Ref. No.: Ex/PG/MEC/PC/T/114A/2025

Master of Mechanical Engineering First Year First Semester Examination 2025

Basics of Finite Element Method

Assume suitable values for any missing data

1. CO1: State the "Brachistochrone" problem and represent it mathematically. Derive the expression for Euler-Lagrange equation in connection to minimizing a 'functional'. [4+6]
2. CO2: From the differential equation $EAu'' + q(x) = 0 \forall x \in [0, L]$ and $u(0) = u(L) = 0$ derive the variational form. Also, derive the expressions for the element stiffness matrix, $[K_{el}]$, and element nodal force vector, $\{f_{el}\} = \{f_q\} + \{f_R\}$, of a bar element with quadratic shape functions, given $q(x) = q_0$. [7+8]
3. CO3: Answer any three.
   a. Show how the serendipity elements drastically reduce DOF count while offering higher order shape functions. [5]
   b. Explain why CST elements are not suitable for modeling bending. [5]
   c. Show that the $[B]$ matrix is a constant for a three node triangle element. [5]
   d. Compare p-refinement vs. h-refinement in terms of computer application. [5]
4. CO4: Answer any three.
   a. Derive the expressions of shape functions for any two of the following.[10]
      i. Six node triangle
      ii. Eight node quadrilateral
      iii. Nine node quadrilateral
   b. Derive the expression for the traction force vector assuming unit thickness. What is the significance of a consistent force vector? [6+4]
   c. Derive the expression for the Jacobian of the quadrilateral element. Is its determinant constant? [7+3]
   d. Derive the variational formulation for the following strong form. [10]
      i. $\nabla . \boldsymbol{\sigma} = \mathbf{0} \in \Omega$
      ii. $\sigma_{ij} = \sum_{k=1}^{3} \sum_{l=1}^{3} \mathbb{D}_{ijkl} \varepsilon_{kl}$
      iii. $\varepsilon_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)$
      iv. $\boldsymbol{u} = \boldsymbol{u}^* \in \Gamma_u$ and $\boldsymbol{t} = \boldsymbol{t}^* \in \Gamma_t$
      v. $\Gamma_u \cup \Gamma_u = \Gamma$ and $\Gamma_u \cap \Gamma_t = \Phi$

[Turn over

5. CO5: Answer any three.

   a. With mathematical derivations show the occurrence of hourglass modes for reduced integration. How are these modes problematic? Discuss about one possible method for prevention of these modes. [5+2+3]

   b. Show how volumetric locking happens in an FE mesh of linear quad elements for a nearly incompressible material ($\nu \to 0.5$). Name atleast two ways this can be prevented. [8+2]

   c. Derive the values of the weights and coordinates of the integration points in a two-point Gauss quadrature stating the basic principles. What is the two-dimensional equivalent of the same in terms of number of integration points, their coordinates and weights? [6+4]

   d. Show that the $[B]$ matrix of a four-node quadrilateral element is linear in its natural coordinates (ξ, η) if the element is a parallelogram in physical coordinates. What are a full and a reduced integration? [6+4]

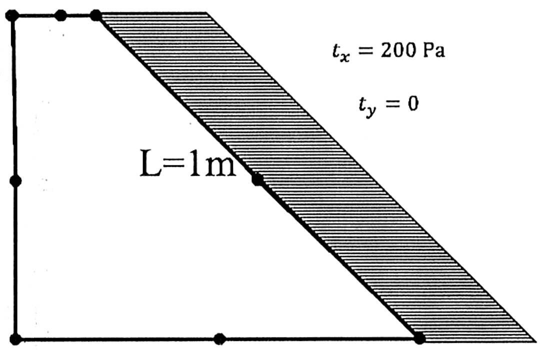

Fig. 4b

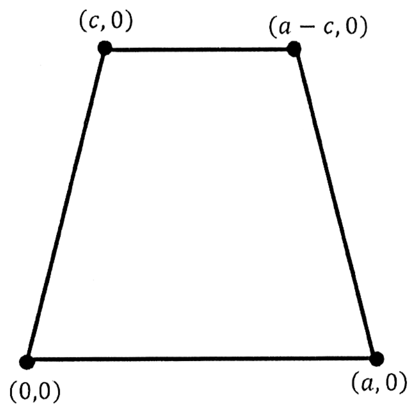

Fig. 4c

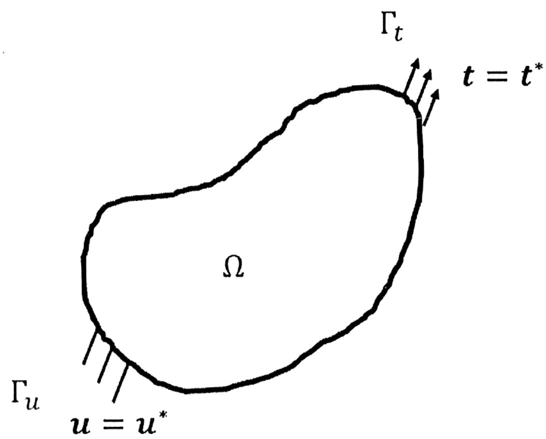

Fig. 4d