A CARDINAL UTILITY BASED APPROACH TO CONSUMER BEHAVIOUR : AN EXPLORATION

Thesis Submitted to Jadavpur University, West Bengal in Partial Fulfillment of the Requirements for the Degree of Master of Philosophy in Economics

Prepared and Submitted by:

SOUMYA GHOSH

Registration No: 114830 of 2011-12

Year: 2019

Supervised by:

Dr. Siddhartha Mitra

Professor

Department of Economics

Jadavpur University

Kolkata-700032

যাদবপুর বিশ্ববিদ্যালয়
কলকাতা-৭০০০৩২, ভারত

*JADAVPUR UNIVERSITY
KOLKATA-700 032, INDIA

FACULTY OF ARTS : DEPARTMENT OF ECONOMICS

To Whom It May Concern

This is to certify that **Mr. Soumya Ghosh** has completed his M.Phil. dissertation for the session 2018-19 under my supervision. His dissertation is entitled, "A Cardinal Utility Based Approach to Consumer Behaviour : An Exploration" and it is now ready for submission at Jadavpur University in partial fulfillment of the requirements for the degree of Master of Philosophy in Economics.

Malabika Roy 22.4.19

Prof. Malabika Roy
Professor and Head
Department of Economics
Jadavpur University
Kolkata-700032

Siddhartha Mitra 22.4.19

Prof. Siddhartha Mitra
Supervisor and Professor
Department of Economics
Jadavpur University
Kolkata-700032

Professor & Head
Department of Economics
Jadavpur University
Kolkata-700032

Professor
DEPARTMENT OF ECONOMICS
JADAVPUR UNIVERSITY
KOLKATA-700 032

Acknowledgements

At the outset, I express my heartfelt gratitude to my supervisor Prof. (Dr.) Siddhartha Mitra. His role is more than that of a mentor. His constant encouragements, valuable advice, guidance and support from the very inception till the completion of the thesis enabled me to develop an intense understanding of the subject. I feel pleasure to complete my M.Phil. dissertation under his supervision.

I am greatly indebted to Dr. Partha Chatterjee who has allowed and made it convenient to continue my studies. I am also indebted to the other faculty members and non-teaching staffs especially Prof. Swapnendu Bandyopadhay, Prof. Tanmoyee Banerjee, Dr. Soma Mandal, Prof. Ambar Nath Ghosh, Prof. Sushil K.Halder, Prof. Arpita Ghose, Prof. Malabika Roy of the department of Economics and Prof. Subenoy Chakraborty, Prof. Abhijit Lahiri, Dr. Sukhendu Kar, Dr. Santosh Biswas of the department of Mathematics, Jadavpur University who have always extended their helping hands towards me whenever I required. I convey my sincere thanks to Sudipto da and Paramita di, research scholars of Jadavpur University for their assistance and co-operation. I would like to thank Dr. A.K.Sarkar, Partha Sen, Dr. Sudeshna Chakraborty, Anupama Sengupta, Sudipta Mandal, Gourab Das, A.N.Singh, Prabir sir, Suparna madam, Dr. Sukanta Ghosh, A.Nag, Ghonoshree Bagh, Subhendu da, Shanti da, Soumendra da, A.Saha, B.Sarder, T.K.Nandi. I am grateful to my mother for her constant support and motivation. Last but not the least, I would like to thank my friends both in Jadavpur University and beyond.

SOUMYA GHOSH

Contents :- *Page No.*

Chapter 1:-

Introduction

Broadly speaking, there have been two approaches to the concept of utility and its applications: cardinal utility theory and ordinal utility theory. In regard to cardinal utility theory, one can distinguish between the early cardinalists (Jeremy Bentham, James Mill, John Stuart Mill, Francis Ysidro, Edgeworth etc.) who thought utility was a variable amenable to exact measurement such as length and time and the more recent cardinalists who believe that affine transformations of a utility function (which amounts to a change in the scale and origin for measurement of utility) leave preferences intact. Recently, there have been attempts to revive the old cardinal utility theory under the *'Back to Bentham'* movement, notable participants in this movement being the Nobel laureate, Daniel Kahneman and the entire neuroeconomics school [1,2].

This thesis is an attempt to establish firm foundations for consumer behaviour on the basis of the old school of cardinal utility theory. Note that the old cardinal school was discarded by economists in favour of the ordinal theory of utility because of two facts: a) the old cardinal school believed that utility was exactly measurable but was unable to find a way to measure it and; b) the ordinal school, which did not rely on the measurability of utility but used the utility function as a generator of numbers to rank commodity bundles [1], was quite successful in explaining consumer behaviour i.e. the consumer choice of the most preferred bundle which obviously remained the same when one utility function was replaced by another that corresponded to the same preference ordering i.e. ranking of commodity bundles.

It must be remembered though that prediction of consumer choice might not be the only function of a theory of utility. Take for example, the cost-benefit analysis of a policy change. The successful implementation of cost benefit analysis depends on our ability to obtain quantitative measures of utility, akin to what Bentham and his fellow early cardinalists desired, for various persons affected by the policy that possess the properties of a) comparability across persons and b) amenability to aggregation for generating a measure of welfare change brought about by the policy . It is for this reason, among others, that the search for functions that can measure cardinal utility and a theory relating to it remains relevant. Note that the early cardinal school implied that each utility function corresponded to a different set of preferences i.e. if one changed the utility function one always associated changed preferences with the changed utility function. This is because the preferences in cardinal utility theory not only indicated the ranking of commodity bundles but also other properties such as, for any three bundles A, B and C, the ratio of the change in utility in going from A to B to that in going from B to C. This, as mentioned, was not the case with ordinal utility theory, as explained above.

[1]Two utility functions that produced different numbers for indexing a given bundle but led to rankings of commodity bundles that were identical were assumed to correspond to the same set of preferences.

The indifference curve theory, which was a refinement under the ordinal school, showed how consumers arrived at the choice of the optimal consumption bundle. Only two assumptions are crucial to the indifference curve theory: a) 'more is better' implying that a commodity bundle associated with more of at least one commodity and no less of the other(s) than another commodity bundle is always preferred to the latter; and b) diminishing marginal rate of substitution which implies that the negative slope of an indifference curve[2] in the two goods diminishes in magnitude as the quantity of the good measured on the horizontal axis increases i.e. the indifference curve is convex to the origin. The negative slope of the indifference curve, its convexity to the origin and the property that one moves to more preferred commodity bundles as one moves in a north-easterly direction or a easterly direction from one indifference curve to another, again implied by assumption a), were enough to guarantee that the point of tangency between the budget line and the highest possible indifference curve corresponded to consumer choice of the most preferred commodity bundle, if such a point of tangency existed.[3]Moreover, this consumer equilibrium obviously did not change if one replaced the utility function with another which preserved the rankings of commodity bundles obtained from the first utility function i.e. one utility function was a positive monotonic transformation of the other. Thus, the ordinal school presented a robust analysis of consumer behaviour without making any demands regarding the measurability of utility. But as pointed out earlier, the application of a utility theory might lie beyond the prediction of consumer choice in the realm of interpersonal comparison of utilities and their aggregation. These applications require measurement of utility in the way that early cardinalists desired.

This thesis can be termed as part of a programme which attempts to prevent throwing of the baby of early cardinal utility theory with the bath water. No comments will be made in this thesis regarding how utility can be measured exactly, compared interpersonally, or aggregated. This part of the programme to revive the early cardinal school lies in the capable hands of Daniel Kahneman and neuroeconomists, among others [3,4]. Rather this thesis will derive an alternative robust theory of consumer choice, based on a refinement of the cardinalist concept of marginal utility schedules (referred to as 'budget constrained marginal utility') and the associated law of diminishing marginal utility, which would make the same predictions regarding consumer choice as those made by the indifference curve theory. After pointing out the inadequacies of the early cardinal theory of consumer behaviour with examples, the concept of Budget Constrained Marginal Utility Schedules (henceforth BCMUSs) has been introduced and various possible BCMUSs have been derived under different set of conditions in the second chapter of the thesis. In the third chapter, the concept of BCMUS has been used to derive consumer equilibria under different conditions. Prices of both goods in the 2 good case are fixed at unity throughout this chapter except at the very end by choosing units suitably. The obvious drawback of this approach

[2]An indifference curve, defined as a locus of commodity bundles among which the consumer is indifferent, is obviously negatively sloping, given a 2 good world, because of assumption a).

[3]It is however quite possible that there is no such point of tangency – this happens when every point on the budget line is associated with an indifference curve which is steeper or flatter than the budget line at that point.

is that the impact of changing relative prices on consumer equilibrium cannot be analysed. However, the special assumptions made here simplify the depiction of consumer equilibrium and render it easily understandable without robbing it of its essence. In the fourth chapter, the concept of BCMUS has been discussed algebraically and several propositions have been derived in regard to their nature and the equilibria these imply. In the fifth chapter, the concept of consumer equilibrium has been described using BCMUSs even under the assumption of non-unit prices of commodities and under differing assumptions regarding nature of utility function. The results found from the entire analysis have been discussed in the sixth and last chapter.

Chapter 2:-

Derivation of Budget Constrained Marginal Utility Schedule as a Response to the Inadequacies of the Early Cardinal Theory of Consumer Behaviour

The contributions of the early cardinalists lay in the fact that they suggested that utility could be measurable; the introduction of the concept of marginal utility or the utility derived from an additional unit of the commodity without any change in the consumption of other commodities or more precisely, the utility addition per unit increase of the commodity measured on the basis of an infinitesimally small increase in the consumption of that commodity, keeping the consumption of other commodities constant; and finally the law of diminishing marginal utility which said that successive equal increments in the consumption of a commodity, keeping the consumption of other commodities constant, were associated with diminishing additions to utility. There was a crude attempt to predict consumer choice by saying that consumer equilibrium or the maximization of utility was obtained where the marginal utility to price ratio was equated across commodities. Two points deserve mention here. First, this condition happens to be a half truth and might not be associated with consumer equilibrium unless it is accompanied by other assumptions.

For example, in a two commodity world, consumer choice according to early cardinalists is characterized by $\frac{MU_X}{P_X} = \frac{MU_Y}{P_Y}$ -- where X and Y are the two commodities i.e. the consumer allocates his/her expenditure so that the utility gained from the last dollar spent on each commodity is equal [5]. Each consumer demands each commodity upto the point at which the marginal utility of a dollar spent on it is the same as the marginal utility of a dollar spent on every other commodity. MU_irefers to the change in utility per unit change in commodity i but for a change which is infinitesimally small; P_i refers to the price of commodity *i* and therefore $\frac{MU_i}{P_i}$ refers to the change in utility per unit increase in expenditure on commodity i for infinitesimally small increase in expenditure on commodity i. However, as mentioned, this might not be associated with the choice of the utility maximizing consumer bundle. This absence of association might be true for cases where the law of diminishing marginal utility holds and where it does not hold. Second, this condition could be associated often with the utility maximizing consumer bundle, even where the law of diminishing marginal utility does not hold. This thesis helps us to understand the analytical basis of these two assertions.

Rearranging the terms of $\frac{MU_X}{P_X} = \frac{MU_Y}{P_Y}$ in the following manner yields additional insight into consumer behaviour [6] :

$$\frac{MU_X}{MU_Y} = \frac{P_X}{P_Y}$$

The relative price of the two commodities, i.e. the right hand side of the above equation, is market-determined and exogenous to the individual consumers. Let us assume that each consumer chooses the commodity bundle such that the ratio of her marginal utilities for x and y is equated to the corresponding price ratio and all her income is spent. Let us call this bundle the assumed equilibrium. Note that the assumed equilibrium might not be the actual equilibrium if a small increase in X accompanied by a small decrease in Y, such that the consumer remains on the budget line, is quite successful in enhancing total utility. First, note that the search for the utility maximizing commodity bundle should only be restricted to the budget line as long as marginal utility from consumption of both commodities is always positive. Now consider the change in utility occurring in 2 steps from the above equilibrium – first, the amount of X increases by a small amount keeping the amount of Y constant; second, the amount of Y decreases by appropriate amount at the changed consumption level of X so that the mentioned increase in X coupled with a decrease in Y brings the consumer back to the budget line. Note that there is an increase in utility associated with the first step given by $MU_X \Delta X$, where ΔX is the increase in X and a decrease in utility associated with the decrease in Y through the second step given by $MU_Y\,\Delta X\,\frac{P_X}{P_Y}$. The algebraic sum of the mentioned utility gain and loss corresponding to the two steps tends to 0 as ΔX tends to zero but for finite, small and positive ΔX this might not be the case. MU_Y and MU_X might have both changed from the levels corresponding to the assumed equilibrium.

For example, if we assume that there is a negative complementarity between X and Y in regard to utility (that is the cross partial derivative of the utility function is negative), the value of MU_Y at the enhanced level of X will be less than the value of MU_Y at the assumed equilibrium and therefore $MU_Y\,\Delta X\,\frac{P_X}{P_Y}$ will be less than $MU_X \Delta X$. If we repeat the mentioned couple of steps under the assumption of positive complementarity and increasing marginal utility of X and Y, the assumed equilibrium might still be not an 'equilibrium'.

Of course, if the validity of the law of diminishing marginal utility is accompanied by positive complementarity between the two commodities, the mentioned equality would correspond to the equilibrium condition, a conclusion which the readers would grasp intuitively but would still be explained in detail when we discuss the concept of the 'Budget Constrained Marginal Utility Schedule'(BCMUS).

It also deserves mention that the 'law of diminishing marginal utility' is not a sacred cow for the practitioners of indifference curve theory under the ordinal school of utility. This is because increasing marginal utility is perfectly compatible in many cases with diminishing marginal rate of substitution (MRS) along the indifference curve (IC) where MRS is defined as the absolute value of the slope of the IC. Note that the same equilibrium condition, as mentioned before for the cardinal case, applies for the ordinal case ($\left|\frac{dY}{dX}\right| = \frac{MU_X}{MU_Y} = \frac{P_X}{P_Y}$), with diminishing MRS implying that ICs are convex to the origin. This property ensures that the point of tangency of the highest possible IC and the budget line, if it exists, satisfies the equality condition mentioned immediately above. Such an equality also implies that there is an interior solution with positive quantities of both goods being consumed. In case such a point of tangency does not exist we have a corner solution with only one of the two goods being consumed in positive quantities and the MRS of the highest IC on the budget line being that at one of the corners of the budget line and exceeding or falling short of the slope of the budget line.

We illustrate our argument through a few cases.

*Case 1:

Consider the utility function, $U = X^2Y^2$.

The reader can easily see that this case corresponds to positive as well as increasing marginal utility for both X and Y. The MRS is given by $\frac{Y}{X}$ which obviously diminishes as X increases along the IC, given its negative slope. Thus diminishing MRS $\nRightarrow$diminishing marginal utility and in this case $MRS_{XY} = \frac{MU_X}{MU_Y} = \frac{P_X}{P_Y}$ is a valid equilibrium condition.

*Case 2: Suppose the utility function is $U = XY$ which is a positive monotonic transformation of the utility function in case 1. This is a case of constant marginal utility with the MRS again being given by $\frac{Y}{X}$ and therefore resulting in the same conditions for consumer equilibrium as in Case 1. This can also be checked by noting that constant level of X^2Y^2, as is true of an indifference curve for the first case, is equivalent to a constant level of XY, the indifference curve for Case 2. As the indifference curve map is the same for both cases the consumer equilibrium in these cases will be identical.

