Ref. No.: Ex/PG/MEC/PC/T/112A/2025

M.E. MECHANICAL ENGINEERING FIRST YEAR FIRST SEMESTER EXAM 2025

SUBJECT: MECHANICAL VIBRATION

Time: 3 Hours **Full Marks: (20 X 5) =100**

Any missing data may be assumed with suitable justification

The symbols/notations carry its usual meanings

(Answer Any Five Questions)

Q1. (a) Write the differential equation of motion for the system shown in **Fig Q1(a)**. Also, determine the natural frequency of the damped oscillation and the critical damping coefficient. Assume the link to be massless. **(10)**

(b) Find the Fourier series expansion of the periodic function shown in **Fig. Q1(b).** **(10)**

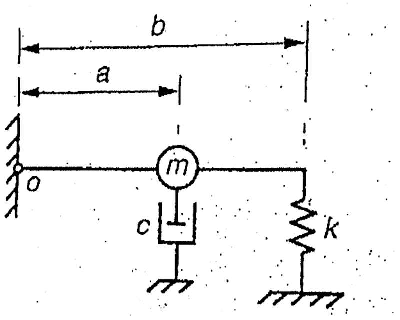

Fig. Q1(a)

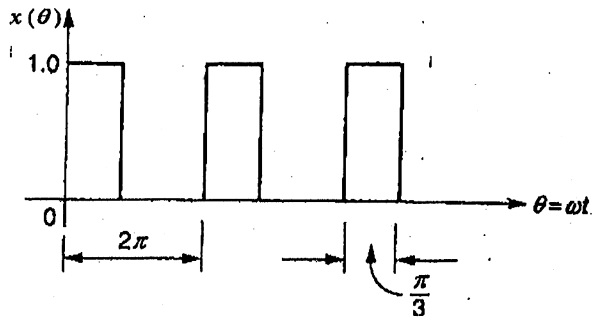

Fig. Q1(b)

Q2. (a) Fig. Q2(a) shows a mass block mounted on a spring and a Coulombic damper is oscilatted under forcing function $F = F_0 \sin \omega t$. Prove that the steady state response amplitude of the system is,

$$X = \frac{F_0}{k}\left[\frac{1-\left(\dfrac{4F_l}{\pi F_0}\right)^2}{\left\{1-\left(\dfrac{\omega}{\omega_n}\right)^2\right\}^2}\right]^{1/2}$$

(b) What is convolution integral? Using the integral, find the response history of the single-degree undamped oscillator for the forcing function, shown in **Fig. Q2(b).** **(10)**

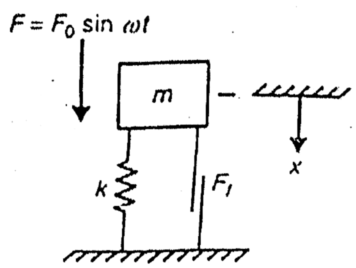

Fig. Q2(a)

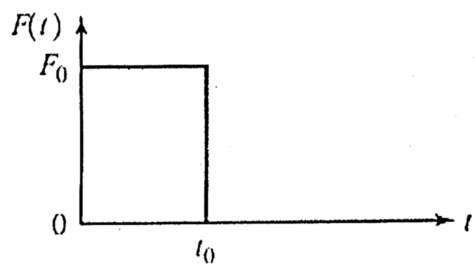

Fig. Q2(b)

[Turn over

Q3. Use static equilibrium to determine the stiffness matrix of the system shown in **Fig. Q3**. **(20)**

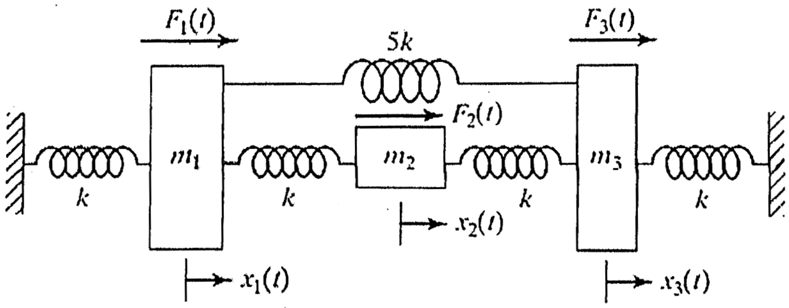

Fig. Q3

Q4. A machine tool, having a mass of m = 1000 kg and a mass moment of inertia J_0 = 300 kg-m^2, is supported on elastic supports, as shown in **Fig. Q4**. If the stiffnesses of the supports are given by k_1 = 3000 N/mm and k_2 = 2000 N/mm, and the supports are located at l_1 = 0.5 m and l_2 = 0.8 m, find the natural frequencies and mode shapes of the machine tool. **(20)**

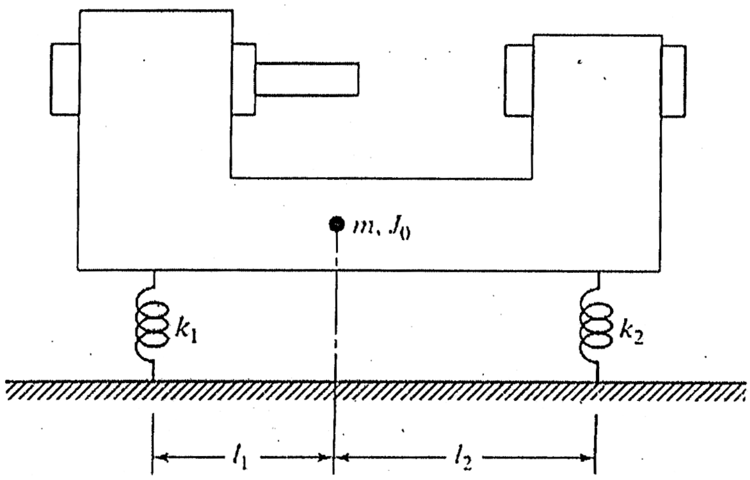

Fig. Q4

Q5. Using modal analysis, calculate the natural frequencies, mode shapes of the system whose equation of motion given below. Also, find the free-vibration responses of the individual masses of the system.

$$\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix} \begin{Bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{Bmatrix} + \begin{bmatrix} k_1 + k_2 & -k_2 \\ -k_2 & k_2 + k_3 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

Assume the following data: m_1 = 10, m_2 = 1, k_1 = 30, k_2 = 5, k_3 = 0, and

$$\vec{x}(0) = \begin{Bmatrix} x_1(0) \\ x_2(0) \end{Bmatrix} = \begin{Bmatrix} 1 \\ 0 \end{Bmatrix},\ \dot{\vec{x}}(0) = \begin{Bmatrix} \dot{x}_1(0) \\ \dot{x}_2(0) \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$ **(20)**

Q6. What is a string? Deduce the equation of motion of a string. Solve the equation to determine the natural frequencies and mode shape of a both ends fixed string. **(20)**

Q7. Prove the following relations in connection with the free transverse vibration of a beam with uniform cross-section:

(i) $EI\dfrac{\partial^4 w}{\partial x^4}(x,t) + \rho A\dfrac{\partial^2 w}{\partial t^2}(x,t) = 0$

(ii) $\int_0^l W_i(x) W_j(x)\, dx = 0$ **(10 +10)**

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