$	$\frac{2\omega^3}{(s^2+\omega^2)^2}$
28	$\frac{1}{2\omega}t\sin\omega t$	$\frac{s}{(s^2+\omega^2)^2}$
29	$t\cos\omega t$	$\frac{s^2-\omega^2}{(s^2+\omega^2)^2}$
30	$\frac{1}{\omega_2^2-\omega_1^2}(\cos\omega_1 t-\cos\omega_2 t) \qquad (\omega_1^2\neq\omega_2^2)$	$\frac{s}{(s^2+\omega_1^2)(s^2+\omega_2^2)}$
31	$\frac{1}{2\omega}(\sin\omega t+\omega t\cos\omega t)$	$\frac{s^2}{(s^2+\omega^2)^2}$