This case also illustrates that diminishing MRS $\nRightarrow$ diminishing MU_X but the case $U = X^{1/2}Y^{1/2}$ also shows that diminishing MRS is perfectly compatible with decreasing marginal utility.

*Case 3: Suppose the two goods in question, X and Y, are perfect substitutes. Then the utility function of a consumer will be $U = aX + b$Y with MRS given by $\frac{a}{b}$, a constant. Thus, constant marginal utility is associated with constant MRS. In that case we again usually have a corner solution with the slope of the budget line exceeding or falling short of the slope of the indifference curve at the consumer equilibrium. Thus, no tangency point exists. An exception is the case where the linear indifference curve coincides with the budget line so that every allocation on the budget line is a consumer equilibrium giving the same level of utility to the consumer.

We know that increasing marginal utility is compatible with diminishing MRS, which implies that the equality of MRS and price ratio if that exists on the budget line is associated with consumer equilibrium. But how do we characterize such equilibria using cardinalist language and not the language of indifference curves which involves concepts such as 'marginal rate of substitution'? To understand this we have to note that the marginal utility schedule, constructed with the level of X changing and the level of Y remaining constant, cannot be directly applied to find consumer equilibrium on the budget line, the reason being that X is traded off against Y as we move along the budget line. Instead we can view this movement as the locus of points on shifting schedules of marginal utility normalized by price, each point corresponding to a different level of X and the shift occurring due to the change in Y which accompanies every such change in X such that each (X, Y) is always a point on the budget line. We call this $X - MU_X$ - locus, with X varying from 0 to $\frac{M}{P_X}$ where M is income, as the 'Budget Constrained Marginal Utility Schedule' (BCMUS) of X.

Thus, the different factors determining the slope of the BCMUS are (a) whether the marginal utility schedule is positively (negatively) sloping because of increasing (diminishing) marginal utility (b) whether there is positive (negative) complementarity between X and Y i.e. the case of positive (negative) cross partial derivatives characterising the utility function. A positive (negative) slope of the marginal utility schedule will tend to make the BCMUS upward (downward) sloping whereas positive (negative) complementarity will tend to make it downward (upward) sloping. As the diagrams below illustrate, the cases of constant or diminishing (increasing) marginal utility and positive or zero (negative) complementarity are associated with non-positive (positive) slope of the BCMUS. On the other hand, any combination of diminishing (increasing) marginal utility with negative (positive) complementarity will be associated with ambiguity regarding the sign of the slope of the BCMUS which will ultimately depend on how strong such complementarity is relative to the absolute value of the slope of the marginal utility schedule.

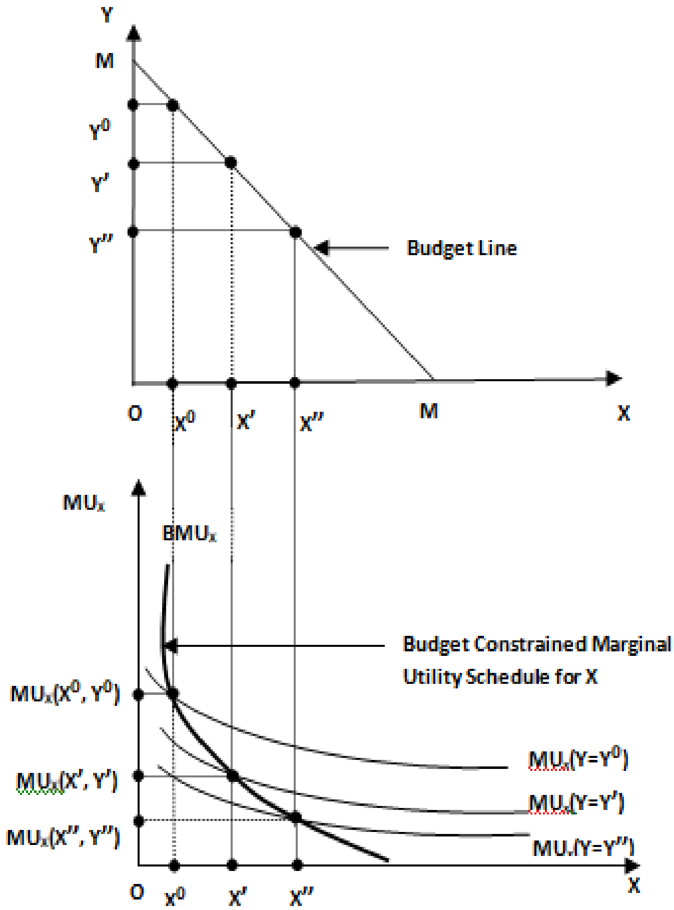

Figure 1: Budget constrained marginal utility schedule under the assumptions of (a) validity of law of diminishing marginal utility and (b) positive complementarity in regard to utility between the two commodities.

Figure 1 illustrates the tracing of the locus which we call the BCMUS. The diagram is divided into two panels, one above the other. In the upper panel we have the budget line. Each point on the budget line corresponds to unique quantities of X and Y. Thus, if we are at the point or consumer bundle (X^0, Y^0) on the budget line this maps onto a point associated with a consumption level of X = X^0 on the marginal utility schedule of X that corresponds to $Y = Y^0$. Now allow X to increase to $X^{'}$; there is automatically a decrease in the consumption of Y to $Y^{'}$as the budget constraint continues to hold with prices and income remaining constant. The new consumption bundle $(X^{'}, Y^{'})$ now maps onto a point associated with a consumption level of X = $X^{'}$on a different marginal utility schedule of X, that corresponding to $Y = Y^{'}$. A further increase in consumption of X to $X^{''}$ is accompanied by further decrease in consumption of Y to $Y^{''}$and the resulting bundle $(X^{''}, Y^{''})$ maps onto a point associated with a consumption level of X = $X^{''}$ on yet another marginal utility schedule of X, that corresponding to $Y = Y^{''}$. The BCMUS is the locus of the three mentioned points, each on a different marginal utility schedule of X (each marginal utility schedule corresponding to a different level of Y) as well as other points which may be obtained by trading off X against Y on the budget line and reading off the marginal utility of the associated consumption of X from the marginal utility schedule of X corresponding to the associated consumption of Y. In this way, each point/consumer bundle on the budget line corresponds to a point on a different marginal utility schedule of X, each corresponding to the level of consumption of Y associated with the mentioned consumer bundle. Thus, the BCMUS is a locus of points, with each point lying on a unique marginal utility schedule of X characterized by a unique assumed level of Y and corresponding to a unique level of X. Some additional points deserve mention here. First, in Fig.1, the marginal utility schedules for different levels of consumption of Y are drawn as downward sloping curves in the $X - MU_X$ space i.e. it is assumed that the law of diminishing marginal utility is obeyed. It is also observed that in Fig.1, an increase in the consumption of X, which is associated with a decrease in the consumption of Y, is associated with a downward shift of the marginal utility schedule of X. This is because of the assumption that a decrease in the consumption of Y leads to a decline in the marginal utility from a given consumption of X i.e. there are positive complementarities in regard to utility between X and Y or in other words, the partial derivative of the marginal utility of X with respect to Y is positive. The two assumptions (that of diminishing marginal utility of X for a given level of Y and positive complementarities in regard to utility between X and Y) reinforce each other and leads to a downward sloping BCMUS, one that is in fact steeper than the conventional or pure marginal utility schedule.

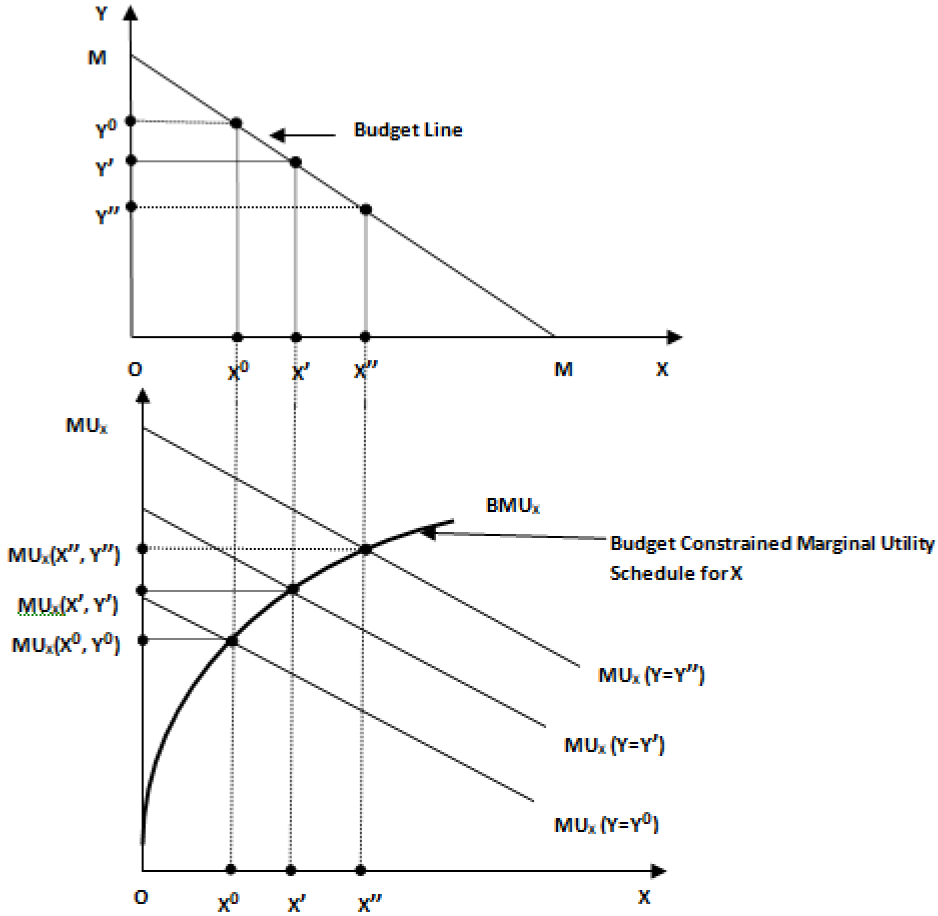

Figure 2: Upward sloping budget constrained marginal utility schedule of commodity X under the assumptions of (a) validity of the law of diminishing marginal utility, (b) negative complementarity in regard to utility between the two commodities i.e. X and Y and (c) positive net change of MU_X for all increases in consumption of X.

Fig. 2 presents the case where the validity of the law of diminishing marginal utility is coupled with negative complementarity in regard to utility between the two commodities X and Y. Thus, the level of MU_X corresponding to the consumer bundle (X^0, Y^0) on the budget line or more simply $(X^0, M - X^0)$ (see the upper panel of Fig. 2) can be read off from the point on the 'marginal utility schedule of X corresponding to $Y = Y^0$, that is associated with a level of consumption of X equalling X^0 (see the lower panel of Fig. 2) When we travel to another point on the budget line, $(X^{'}, Y^{'})$ where $X^{'} > X^0$ and $Y^{'} < Y^0$, MU_X is given by the level of marginal utility read off from the 'marginal utility schedule of X corresponding to $Y = Y^{'}$, at a level of consumption of $X = X^{'}$. Note that the 'marginal utility schedule of X corresponding to $Y = Y^{'}$is obtained by an upward shift of the 'marginal utility schedule of X corresponding to $Y = Y^0$, given the assumed negative complementarity between X and Y. This tends to push MU_X at

$X = X^0$ upwards but to reach $X = X^{'}$ from $X = X^0$ on this schedule ('marginal utility schedule of X corresponding to $Y = Y^{'}$) a decline in MU_X is experienced. The total change in MU_X from (X^0, Y^0) to $(X^{'}, Y^{'})$ is the sum of the increase in marginal utility of X due to the mentioned upward shift and the subsequent fall in the marginal utility while moving down the shifted marginal utility schedule from X^0 to $X^{'}$. The net change in marginal utility is positive or negative. Fig. 2 illustrates the case where this net change is not only positive for the given increase in consumption of X (from X^0 to $X^{'}$) but for all increases in consumption of X. Accordingly, we have an upward sloping BCMUS of X (in the second panel of Fig.3) and for similar reasons (not illustrated by a diagram) we could have an upward sloping BCMUS of Y.

The same assumptions as those corresponding to Fig. 2 can generate downward sloping BCMUSs when the tendency for increase in marginal utility from X following an upward shift in the marginal utility schedule of X (due to a decrease in Y corresponding to an increase in X) is overwhelmed by the tendency for decrease in marginal utility from X due to movement down the shifted schedule. This is illustrated in Fig. 3.

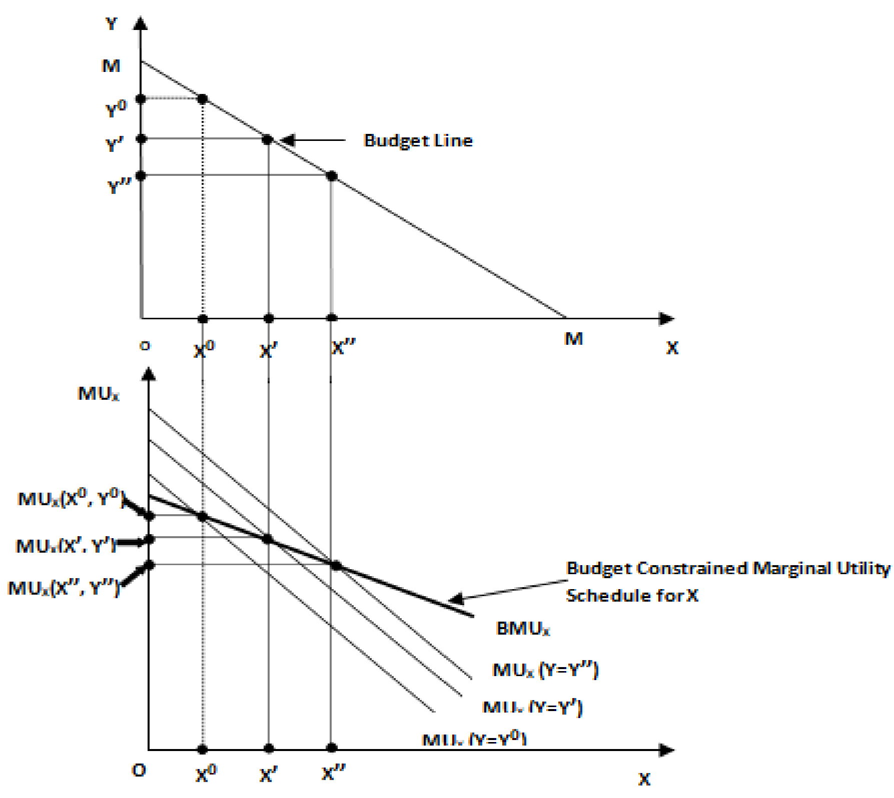

Figure 3: Downward sloping BCMUS of X under the assumptions of (a) validity of the law of diminishing marginal utility, (b) negative complementarity in regard to utility between the two commodities and (c) negative net change of MU_X for all increases in consumption of X.

Moreover, the assumptions underlying Fig. 2 and Fig. 3 (diminishing marginal utility and negative complementarity) can also correspond to a non-monotonic BCMUS. For example, consider the case where the negative cross partial derivative increases in magnitude as Y decreases. This implies that for all equal decreases in Y the upward shift in the MU_X schedule is greater for later decreases. At the same time, assume that the MU_X schedule is convex i.e. the fall in MU_X for a given increase in X decreases or remains constant as the value of X, before the increase in X, increases. Initially, as we travel down the budget line the tendency for decrease in MU_X with increase in X corresponding to movement along the shifted marginal utility schedule, the shift occurring due to the decrease in Y that accompanies the increase in X, will overwhelm the tendency for increase in MU_X due to the shift in the marginal utility schedule. Thus, the BCMUS will be initially downward sloping. But as the shifts become larger for successive equal decreases in Y that accompany equal successive increases in X, and the equal successive increases in X correspond to progressively smaller or equal declines in marginal utility along the shifted marginal utility schedule there is likely to be a point where the two tendencies neutralize each other (i.e. a stationary point in BCMUS is reached). Beyond the point, the BCMUS will curve upwards (i.e. BCMUS will increase in X) as the second of the mentioned tendencies will overcome the first tendency. Thus, we shall have a U shaped schedule. This is illustrated in Fig. 4, drawn under the assumption of a strictly convex marginal utility schedule.

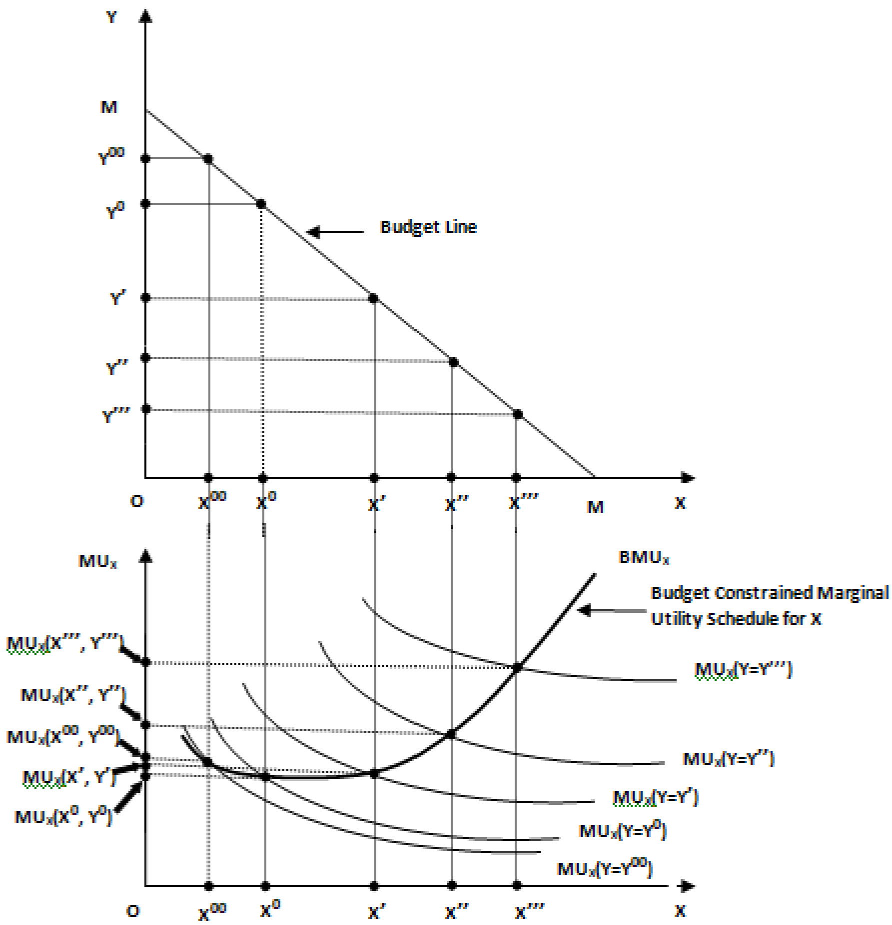

Figure 4: U shaped BCMUS of X under the assumptions of (a) validity of the law of diminishing marginal utility, (b) negative complementarity in regard to utility between the two commodities, (c) increasing negative cross partial derivative in magnitude corresponding to equal successive decreases in Y and (d) strictly convex downward sloping marginal utility schedule of commodity X.

However, non-monotonic BCMUSs corresponding to the mentioned assumptions can also be of an inverted U shape. The reader can see this by fashioning an argument similar to that in the previous paragraph but this time assuming concave downward sloping marginal utility schedules (the decreases in marginal utility for equal successive increases in X become progressively larger or remain equal in magnitude) and upward shifts in the MU_X schedule corresponding to equal successive decreases in Y that are progressively smaller in magnitude. Thus, the second of the mentioned effects initially outweighs the first effect, giving rise to an initial rising segment of the BCMUS; but as the shifts become smaller and the magnitudes of decreases in marginal utility remain the same or increase in magnitude, there will come a stationary point followed by a downward sloping portion. Thus, we shall have an inverted U shaped schedule, as illustrated by Fig. 5 which is under the assumption of a concave marginal utility schedule.

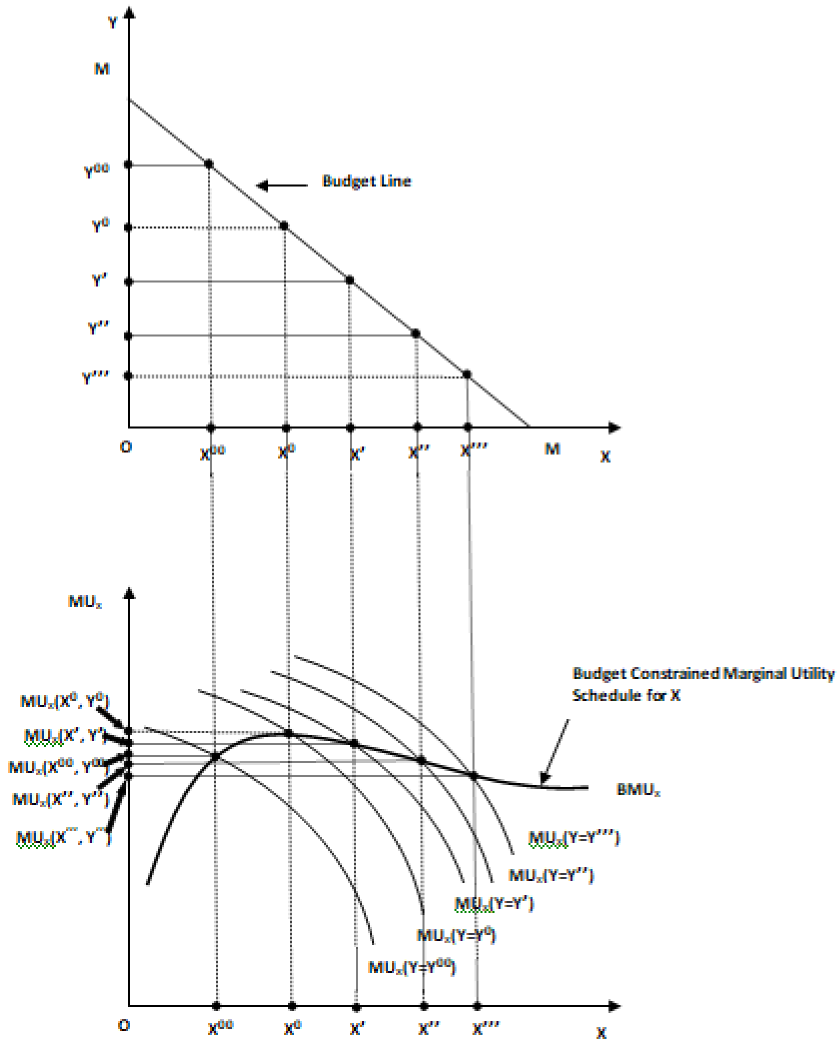

Figure 5: Inverted U-shaped BCMUS of X under the assumptions of (a) validity of the law of diminishing marginal utility, (b) negative complementarity in regard to utility between the two commodities, (c) decreasing negative cross partial derivative in magnitude corresponding to equal successive decreases in Y and (d) strictly concave downward sloping marginal utility schedule of commodity X.

This chapter is not exhaustive in terms of deriving BCMUSs under different set of conditions. Derivations of residual BCMUSs, if the underlying assumptions change, have been depicted in from Fig. 25 to Fig. 31 in the appendix.

To conclude this chapter, the foundations of the theory of consumer behaviour under the early cardinal school of utility are weak. This is in stark contrast to the ordinal utility school and under it, the theory of indifference curves, which clarifies the assumptions under which the equality of the ratio of marginal utilities and the ratio of commodity prices in a two good world (or its equivalent, the equality of the ratio of marginal utility and price of the two goods under consideration) indeed corresponds to the maximization of consumer utility subject to the budget constraint (i.e. attainment of consumer equilibrium): the "more is better" assumption and the assumption of "diminishing marginal rate of substitution". The early cardinal school also states the same condition as the equilibrium condition (i.e. the equality of the ratio of marginal utility and price of the two goods under consideration) but does not explicitly state in cardinalist language the assumptions that are necessary to rule out cases in which the condition is not compatible with the maximization of utility. The language of the early cardinal school involves terms such as 'marginal utility', 'law of diminishing marginal utility' and associated 'declining marginal utility schedules' which alone are inadequate for ruling out these cases. To remove this ambiguity, we have introduced the concept of the BCMUS which shows how marginal utility of the good changes as we change the level consumed while moving on the budget line.

Note that the law of diminishing marginal utility says that the marginal utility from a commodity, holding the quantities of all other commodities constant, should be declining in the quantity consumed of that commodity by the consumer. Yet it is possible to imagine its violation in real life, such as in listening to music or for the consumption of ice-cream by gourmands. Further, diminishing marginal utility is not even necessary for the satisfaction of the two conditions of diminishing MRS and positive marginal utility which guarantee that the point of tangency of the highest indifference curve with the budget line is that of consumer equilibrium.

Chapter 3:-

Determining Consumer Equilibrium Using Budget Constrained Marginal Utility Schedules

Let us choose, for the sake of simplicity of illustration, units of X and Y such that the price of each is unity. Thus, the condition for consumer equilibrium reduces to $MU_X = MU_Y$ under appropriate conditions. In the diagram below we draw a horizontal line which is M units long, M being denoted by income, and then measure the quantities of X and Y from the leftward and rightward origin respectively. Each point on this line thus gives a unique allocation of M between X and Y and maps onto two marginal utilities one for X and one for Y, thus generating the BCMUSs of X and Y respectively. The intersection of the two BCMUSs, if it exists, might be associated with the attainment of consumer equilibrium but not necessarily. The downward slope of both BCMUSs, which as we have seen above can be generated from different sets of assumptions regarding slopes of marginal utility schedules and complementarity, is obviously a sufficient condition for such consumer equilibrium to be located at the point of intersection, if such a point exists. This is because there is no way the consumer can increase his utility by deviating from the allocation corresponding to the intersection as utility gain associated with an increase in X(Y) will always be less than the magnitude of utility loss associated with an decrease in Y(X). This is the case illustrated below. But the downward slope is not a necessary condition, as illustrated by the diagrams in this chapter and thereafter in the appendix (Fig. 32 to Fig. 43) and the table of cases in the next chapter.

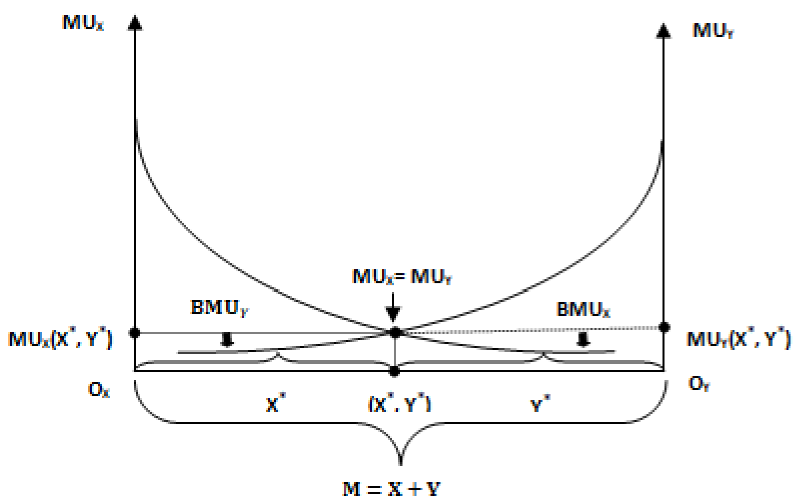

Figure 6 : Consumer equilibrium associated with downward sloping BCMUSs and unit prices for both commodities.

In the above diagram, the BCMUS of X is a negatively sloping curve. The same is true for the BCMUS of Y in $Y - MU_Y$ space. Given our definition of the units of X and Y, the budget constraint reduces to $M = X + Y$. The simplest way to define a consumer bundle is to consider a horizontal line which is M monetary units long. The consumer bundle (X^0, Y^0) or more simply $(X^0, M - X^0)$(as $X^0$and $Y^0$are related by the given budget constraint) can be represented as a point on the given horizontal straight line which is X^0units distant from the leftward origin of the line (we call this the origin for measuring X) and automatically $M - X^0 = Y^0$units distant from the rightward origin of the line (we call this the origin for measuring Y). In a similar way, any other consumption bundle on the budget line can be represented by a single point on the given horizontal line. Let O_X be the origin for measuring X and let O_Y be the origin for measuring Y. We can then measure MU_X and MU_Y along the vertical axes emerging from O_X and O_Y. What we have after the construction of the horizontal scale and the mentioned vertical axes is a device that resembles the Edgeworth box (see Fig. 6). Each point on the horizontal scale $(X, Y = M - X)$, representing a consumer bundle that is attainable through the full disbursal of the given money income M, maps on to two points, (X, MU_X) and (Y, MU_Y), the first a point on the BCMUS of X and the second on the BCMUS of Y. In this way, the two BCMUSs are mapped in the same diagram, with the one for X drawn with O_X as origin and the one for Y drawn with O_Y as origin. Given that both marginal schedules are downward sloping it is very likely that they will intersect within the open box defined by the horizontal scale and the vertical axes emanating from the two origins,O_X and O_Y. The point of intersection implies that the corresponding consumer bundle, located by dropping a vertical line from the point of intersection, is associated with marginal utilities of X and Y that are equal. That is, the condition $MU_X = MU_Y$ is satisfied. This in fact is clearly the allocation on the budget line that maximizes total utility subject to the budget constraint i.e. the consumer equilibrium. Allocations to the right of this allocation $(X^*, Y^* = M - X^*)$ correspond to higher levels of X and lower levels of Y. Since the BCMUS of X lies everywhere below the BCMUS of Y for bundles to the right of (X^*, Y^*) the additional utility gained because of a higher X is lower in magnitude than the utility loss caused by a lower Y i.e. all these bundles yield a lower total utility than (X^*, Y^*). The same is true when we move to the left of the bundle (X^*, Y^*) on the horizontal scale by decreasing X and increasing Y i.e. the BCMUS of Y lies everywhere below that of X, which implies that the total utility for any consumer bundle on the horizontal scale located to the left of (X^*, Y^*) is less than that corresponding to (X^*, Y^*). In short, the condition $MU_X = MU_Y$ stands for the equilibrium condition when BCMUSs of X and Y are downward sloping in the $X - MU_X$ and $Y - MU_Y$ space respectively, and is attained where the mentioned BCMUSs intersect. Such downward sloping schedules result in the case where validity of the law of diminishing marginal utility is reinforced by positive complementarity between X and Y but, as we have seen in chapter 2 and in the related diagrams in the appendix, downward sloping BCMUSs are by no means restricted to this case. Further, we shall see that it is possible to have non-monotonic BCMUSs, which are downward sloping in the neighbourhood of their intersection, which implies that the condition, $MU_X = MU_Y$ might again correspond to consumer equilibrium. Again if this condition is not fulfilled by any bundle on the horizontal scale (budget line) because of one schedule lying completely below the other then the entire budget will be spent on one of the two commodities in equilibrium, with the consumption of the other commodity being 0.

Note that corner solutions occur in the case of indifference curve analysis, under the assumption of both prices equalling unity, when $\frac{MU_X}{MU_Y} \neq 1$ at every point on the budget line. A corner solution corresponds to the entire income being spent on X (Y) when $\frac{MU_X}{MU_Y} > 1$ ($\frac{MU_X}{MU_Y} < 1$) at every point on the budget line. Similarly, consider the BCMUS of X (Y) lying everywhere above that of Y (X) i.e. $MU_X > MU_Y \Leftrightarrow \frac{MU_X}{MU_Y} > 1$ ($MU_X < MU_Y \Leftrightarrow \frac{MU_X}{MU_Y} < 1$). Obviously this means that any interior consumption bundle (positive levels of X and Y) can be improved upon in terms of utility by increasing X (Y). Thus, we have the corner solution, $X = \frac{M}{P_X}, Y = 0$ in case the following condition characterising every allocation on the budget line: $MU_X > MU_Y \Leftrightarrow \frac{MU_X}{MU_Y} > 1$. Note that this means that the BCMUS of X lies everywhere above that of Y and equivalently that each point on the budget line is intersected by an IC which is steeper than the budget line. Similarly, we have the corner solution $X = 0, Y = \frac{M}{P_Y}$, in case of $MU_X < MU_Y \Leftrightarrow \frac{MU_X}{MU_Y} < 1$.

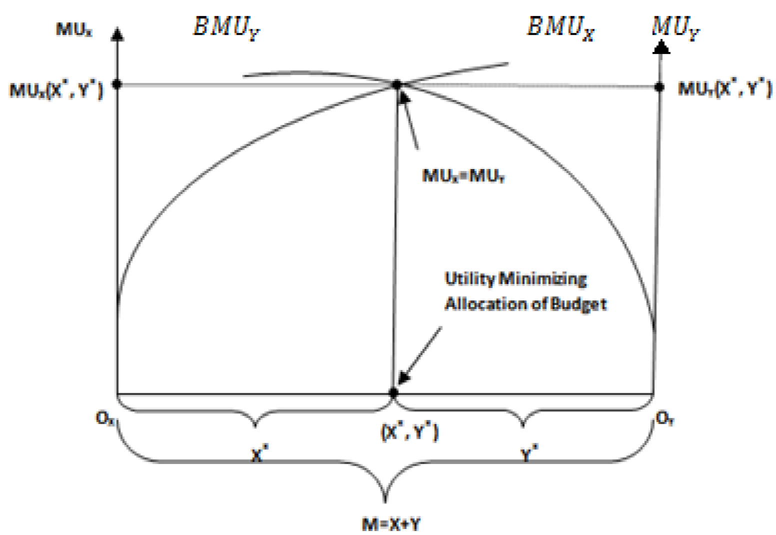

Figure 7 : Consumer equilibrium associated with upward rising BCMUSs and unit prices for both the commodities.

Fig. 7 depicts the case of consumer equilibrium in which BCMUSs are upward sloping. The point of intersection of the BCMUSs corresponds to the consumption bundle (X^*, Y^*) and implies that $MU_X(X^*, Y^*) = MU_Y(X^*, Y^*)$. However, note that any movement along the budget line by increasing X and therefore decreasing Y, starting from $X = X^*$, will imply $MU_X > MU_Y$. Thus, the utility gain from increase in X will more than compensate for the utility loss from decrease in Y, implying a net gain in utility. Similarly for all bundles involving a lower amount of X and a higher amount of Y than (X^*, Y^*) , $MU_Y > MU_X$ i.e. a transition to these bundles from (X^*, Y^*) will also involve a gain in utility. Thus (X^*, Y^*), the point of intersection of the BCMUSs of X and Y marked by $MU_X(X^*, Y^*) = MU_Y(X^*, Y^*)$ corresponds to utility minimization and not utility maximization ($MU_X = MU_Y$ is not an equilibrium condition) on the budget line. In fact, higher the deviation from (X^*, Y^*), either towards the left or to the right, greater is the utility obtained from the consumer bundle. Consumer equilibrium will be a corner solution. Given this fact, utility is either maximized at $(\frac{M}{P_X}, 0)$ or $(0, \frac{M}{P_Y})$.

Fig. 8, Fig. 9 and Fig. 10 depict consumer equilibrium when the BCMUSs of X and Y are of the U shape depicted by Fig. 4 in chapter 2 or by Fig. 28 in the appendix. In Fig.8, the allocation corresponding to the point of intersection of the two BCMUSs, (X^*, Y^*) satisfies $MU_X(X^*, Y^*) = MU_Y(X^*, Y^*)$. However, the point of intersection lies on the upward sloping segments of both the BCMUSs of X and Y. This implies that gains in utility can be achieved by increasing X or Y starting from (X^*, Y^*). Thus, the condition $MU_X(X^*, Y^*) = MU_Y(X^*, Y^*)$ corresponds to the minimization of total utility subject to the budget constraint.

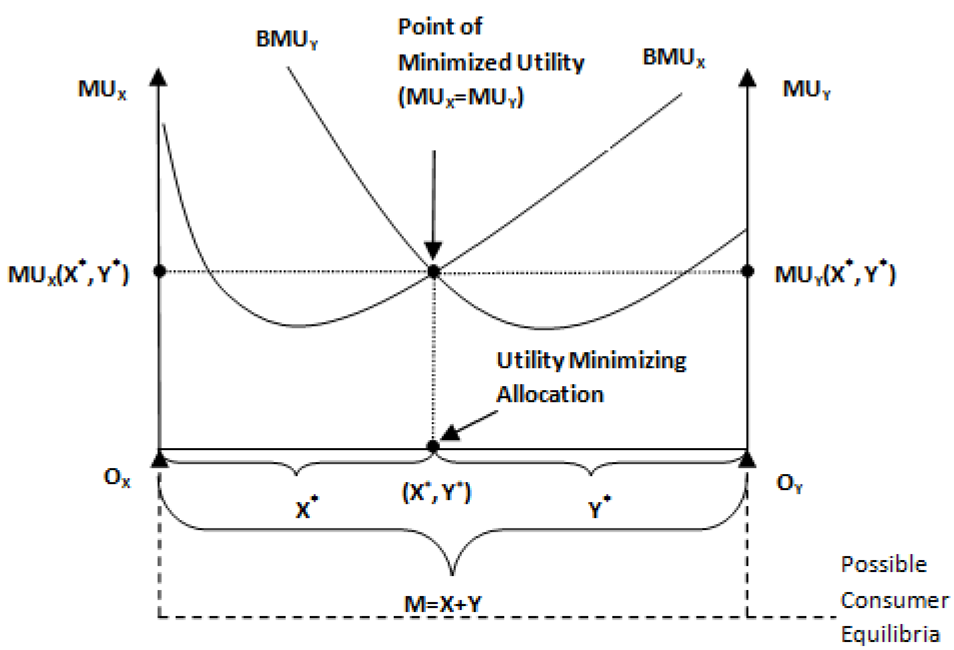

Figure 8: Consumer equilibrium when the BCMUSs of both the commodities are U-shaped.

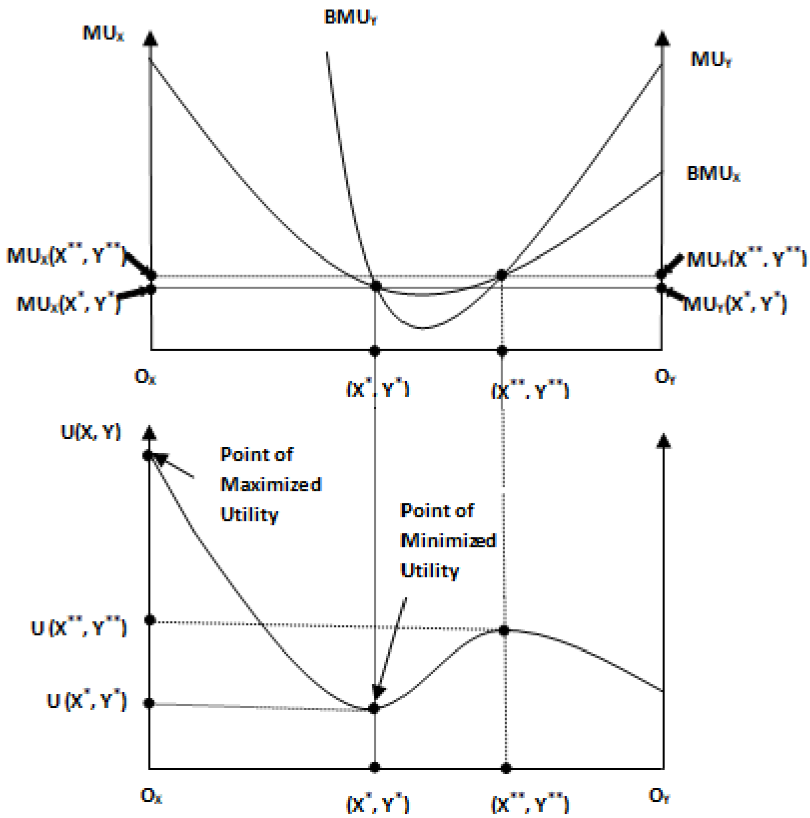

Figure 9: Consumer equilibrium in case of multiple stationary points when the BCMUSs of both X and Y are of U-shaped and BMU_Y is steeper everywhere than BMU_X.

In the upper panel of Fig. 9, as drawn, the U shaped BCMUS of Y is steeper everywhere than the U shaped BCMUS of X. As drawn, the former schedule intersects the latter at two points, (X^*, Y^*) and (X^{**}, Y^{**}) -- one on the downward sloping segment of the latter and another on the upward sloping segment of the latter i.e. $MU_X = MU_Y$ at both these allocations. The lower panel shows the shape of the utility profile for various allocations of X and Y on the budget line on the basis of the shape of the BCMUSs and their relative positions over different ranges of X and Y. The curve has two stationary points, one a maxima and the other a minima, each of these corresponding to the allocations at the points of intersection of the BCMUSs. As drawn, the maxima does not correspond to maximized utility under the budget constraint. The maximized utility corresponds to a corner solution. But the local interior maxima could also possibly correspond to a global maximum if the peak in the interior had gone above the utility levels at both the corners.

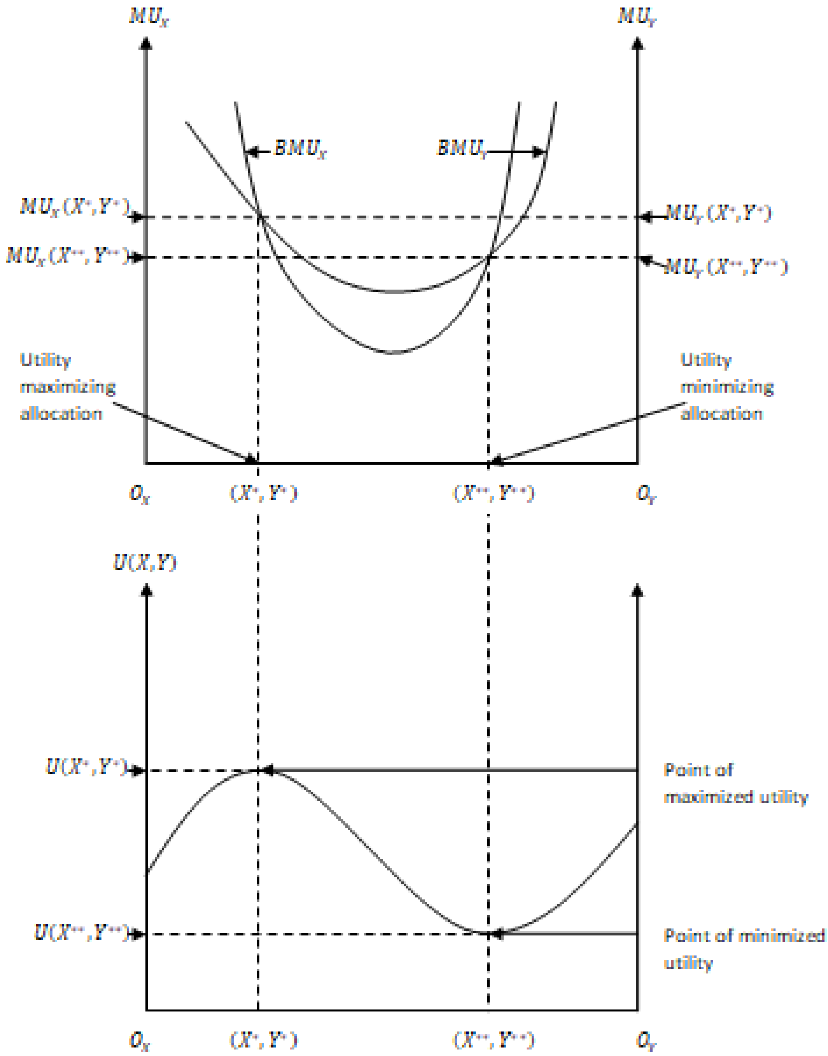

Figure 10: Consumer equilibrium in case of multiple stationary points when the BCMUSs of both the commodities are U-shaped and BMU_X is steeper everywhere than BMU_Y.

Fig. 10 also shows U shaped BCMUSs intersecting each other at two allocations. In the upper panel, BCMUS of X is steeper everywhere than the BCMUS of Y. The former intersects the latter at two points; one on the downward sloping segment of the latter and another on the upward rising segment of the latter i.e. $MU_X = MU_Y$ at both (X^*, Y^*) and (X^{**}, Y^{**}). The lower panel shows the shape of the utility profile for various allocations of X and Y on the budget line. The curve has two stationary points, one a maxima corresponding to (X^*, Y^*) and the other a minima corresponding to (X^{**}, Y^{**}). As drawn, the allocation corresponding to one of the points of intersection yields a maximum on the utility profile and therefore also the maximized utility on the budget constraint. Thus, out of the two allocations that yield $MU_X = MU_Y$, one corresponds to the maximization of utility subject to the budget constraint. It is also visible from

Fig. 10 that here the local minima is also the global minimum i.e. (X^{**}, Y^{**}) is indeed the utility minimizing bundle subject to the budget constraint.

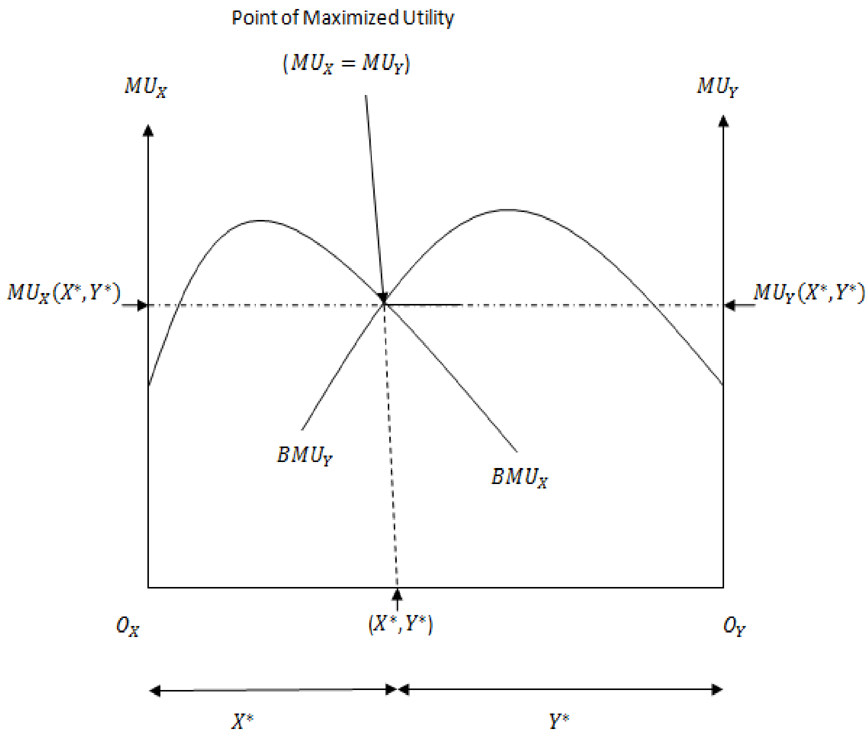

Figure 11: Consumer equilibrium when the BCMUSs of both the commodities are inverted U-shaped.

Fig. 11 depicts the consumer equilibrium when the BCMUSs of X and Y are inverted U-shaped, as depicted by Fig. 5 in the second chapter and by Fig. 27 in the appendix. In Fig.11, the allocation corresponding to the point of intersection of the two BCMUSs, (X^*, Y^*) satisfies $MU_X(X^*, Y^*) = MU_Y(X^*, Y^*)$. However, the point of intersection lies on the downward sloping segments of both the BCMUSs of X and Y and hence corresponds to the actual consumer equilibrium i.e, a point of maximized utility. This implies that gains in utility cannot be achieved by increasing X or Y starting from (X^*, Y^*). This is because the additional gain in utility by increasing X is less than the additional loss in utility by decreasing Y corresponding to any allocation to the right of (X^*, Y^*) and vice-versa corresponding to any allocation to the left of (X^*, Y^*). Thus the condition $MU_X(X^*, Y^*) = MU_Y(X^*, Y^*)$ corresponds to the maximization of total utility subject to the budget constraint. This case is extremely important as it shows that when the BCMUSs are both of an inverted U shape, with the peak of X lying to the left of that of Y, the point of intersection of the two schedules is the actual consumer equilibrium. As it turns out in the later chapters, this case holds for utility functions exhibiting increasing marginal utility with positive complementarity.

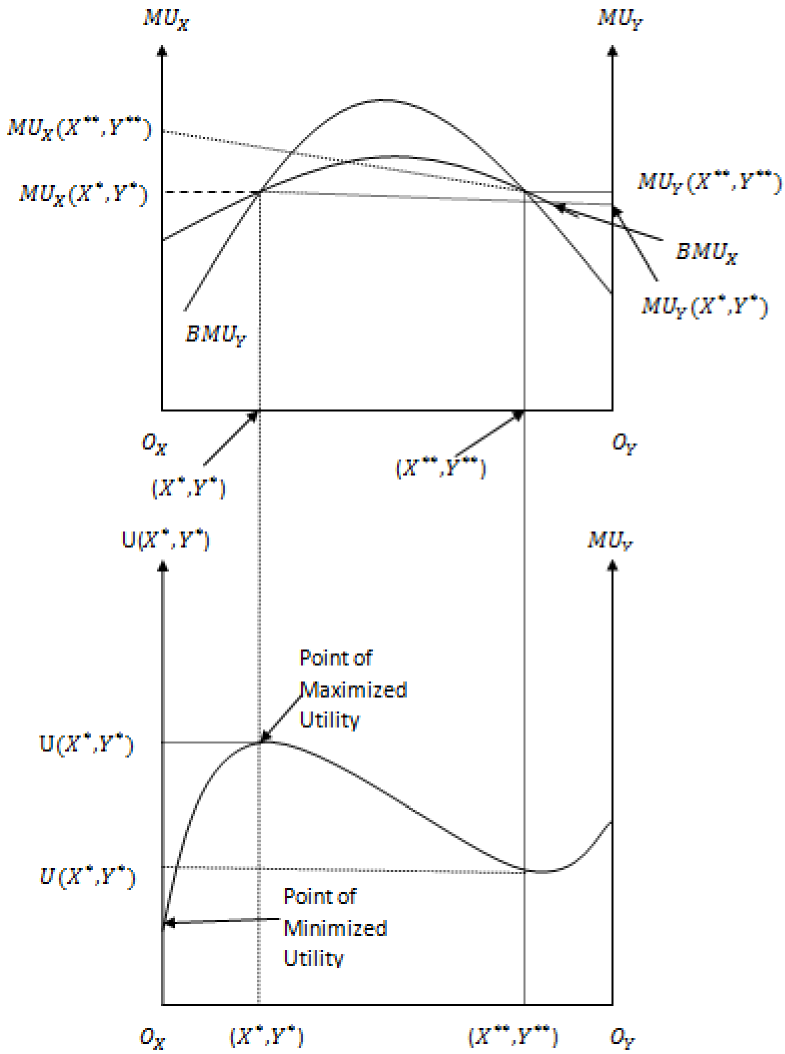

Figure 12: Consumer equilibrium in case of multiple stationary points when the BCMUSs of both the commodities are of the inverted U shaped and BMU_Y is steeper everywhere than BMU_X.

In the upper panel of Fig.12, as drawn the inverted U-shaped BCMUS of Y is steeper everywhere than the inverted U-shaped BCMUS of X. As drawn, the former schedule intersects the latter at two points (X^*, Y^*) and (X^{**}, Y^{**}) one on the upward rising segment of the latter and another on the downward sloping segment of the latter i.e. $MU_X = MU_Y$ at both these allocations. The lower panel shows the shape of the utility profile for various allocations of X and Y on the budget line on the basis of the shape of the BCMUSs and their relative positions over different ranges of X and Y. The curve has two stationary points, one a maxima and the other a minima, each of these corresponding to the allocations at the points of intersection of the BCMUSs. As drawn, the maxima in the interior corresponds to a global maximum, but the interior minima does not correspond to a global minimum.

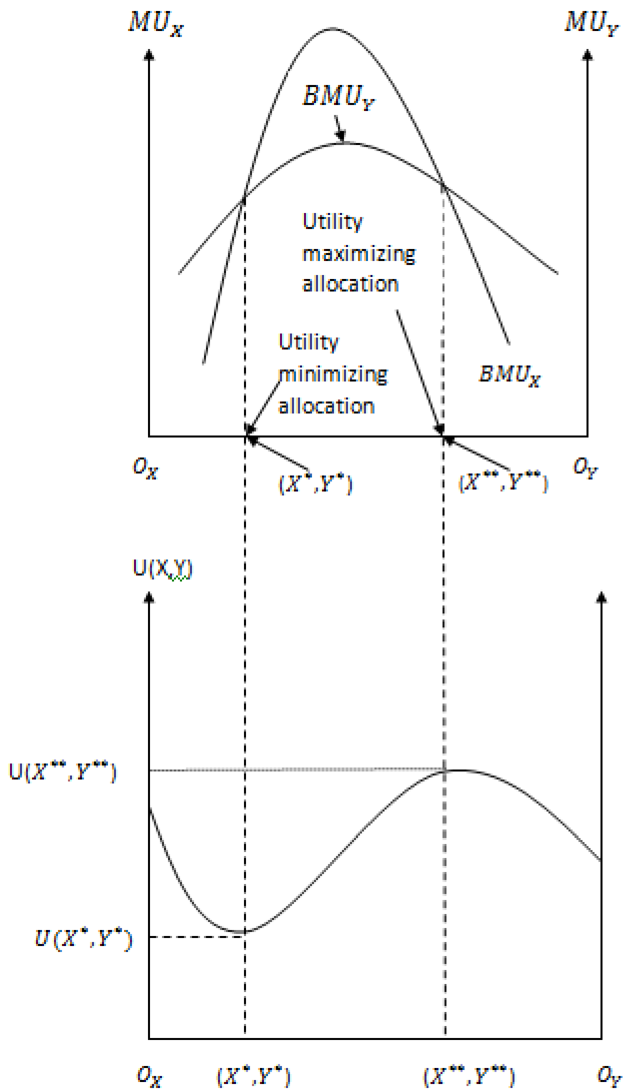

Figure 13: Consumer equilibrium in case of multiple stationary points when the BCMUSs of both the commodities are of inverted U-shaped and BMU_X is steeper everywhere than BMU_Y.

The exact opposite case of the previous case is depicted in Fig.13. In the upper panel of the figure, both the BCMUSs of X and Y are of inverted U-shaped and BMU_X is steeper everywhere than BMU_Y. The former intersects the latter at two points. In the lower panel, the utility profile for various allocations of X and Y on the budget line has been drawn. The curve has two stationary points one a minima corresponding to (X^*, Y^*) which corresponds to the minimized utility under the budget constraint and the other a maxima corresponding to (X^{**}, Y^{**}) which also corresponds to the maximized utility under the budget constraint. Thus, Fig.13 depicts multiple stationary points (intersections of the BCMUSs) with one allocation corresponding to a maximum on the utility profile and another allocation corresponding to a minimum on the utility profile.

If the BCMUS of Y is inverted U-shaped and that of X is U-shaped, then the two marginal utility schedules may intersect each other at two points corresponding to allocations (X^*, Y^*) and (X^{**}, Y^{**}) as depicted in the upper panel of Fig. 14. If we shift a little to any allocation to the right or to the left of (X^*, Y^*), loss in utility is incurred. Similarly, if we shift a little to any allocation to the right or to the left of (X^{**}, Y^{**}) gain in total utility is experienced. Hence, in the lower panel of Fig. 14, the utility profile curve has a maxima corresponding to (X^*, Y^*) and a minima corresponding to (X^{**}, Y^{**}). As drawn, the maxima indeed corresponds to maximized utility and the minima indeed corresponds to minimized utility under the budget constraint. Consumer equilibrium is achieved corresponding to allocation (X^*, Y^*).

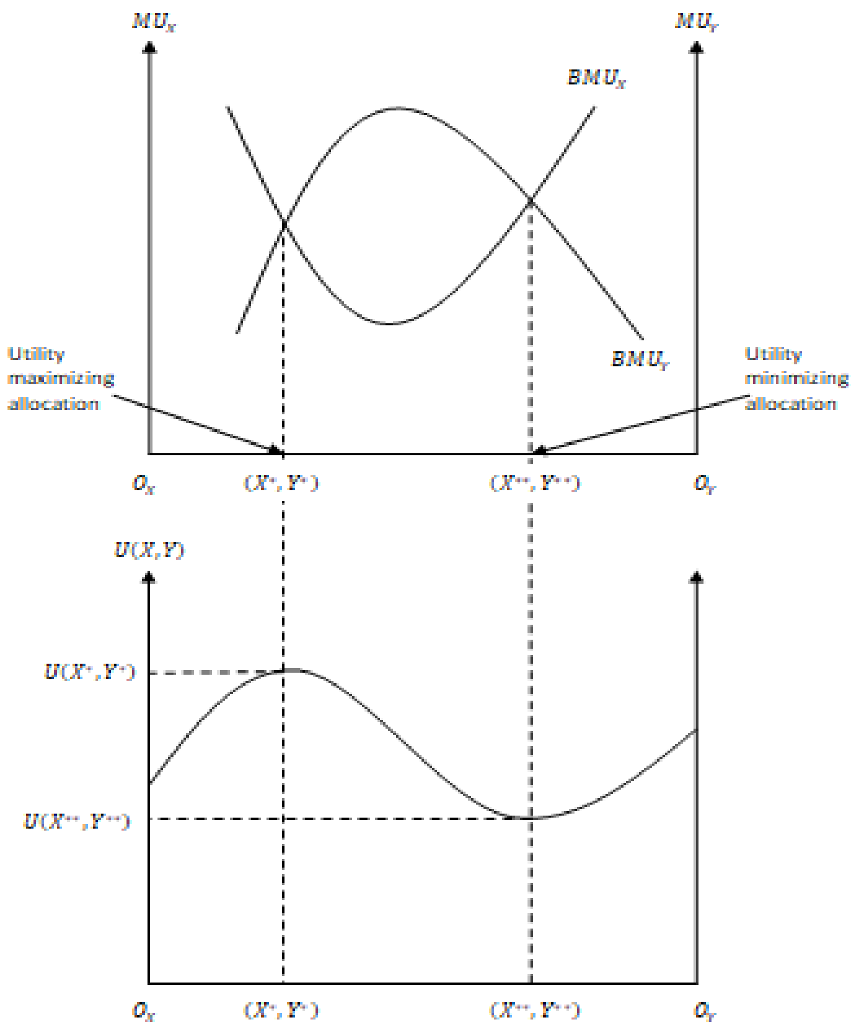

Figure 14 : Consumer equilibrium when BMU_X is U-shaped and BMU_Y is inverted U-shaped.

The exact opposite case of the afore-mentioned case is depicted in the upper panel of Fig. 15. The two BCMUSs of X and Y may intersect each other at two points corresponding to the two allocations (X^*, Y^*) and (X^{**}, Y^{**}) respectively. If we shift to any allocation to the left or to the right of (X^*, Y^*), total utility increases. Similarly, if we shift to any allocation to the left or to the right of (X^{**}, Y^{**}), then loss in utility is incurred. Hence, in the lower panel of Fig. 15, the utility profile curve has a minima corresponding to (X^*, Y^*) and a maxima corresponding to (X^{**}, Y^{**}). As drawn, the allocation (X^*, Y^*) is also indeed the utility minimizing allocation and the allocation (X^{**}, Y^{**}) is the utility maximizing allocation. Hence, consumer equilibrium occurs corresponding to (X^{**}, Y^{**}).

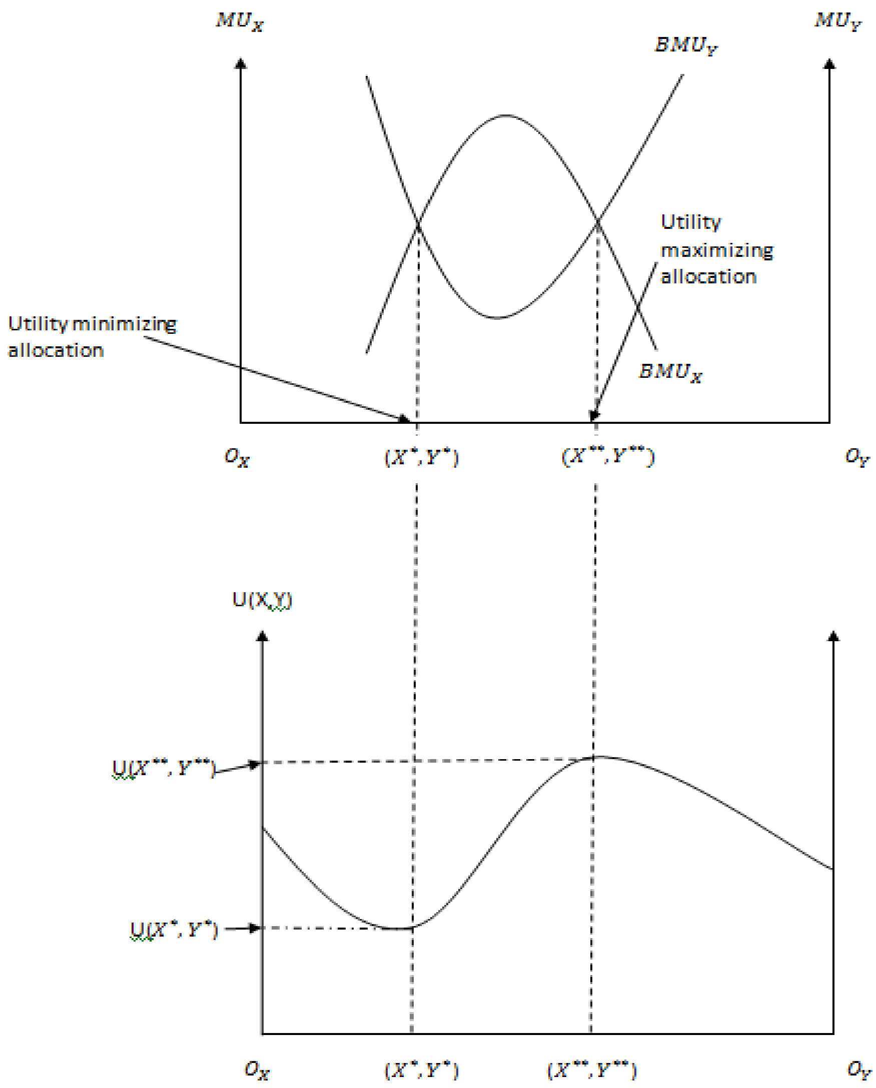

Figure 15 : Consumer equilibrium when the BMU_X is inverted U-shaped and BMU_Y is U-shaped.

Another possibility is a horizontal BCMUS of X and a positively sloped BCMUS of Y. The positively sloped BCMUS of Y and horizontal BCMUS of X intersect each other at the allocation (X^*, Y^*) as shown in Fig. 16 below. If we shift from (X^*, Y^*) to the right, then the gain in utility due to increase in X outweighs the loss in utility due to decrease in Y. Similarly, if we shift to the left of (X^*, Y^*), then the gain in utility due to increase in Y outweighs the loss in utility due to decrease in X compared to (X^*, Y^*). Thus if we shift to the right or to the left of (X^*, Y^*), the total utility increases. So, the allocation corresponding to (X^*, Y^*) is indeed the utility minimizing allocation.

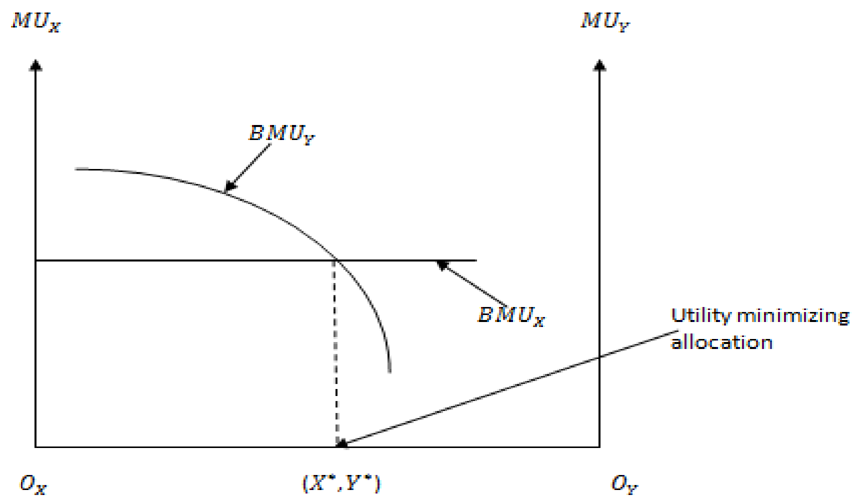

Figure 16: Consumer equilibrium in case of horizontal BCMUS of X and positively sloped BCMUS of Y.

In yet another case we consider a horizontal BCMUS of X and a negatively sloped BCMUS of Y. These two BCMUSs intersect each other at the allocation (X^*, Y^*) as shown in Fig. 17. If we shift to any allocation to the right of (X^*, Y^*), the loss in utility due to decreases in consumption of Y outweighs the gain in utility due to the increase in consumption of X. Thus, loss in utility is incurred. The reader can also easily verify that loss in utility is also incurred in moving to any allocation to the left of (X^*, Y^*). Therefore, the allocation (X^*, Y^*) is the utility maximizing allocation and hence, consumer equilibrium occurs at (X^*, Y^*).

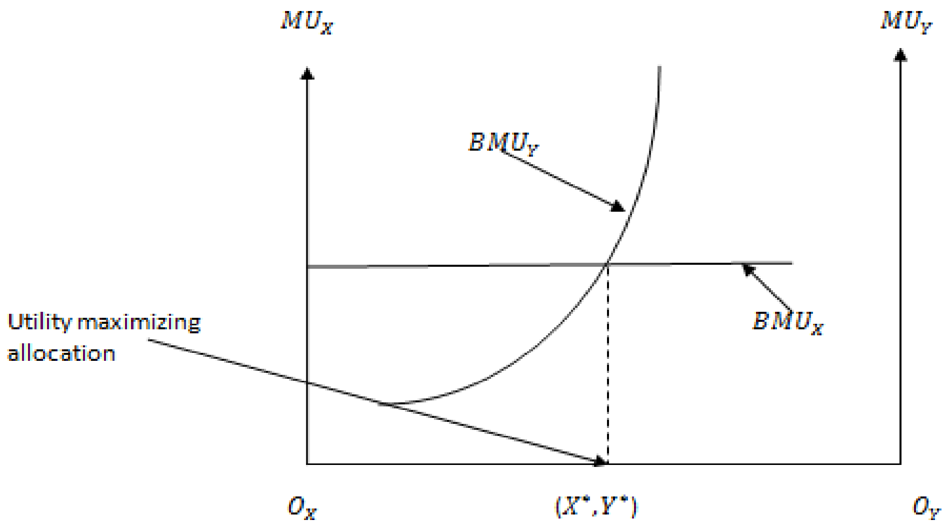

Figure 17: Consumer equilibrium in case of horizontal BCMUS of X and negatively sloped BCMUS of Y.

Similarly, the BCMUS of Y can also be horizontal. Determining consumer equilibrium in the two sub-cases under this case is depicted in Fig. 32 and Fig. 33 in the appendix. It is also possible that the slopes of the BCMUSs of the two goods are of opposite sign. Determining consumer equilibrium in those cases has been depicted from Fig. 34 to Fig. 43 in the appendix. Fig. 34 to Fig. 38 depict consumer equilibria associated with negatively sloped BCMUS of X and positively sloped BCMUS of Y, whereas Fig. 39 to Fig. 43 exhibit consumer equilibria associated with positively sloped BCMUS of X and negatively sloped BCMUS of Y. Next, we shall show that consumer equilibrium can also be depicted in the pseudo-Edgeworth box kind of diagram even if the prices of the two commodities are not unity.

Inference 1: If BCMU of X and BCMU of Y are downward (upward) sloping with respect to X and Y respectively, then $BCMU_X$ and $BCMU_Y$ are also downward (upward) sloping with respect to the expenditure on X $(E_X = XP_X)$ and expenditure on Y $(E_Y = YP_Y)$ respectively (where BCMU denotes budget constrained marginal utility).

Proof: $\frac{\boldsymbol{\partial(BCMU_X)}}{\boldsymbol{\partial(XP_X)}} = \frac{\boldsymbol{\partial(BCMU_X)}}{\boldsymbol{\partial X}} \frac{\boldsymbol{\partial X}}{\boldsymbol{\partial(XP_X)}} = \frac{\boldsymbol{\partial(BCMU_X)}}{\boldsymbol{\partial X}} \frac{\mathbf{1}}{\frac{\boldsymbol{\partial(XP_X)}}{\boldsymbol{\partial X}}} = \frac{\mathbf{1}}{\boldsymbol{P_X}} \frac{\partial(BCMU_X)}{\partial X}$

Therefore, $\frac{\partial(BCMU_X)}{\partial X} < (>)0 \Rightarrow \frac{\boldsymbol{\partial(BCMU_X)}}{\boldsymbol{\partial(XP_X)}} = \frac{\mathbf{1}}{\boldsymbol{P_X}} \frac{\partial(BCMU_X)}{\partial X} < (>)0$

Similarly, $for\ Y$

Inference 2: It follows that $\frac{BCMU_X}{P_X}$ and $\frac{BCMU_Y}{P_Y}$ are also downward (upward) sloping with respect to the expenditure on X and expenditure on Y respectively if $BCMU_X$ and $BCMU_Y$ are downward (upward) sloping with respect to X and Y respectively.

Given inference 2, even if, prices of the two commodities are not equal to unity, then also the pseudo Edgeworth kind of diagram can be drawn. In the following diagram, the horizontal line segment is M monetary units long. The leftward origin is the origin for measuring the expenditure on commodity X and the rightward origin is the origin for measuring the expenditure on commodity Y. Any point (E_X^0, E_Y^0) on the horizontal line segment, which is E_X^0 units distant from the leftward origin and E_Y^0 units distant from the rightward origin, represents the expenditure on commodities X and Y respectively. Similarly, any other allocation of budget on the two commodities can be represented by the specific point on the horizontal segment of the diagram given below. Assuming that the BCMU of both X and Y are downward sloping with respect to X and Y respectively, the negatively sloped schedules for $\frac{BCMU_X}{P_X}$ and $\frac{BCMU_Y}{P_Y}$ with respect to the expenditure on X and Y respectively, are drawn in the diagram below. In this general case, the equilibrium condition corresponding to consumer equilibrium is obviously $\frac{BCMU_X}{P_X} = \frac{BCMU_Y}{P_Y}$. (E_X^*, E_Y^*) is the point on the mentioned horizontal segment corresponding to the intersection point of the two curves showing $\frac{BCMU_X}{P_X}$ and $\frac{BCMU_Y}{P_Y}$. Thus,(E_X^*, E_Y^*) is the optimal or utility maximizing allocation of expenditure among X and Y. The optimal consumption bundle (X^*, Y^*) can then be found by dividing E_X^* and E_Y^* by P_X and P_Y respectively.

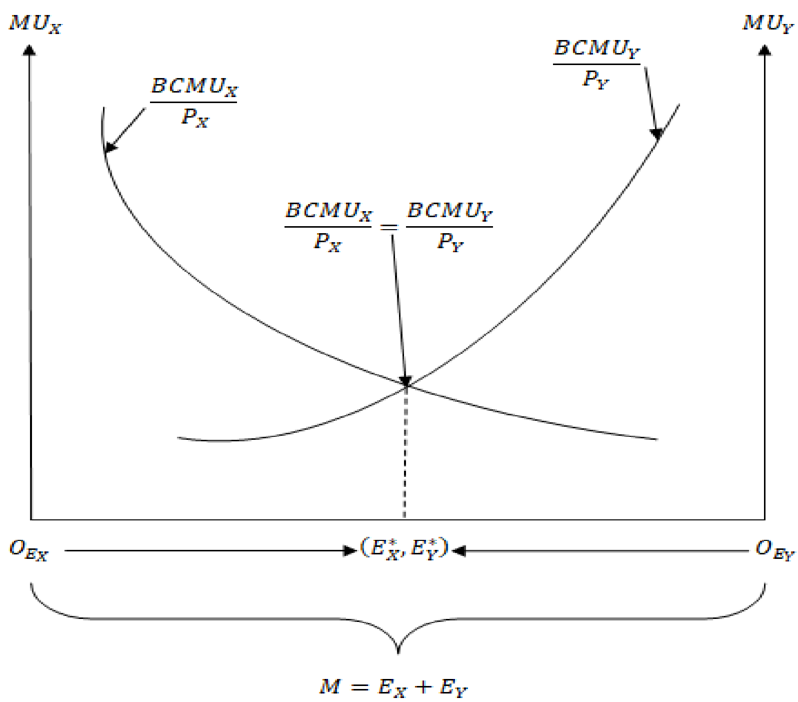

Figure 18 : Consumer equilibrium associated with downward sloping BCMUSs of X and Y when prices of the commodities are not unity.

Chapter 4:-

The Simple Algebra of Budget Constrained Marginal Utility Schedules

Let us consider the following utility function:

$$U = U(x, y)$$

where x denotes the quantity of commodity X and y denotes the quantity of commodity Y.

Let us simplify matters at the beginning by assuming that the units for measuring x and y are defined so that the price of both is equal to unity. As mentioned before, we can think of X as a narrowly defined commodity, say apples and Y as a composite commodity, which is measured by the expenditure (say, in dollars) on all commodities other than X. Given our stated assumptions, the budget constraint may be written as

$$M = x + y \Leftrightarrow y = M - x \tag{1}$$

Given (1), the marginal utility of X at $x = x^0$ can be written as

$$MU_x(x^0, M - x^0) = U_x(x^0, M - x^0)$$

where $U_x(.)$ stands for the partial derivative of $U(x, y)$ with respect to x. Given that we are at the bundle $(x^0, M - x^0)$ on the budget line, the total derivative of U_x with respect to x is given by

$$\frac{d[U_x(x^0, M-x^0)]}{dx} = U_{xx}(x^0, M - x^0) - U_{xy}(x^0, M - x^0) \tag{2}$$

where U_{xx} and U_{xy} represent the partial derivatives of U_x with respect to x and y respectively.

In general we may write

$$\frac{d[U_x(x, M-x)]}{dx} = U_{xx}(x, M - x) - U_{xy}(x, M - x) \tag{3}$$

Similarly, the total derivative of the marginal utility of Y with respect to y may be written down as

$$\frac{d[U_y(y, M-y)]}{dy} = U_{yy}(y, M - y) - U_{xy}(y, M - y) \tag{4}$$

The expressions given by (3) and (4) refer to slopes of the BCMUSs of X and Y with respect to x and y respectively. In other words, these represent the rates of change of the marginal utility of X (Y) as we move down or up the budget line. Equations (3) and (4) give rise to the following propositions for constant U_{xx}, U_{yy} and U_{xy}.

Proposition 1: *Given $U_{xx}\left(U_{yy}\right) < 0$ and $U_{xy} > 0$, the slope of the BCMUS of X(Y) with respect to x (y), given by $\frac{d[U_x(x,M-x)]}{dx}$ $\left(\frac{d\left[U_y(y,M-y)\right]}{dy}\right)$, is negative. Thus, the condition $U_x = U_y$ when coupled with satisfaction of the constraint $M = x + y$ does indeed correspond to consumer equilibrium.*

Thus, Proposition 1 says that given the validity of law of diminishing marginal utility and positive complementarity in regard to utility between X and Y, (in the sense that an increase in x raises the marginal utility of Y and an increase in y raises the marginal utility of X), the marginal utility of X (Y) decreases as we move down (up) the budget line in x-y space. In short, the BCMUS of $X(Y)$ is negatively sloped. As seen in the diagrammatic representation of downward sloping BCMUSs of X and Y and the related discussion, the point of intersection of these schedules (corresponding to $U_x = U_y$) yields consumer equilibrium.

Proposition 2: *If $U_{xx}, U_{yy}, U_{xy} > 0$; and $|U_{ii}| < U_{xy}\,(i = x, y)$ then the slope of the BCMUS of X(Y) with respect to x (y), given by $\frac{d[U_x(x,M-x)]}{dx}$ $\left(\frac{d\left[U_y(y,M-y)\right]}{dy}\right)$, is negative. Thus, the condition $U_x = U_y$ coupled with satisfaction of the constraint $M = x + y$ does indeed correspond to consumer equilibrium. On the other hand, if $|U_{ii}| \geq U_{xy}\,(i = x, y)$ then $\frac{d[U_x(x,M-x)]}{dx}\left(\frac{d\left[U_y(y,M-y)\right]}{dy}\right) \geq 0$ i.e. the BCMUSs of X(Y) are either positively sloped or have zero slope. In case these are positively sloped (i.e. the condition $|U_{ii}| > U_{xy}$ holds) then the allocation corresponding to the condition $U_x = U_y$ and the satisfaction of the constraint $M = x + y$ is associated with utility minimization and therefore does not correspond to consumer equilibrium i.e. the consumer equilibrium in this case is either $(M, 0)$ or $(0, M)$ or both, i.e. whichever of these bundles provides the higher utility or both if these bundles provide the same utility. In case $|U_{ii}| = U_{xy}\,(i = x, y)$, the BCMUSs have zero slope and three results are possible: a) the BCMUS of X lies above that of Y in the pseudo-Edgeworth apparatus, implying that $(M, 0)$ is the equilibrium; b) the BCMUS of X lies below that of Y, implying that $(0, M)$ is the equilibrium; and c) the two BCMUSs (of X and Y) coincide with each other, thus implying that any allocation satisfying the constraint $M = x + y$ is an equilibrium.*

In discussing Proposition 2 it is obvious that "$|U_{ii}| < U_{xy}\,(i = x, y)$ where $U_{ii} > 0$ and $U_{xy} > 0$" involves downward sloping BCMUSs (see equations (3) and (4)) and, therefore, the intersection of these corresponds to the utility maximizing allocation (see Fig.6 and the related discussion). If $|U_{ii}| > U_{xy}$ *for* $U_{ii}, U_{xy} > 0$ then we know from equations (3) and (4) that the BCMUSs are upward sloping. Thus, the consumer equilibrium is given by the discussion related to Fig.7 which is consistent with the statement of Proposition 2.

Proposition 2 also provides the condition for BCMUSs to be horizontal. In case BCMUSs are both horizontal and the schedule for X (Y) lies above that for Y(X), then any increase in x(y) brings about a gain in utility which is more in terms of magnitude than the loss in utility associated with a decrease in $y(x)$. Thus, the net change in utility corresponding to an increase in x(y) is always positive. This leads to the conclusions stated in a) and b) of Proposition 2. In addition, Proposition 2 also states that coincidence of the BCMUSs leads to an infinite number of equilibria given by all those bundles which lie on the budget line. This is because at any allocation, $U_x = U_y$, which implies that a rightward or leftward movement, provided these are possible, from any bundle on the budget line always results in a change of utility that equals 0, the gain in utility from an increase in x(y) always equalling the loss in utility caused by the associated decrease in y(x). Thus, every bundle on the horizontal scale in the pseudo-Edgeworth apparatus will provide equal utility i.e. there are multiple consumer equilibria.

The reader is being referred to the table at the end of this chapter and to Fig. 16, Fig. 17 in the third chapter and from Fig. 32 to Fig. 43 in the appendix for depiction of another possible six sub-cases under this case.

Proposition 3: *If* $U_{xx}, U_{yy} > 0$*; and* $U_{xy} < 0$ *then the slope of the BCMUS of X(Y) with respect to x (y), given by* $\frac{d[U_x(x,M-x)]}{dx}$ $\left(\frac{d[U_y(y,M-y)]}{dy}\right)$*, is positive. Thus, the condition* $U_x = U_y$ *when coupled with satisfaction of the constraint* $M = x + y$ *does not correspond to consumer equilibrium but minimization of utility subject to the mentioned constraint.*

Proposition 3 follows straightforwardly from (3) and (4). As stated and explained above, given the positive slope of the BCMUSs of X and Y, consumer equilibrium is achieved at one of the corner commodity bundles, $(M, 0)$ or $(0, M)$; or both if the utility corresponding to both these bundles is equal.

Proposition 4: *Given* $U_{xx}(U_{yy}) < 0$ *and* $U_{xy} < 0$*, the slope of the BCMUS of X(Y) with respect to x(y), given by* $\frac{d[U_x(x,M-x)]}{dx}$ $\left(\frac{d[U_y(y,M-y)]}{dy}\right)$*, is ambiguous. In the case where* $|U_{xy}| > |U_{ii}| (i = x, y)$ *the BCMUSs of both X and Y are positively sloped. Thus, the condition,* $U_x = U_y$*, when coupled with satisfaction of the constraint* $M = x + y$*, is associated with minimization of utility subject to satisfaction of the mentioned constraint* as depicted in Fig. 7. *The consumer equilibrium is at* $(M, 0)$ *or* $(0, M)$ *(depending on which of these corner commodity bundles provides greater utility than the other) or both (if both commodity bundles provide the same level of utility). In the case where* $|U_{xy}| < |U_{ii}| (i = x, y)$ *the BCMUSs of both X and Y are negatively sloped. Thus, the condition,* $U_x = U_y$ *, when coupled with satisfaction of the constraint* $M = x + y$ *, does indeed correspond to consumer equilibrium as depicted in Fig. 6 in chapter 3. In the case where* $|U_{xy}| = |U_{ii}|$ $(i = x, y)$*, the BCMUS of X (Y) has zero slope (it is a horizontal line in x-*U_x *or* $y-U_y$ *space). Such a case is characterized by the condition i)*$U_x > U_y$ *or ii)* $U_x < U_y$ *or iii)*$U_x = U_y$ *being satisfied for all commodity bundles which are on the budget line,* $M = x + y$*. In sub-case (i) or (ii), obviously satisfaction of* $U_x = U_y$ *is not possible for any commodity bundle on the budget line,* $M = x + y$*. In sub-case (i), the commodity bundle* $(M, 0)$

maximizes utility subject to the budget constraint while in sub-case (ii) the same is done by $(0, M)$ *i.e.* $(M, 0)$ *and* $(0, M)$ *are the consumer equilibria in sub-cases (i) and (ii) respectively. In sub-case (iii) utility is the same at all consumer bundles on the budget line,* $M = x + y$*, as the marginal utility schedules of X and Y are coincident at all consumer bundles in the pseudo-Edgeworth apparatus i.e. there are an infinite number of consumer equilibria, one corresponding to each consumer bundle on the budget line.*

Proposition 4 follows straightforwardly from (3) and (4) stated above. Another six sub-cases are possible under this proposition which would be clear from the table at the end of this chapter and those sub-cases can be illustrated by Fig. 16, Fig. 17 of chapter 3 and from Fig. 32 to Fig. 43 in the appendix.

Proposition 5: *Given* $U_{xx}(U_{yy}) = 0$ *and* $U_{xy} > 0$*, the slope of the BCMUS of X(Y) with respect to x(y), given by* $\frac{d[U_x(x,M-x)]}{dx}$ $(\frac{d[U_y(y,M-y)]}{dy})$*, is negative. The consumer equilibrium is given by the condition* $U_x = U_y$ *i.e. the allocation corresponding to the intersection of the BCMUSs of X and Y. On the other hand, if* $U_{xx}(U_{yy}) = 0$ *and* $U_{xy} < 0$ *then the slope of the BCMUS of X(Y) with respect to x(y) is positive. The condition,* $U_x = U_y$*, now yields the utility minimizing allocation on the budget line. Consumer equilibrium is given by i)* $(M, 0)$ *if* $U(M, 0) > U(0, M)$*; or ii)* $(0, M)$ *if* $U(M, 0) < U(0, M)$*; or iii) both* $(M, 0)$ *and* $(0, M)$ *if* $U(M, 0) = U(0, M)$*. Finally, if* $U_{xx}(U_{yy}) = U_{xy} = 0$ *then the BCMUS of X(Y) has zero slope (i.e the BCMUSs are horizontal lines in* $x - U_x$ *and* $y - U_y$ *space respectively). The consumer equilibrium, given constant marginal utilities of X(Y) along the BCMUS, is given by i)* $(M, 0)$ *if* $U_x > U_y$*; or ii)* $(0, M)$ *if* $U_x < U_y$*; or iii) each allocation on the budget line if* $U_x = U_y$*.*

Note that Proposition 5 follows straightforwardly from equations (3) and (4) as well as the properties of consumer equilibrium derived before for upward sloping, downward sloping and horizontal BCMUSs of X (Y).

Finally note that if $U_{xy} = 0$, the BCMUS and the conventional marginal utility schedule for either X or Y are identical (as there is a unique conventional marginal utility schedule for both X and Y). Our discussion and conclusions about the properties and conditions underlying consumer equilibrium would hold.

Proposition 6: Given $U_{xx} > 0, U_{yy} < 0, U_{xy} > 0$, the slope of the BCMUS of Y with respect to y, given by $\frac{d[U_y(y,M-y)]}{dy}$ is unambiguously negative. But the slope of the BCMUS of X with respect to x given by $\frac{d[U_x(x,M-x)]}{dx}$ is $\begin{cases} positive \\ zero \\ negative \end{cases}$ if $\begin{cases} |U_{xx}| > |U_{xy}| \\ |U_{xx}| = |U_{xy}| \\ |U_{xx}| < |U_{xy}| \end{cases}$. Thus the condition $U_x = U_y$

when coupled with satisfaction of the constraint M=x+y does indeed correspond to consumer equilibrium if $|U_{xx}| \leq |U_{xy}|$.

In discussing Proposition 6, note that under the assumption of positive complementarity between the two goods i.e. the marginal utility of X(Y) decreases as we move down (up) the budget line, consumer equilibrium is achieved corresponding to the intersection point of the two BCMUSs if the law of diminishing marginal utility holds for commodity Y and the marginal utility of commodity X is an increasing function of x, but the rate of increase is overwhelmed by positive complementarity. Consumer equilibrium can be illustrated by Fig. 6 in that case. If $|U_{xx}| > |U_{xy}|$, then different possibilities of consumer equilibria are depicted from Fig. 39 to Fig. 43 in the appendix.

Proposition 7: If $U_{xx} > 0, U_{yy} < 0, U_{xy} < 0,$ the slope of the BCMUS of X with respect to x, given by $\frac{d[U_x(x,M-x)]}{dx}$ is unambiguously positively sloped; whereas the slope of the BCMUS of Y with respect to y, given by $\frac{d[U_y(y,M-y)]}{dy}$ is $\begin{cases} positive \\ zero \\ negative \end{cases}$ depending on $\begin{cases} |U_{xy}| > |U_{yy}| \\ |U_{xy}| = |U_{yy}| \\ |U_{xy}| < |U_{yy}| \end{cases}$.Thus the condition $U_x = U_y$ when coupled with satisfaction of the constraint M=x+y does indeed correspond to utility minimizing bundle on the budget line if $|U_{xy}| \geq |U_{yy}|$ and therefore definitely does not correspond to consumer equilibrium.

Thus Proposition 7 tells us that given the assumption of negative complementarity between the two goods i.e. the marginal utility of X(Y) increases as we move down (up) the budget line, utility minimization occurs corresponding to the intersection point of the two BCMUSs if the law of diminishing marginal utility holds for commodity Y and the marginal utility of commodity X is an increasing function of x, given that the absolute value of the rate of decrease of marginal utility of commodity Y does not exceed the magnitude of the negative complementarity between the two commodities. Utility minimization can be depicted by Fig. 7 in that case. If $|U_{yy}| > |U_{xy}|$, then different possibilities of consumer equilibria are depicted from Fig. 39 to Fig. 43 in the appendix.

Proposition 8: Given $U_{xx} < 0, U_{yy} > 0, U_{xy} > 0,$ the slope of the BCMUS of X with respect to x, given by $\frac{d[U_x(x,M-x)]}{dx}$ is negative ; whereas the slope of the BCMUS of Y with respect to y, given by $\frac{d[U_y(y,M-y)]}{dy}$ is $\begin{cases} positive \\ zero \\ negative \end{cases}$ if $\begin{cases} |U_{xy}| < |U_{yy}| \\ |U_{xy}| = |U_{yy}| \\ |U_{xy}| > |U_{yy}| \end{cases}$.Thus the condition $U_x = U_y$ when coupled with satisfaction of the constraint M=x+y does indeed correspond to consumer equilibrium when $|U_{xy}| \geq |U_{yy}|$.

Proposition 8 tells us that under the assumption of positive complementarity between the two goods i.e. the marginal utility of X(Y) decreases as we move down (up) the budget line, consumer equilibrium is achieved corresponding to the intersection point of the two BCMUSs if the law of diminishing marginal utility holds for commodity X and the marginal utility of commodity Y is an increasing function of y, but the rate of increase is overwhelmed by positive complementarity. Determining of consumer equilibrium can be depicted by Fig. 6 in that case. If $\left|U_{xy}\right| < \left|U_{yy}\right|$, then the different possibilities of consumer equilibria are depicted from Fig. 34 to Fig. 38 in the appendix.

Proposition 9: If $U_{xx} < 0, U_{yy} > 0, U_{xy} < 0,$ the slope of the BCMUS of Y with respect to y, given by $\frac{d[U_y(y,M-y)]}{dy}$ is unambiguously positive; whereas the slope of the BCMUS of X with respect to x, given by $\frac{d[U_x(x,M-x)]}{dx}$ is $\begin{cases} positive \\ zero \\ negative \end{cases}$ if $\begin{cases} |U_{xx}| < |U_{xy}| \\ |U_{xx}| = |U_{xy}| \\ |U_{xx}| > |U_{xy}| \end{cases}$. Thus the condition $U_x = U_y$ when coupled with satisfaction of the constraint M=x+y does indeed correspond to utility minimization if $|U_{xx}| \leq |U_{xy}|$.

Thus Proposition 9 tells us that given the assumption of negative complementarity between the two commodities i.e. the marginal utility of X(Y) increases as we move down (up) the budget line, utility minimization occurs corresponding to the intersection point of the two BCMUSs if the law of diminishing marginal utility holds for commodity X and the marginal utility of commodity Y is an increasing function of y, given that the absolute value of the rate of decrease of marginal utility of commodity X does not exceed the magnitude of the negative complementarity between the two commodities. Utility minimization can be illustrated by Fig. 7 in that case. If $|U_{xx}| > \left|U_{xy}\right|$, then different possibilities of consumer equilibria are depicted from Fig. 34 to Fig. 38 in the appendix.

Three sub-cases lie under each proposition from Proposition 6 to Proposition 9 which is apparent from the table below and those sub-cases can be illustrated by the related diagrams of the third chapter and of the appendix.

The above propositions (1-9) highlight and are encapsulated by the following observation: a necessary condition for consumer equilibrium being achieved with allocations of positive expenditure to both goods is that the marginal utility of expenditure of both goods is equated. However, this is not a sufficient condition. The necessary condition only implies that the schedules for ‘budget constrained marginal utility normalized by price’ intersect in the pseudo-Edgeworth apparatus described earlier in the thesis. But to have a consumer equilibrium at this point of intersection we need to impose certain additional conditions on utility functions. The sets of such conditions, for example a) positive complementarity coupled with diminishing marginal utility and b) increasing marginal utility overwhelmed by positive complementarity, which provide for negatively sloping schedules and therefore utility maximization are highlighted by the propositions. On the other hand, the propositions also highlight obvious sufficient conditions – for example, the opposites of a) and b) --which ensure that the BCMUSs are upward sloping --for ruling out the possibility that the intersection corresponds to utility maximization.

Throughout the propositions we have assumed all third derivatives to be zero. This assumption is necessary for making the algebra tractable. However, once we relax this assumption it is possible to have non-monotonic BCMUSs intersecting in a locality where both schedules are downward sloping and corresponding to a consumer equilibrium i.e. a utility maximizing allocation of expenditure.

TABLE: Consumer equilibria in different cases and sub-cases for constant U_{xx}, U_{yy} and U_{xy}

	Cases	Sub-Cases	Slope of Budget Constrained Marginal Utility Schedules(BCMUSs)	Condition for consumer equilibrium i.e. maximization of utility subject to budget constraint
1.	$U_{xx}, U_{yy} < 0$ and $U_{xy} > 0$	None	Negative (as in Fig. 6)	Consumer equilibrium will be attained at an allocation on the budget line corresponding to $U_x = U_y$
2.	$U_{xx}, U_{yy} < 0$ and $U_{xy} < 0$	(i) $\lvert U_{xy}\rvert > \lvert U_{ii}\rvert$ (i=x,y)	Positive (as in Fig.2 and Fig.7)	Consumer equilibrium will be attained at (i)(M,0) if U(M,0)>U(0,M) (ii)(0,M)if U(M,0)<U(0,M) (iii)both (M,0) and (0,M)if U(M,0)=U(0,M) [Note allocation corresponding to $U_x = U_y$ is the utility minimizing bundle on the budget line]
		(ii) $\lvert U_{xy}\rvert < \lvert U_{ii}\rvert$ (i=x,y)	Negative (as in Fig. 3 and Fig. 6)	Same as Case 1 above
		(iii) $\lvert U_{xy}\rvert = \lvert U_{ii}\rvert$ (i=x,y)	Zero	Consumer equilibrium will be at (i)(M,0) if $U_x > U_y$; (ii)(0,M) if $U_x < U_y$; (iii) all commodity bundles on the budget line if $U_x = U_y$ (Note that in this sub-case marginal utilities of X and Y are constant across all commodity bundles on the budget line.)

		(iv) $\lvert U_{xx}\rvert > \lvert U_{xy}\rvert > \lvert U_{yy}\rvert$	Negative for X Positive for Y	Ambiguous
		(v) $\lvert U_{yy}\rvert > \lvert U_{xy}\rvert > \lvert U_{xx}\rvert$	Positive for X Negative for Y	Ambiguous
		(vi) $\lvert U_{yy}\rvert \neq \lvert U_{xy}\rvert = \lvert U_{xx}\rvert$ and $\lvert U_{xy}\rvert > \lvert U_{yy}\rvert$	Zero for X Positive for Y	Same as Case 2,first sub-case
		(vii) $\lvert U_{yy}\rvert \neq \lvert U_{xy}\rvert = \lvert U_{xx}\rvert$ and $\lvert U_{xy}\rvert < \lvert U_{yy}\rvert$	Zero for X Negative for Y	Same as Case 1above
		(viii) $\lvert U_{xx}\rvert \neq \lvert U_{xy}\rvert = \lvert U_{yy}\rvert$ and $\lvert U_{xy}\rvert > \lvert U_{xx}\rvert$	Positive for X Zero for Y	Same as Case 2,first sub-case
		(ix) $\lvert U_{xx}\rvert \neq \lvert U_{xy}\rvert = \lvert U_{yy}\rvert$ and $\lvert U_{xy}\rvert < \lvert U_{xx}\rvert$	Negative for X Zero for Y	Same as Case 1 above
3.	$U_{xx}, U_{yy} > 0 \; and \; U_{xy} > 0$	(i) $\lvert U_{xy}\rvert > \lvert U_{ii}\rvert$ (i=x,y) (as in Fig. 25)	Negative	Same as Case 1 above
		(ii) $\lvert U_{xy}\rvert < \lvert U_{ii}\rvert$ (i=x,y) (as in Fig. 15)	Positive	Same as Case 2,first sub-case
		(iii) $\lvert U_{xy}\rvert = \lvert U_{ii}\rvert$ (i=x,y)	Zero	Same as Case 2,third sub-case
		(iv) $\lvert U_{xx}\rvert > \lvert U_{xy}\rvert > \lvert U_{yy}\rvert$	Positive for X Negative for Y	Same as Case 2, fifth sub-case
		(v) $\lvert U_{yy}\rvert > \lvert U_{xy}\rvert > \lvert U_{xx}\rvert$	Negative for X Positive for Y	Same as Case 2,fourth sub-case
		(vi) $\lvert U_{yy}\rvert \neq \lvert U_{xy}\rvert = \lvert U_{xx}\rvert$ and $\lvert U_{yy}\rvert > \lvert U_{xy}\rvert$	Zero for X Positive for Y	Same as Case 2,first sub-case
		(vii) $\lvert U_{yy}\rvert \neq \lvert U_{xy}\rvert = \lvert U_{xx}\rvert$ and $\lvert U_{yy}\rvert < \lvert U_{xy}\rvert$	Zero for X Negative for Y	Same as Case 1 above
		(viii) $\lvert U_{yy}\rvert = \lvert U_{xy}\rvert \neq \lvert U_{xx}\rvert$ and $\lvert U_{xx}\rvert > \lvert U_{xy}\rvert$	Positive for X Zero for Y	Same as Case 2,first sub-case
		(ix) $\lvert U_{yy}\rvert = \lvert U_{xy}\rvert \neq \lvert U_{xx}\rvert$ and $\lvert U_{xx}\rvert < \lvert U_{xy}\rvert$	Negative for X Zero for Y	Same as Case 1 above
4.	$U_{xx}, U_{yy} > 0 \; and \; U_{xy} < 0$ (as in Fig.32)	None	Positive	Same as Case 2,first sub-case
5.	U_{xx}, U_{yy} =0	$(i) U_{xy}$ >0 (as in Fig. 30)	Negative	Same as Case 1 above
		$(ii) U_{xy}$ <0 (as in Fig.31)	Positive	Same as Case 2,first sub-case
		$(iii) U_{xy}$ =0	Zero	Same as Case 2,third sub-case
6.	$U_{xx} > 0, U_{yy} < 0 \; and \; U_{xy} > 0$	(i) $\lvert U_{xx}\rvert > \lvert U_{xy}\rvert$	Positive for X Negative for Y	Same as Case 2,fifth sub-case
		(ii) $\lvert U_{xx}\rvert < \lvert U_{xy}\rvert$	Negative for X Negative for Y	Same as Case 1 above
		(iii) $\lvert U_{xx}\rvert = \lvert U_{xy}\rvert$	Zero for X Negative for Y	Same as Case 1 above

7.	$U_{xx} > 0, U_{yy} < 0 \; and \; U_{xy} < 0$	(i)$\lvert U_{xy} \rvert > \lvert U_{yy} \rvert$	Positive for X Positive for Y	Same as Case 2,first sub-case
		(ii)$\lvert U_{xy} \rvert < \lvert U_{yy} \rvert$	Positive for X Negative for Y	Same as Case 2, fifth sub-case
		(iii)$\lvert U_{xy} \rvert = \lvert U_{yy} \rvert$	Positive for X Zero for Y	Same as Case 2,first sub-case
8.	$U_{xx} < 0, U_{yy} > 0 \; and \; U_{xy} > 0$	(i)$\lvert U_{xy} \rvert > \lvert U_{yy} \rvert$	Negative for X Negative for Y	Same as Case 1 above
		(ii)$\lvert U_{xy} \rvert < \lvert U_{yy} \rvert$	Negative for X Positive for Y	Same as Case 2,fourth sub-case
		(iii)$\lvert U_{xy} \rvert = \lvert U_{yy} \rvert$	Negative for X Zero for Y (as in Fig. 33)	Same as Case 1 above
9.	$U_{xx} < 0, U_{yy} > 0 \; and \; U_{xy} < 0$	(i)$\lvert U_{xx} \rvert > \lvert U_{xy} \rvert$	Negative for X Positive for Y	Same as Case 2,fourth sub-case
		(ii)$\lvert U_{xx} \rvert < \lvert U_{xy} \rvert$	Positive for X Positive for Y	Same as Case 2,first sub-case
		(iii)$\lvert U_{xx} \rvert = \lvert U_{xy} \rvert$	Zero for X Positive for Y	Same as Case 2,first sub-case

Chapter 5:-

Simple Algebraic Description of Consumer Equilibrium Using Budget Constrained Marginal Utility Schedules and Variable Relative Prices

Till now we have assumed that the prices of both commodities, X and Y, is unity. This can be done by suitable choice of units. However, such an assumption is not suitable if we are interested in understanding how consumer equilibrium changes with respect to price(s). Thus, in this chapter, we shall denote the price of X as p_x and this can be of any positive magnitude while we shall assume the price of Y to be unity. This makes sense as y, the quantity of Y, can be assumed to be the aggregate of expenditure on goods other than X , thus making it possible to capture the *n* good case through an algebraic illustration in terms of 2 goods. Thus we can write the utility function as $U = U(x, y)$. We also make some further assumptions – all third derivatives of the utility function are taken to be zero i.e. the derivatives U_{xx} , U_{xy} and U_{yy} are assumed to be constants. Now marginal utility with respect to x may be written as $U_x(x, y) = \frac{\partial[U(x,y)]}{\partial x}$. However, this marginal utility with respect to x for a bundle on the budget constraint may be expressed as $U_x(x, M - xp_x)$

Further, the slope of the marginal utility of x for a bundle (x, y) lying on the budget constraint is given by $\frac{d(MU_x)}{dx} = U_{xx} - p_x U_{xy}$ (5)

Now for given U_{xx} and U_{xy} (remember these are constants by assumption) we can write MU_x as a function of x only (the level of y being automatically determined by the constraint $M = xp_x + y$)

$$U_x(x, M - xp_x) = \int \frac{d(MU_x)}{dx} dx = (U_{xx} - p_x U_{xy})\text{x} + \text{c} \quad (6)$$

In equation (6), c is an integration or arbitrary constant. Note that this equation only applies for values of x lying between 0 and $\frac{M}{p_x}$, as the corresponding values of y cannot be negative, given satisfaction of the budget constraint. Now from the equation above, we can see that $c = U_x(0, M)$. Thus, we may write

$$U_x(x, M - xp_x) = (U_{xx} - p_x U_{xy})x + U_x(0, M) \quad (7)$$

Similarly, the marginal utility with respect to y for a bundle on the budget constraint may be written as $MU_y = U_y\left(\frac{M-y}{p_x}, y\right)$.

Again the slope of the marginal utility of y for a bundle (x, y) lying on the budget constraint is given by $\frac{d(MU_y)}{dy} = U_{yy} - \frac{U_{xy}}{p_x}$ (8)

Now this may be integrated with respect to y to get back the marginal utility schedule.

$$U_y(\frac{M-y}{p_x}, y) = \int \frac{d(MU_y)}{dy} dy = (U_{yy} - \frac{U_{xy}}{p_x})y + c_0 \quad (9)$$

where c_0 is an integration constant given by $U_y(\frac{M}{p_x}, 0)$

Thus, we may write

$$U_y(\frac{M-y}{p_x}, y) = (U_{yy} - \frac{U_{xy}}{p_x})y + U_y(\frac{M}{p_x}, 0) \quad (10)$$

Now, given constant U_{xy}, we can express (7) and (10) as follows:

$$U_x(x, M - xp_x) = (U_{xx} - p_x U_{xy})x + U_x(0,0) + U_{xy}M \quad (11)$$

$$U_y(x, M - xp_x) = (U_{yy} - \frac{U_{xy}}{p_x})(M - xp_x) + U_y(0,0) + U_{xy}\frac{M}{p_x} \quad (12)$$

Now note that $\frac{d(MU_x)}{dx}$ and $\frac{d(MU_y)}{dy}$ will be negative as long as $U_{ii} < 0$ $(i = x, y)$ and $U_{xy} \geq 0$. In other words, the BCMUSs will be downward sloping in $i - U_i$ space (i=x,y). This implies that consumer equilibrium will be given by $(x^*, M - x^*p_x)$ where $\frac{U_{x^*}(x^*, M - x^*p_x)}{p_x} = U_{y^*}(x^*, M - x^*p_x)$ if such a value of $x^* \epsilon [0, \frac{M}{p_x}]$. Thus x* will be the value of x which solves the following equation:

$$\frac{(U_{xx} - p_x U_{xy})x + U_x(0,0) + U_{xy}M}{p_x} = (U_{yy} - \frac{U_{xy}}{p_x})(M - xp_x) + U_y(0,0) + U_{xy}\frac{M}{p_x}$$

This yields

$$x^* = \frac{[U_y(0,0)p_x - U_x(0,0)] + M[U_{yy}p_x - U_{xy}]}{[U_{xx} - 2p_x U_{xy} + p_x^2 U_{yy}]}$$

Note that $x *> 0$ under the stated sign of $U_{ii}(i = x, y)$ and U_{xy} iff the numerator of the expression for x^* is negative i.e. $[U_y(0,0)p_x - U_x(0,0)] + M[U_{yy}p_x - U_{xy}] < 0$.

From (7) and (10), the equilibrium condition can be written as follows:

$$\frac{(U_{xx} - p_x U_{xy})x + U_x(0, M)}{p_x} = \left(U_{yy} - \frac{U_{xy}}{p_x}\right)(M - xp_x) + U_y\left(\frac{M}{p_x}, 0\right)$$

$$\Rightarrow x^* = \frac{\left[U_y\left(\frac{M}{p_x}, 0\right) - \frac{U_x(0,M)}{p_x}\right] + M\left[U_{yy} - \frac{U_{xy}}{p_x}\right]}{\frac{U_{xx}}{p_x} - 2U_{xy} + p_x U_{yy}}$$

Note that $x *> 0$ under the stated sign of $U_{ii}(i = x, y)$ and U_{xy} iff

$$\left[U_y\left(\frac{M}{p_x}, 0\right) - \frac{U_x(0, M)}{p_x}\right] + M\left[U_{yy} - \frac{U_{xy}}{p_x}\right] < 0$$

$$\Rightarrow \left[U_y\left(\frac{M}{p_x}, 0\right) - \frac{U_x(0,M)}{p_x}\right] < M\left[-U_{yy} + \frac{U_{xy}}{p_x}\right] \tag{13}$$

Now given that (10) is expressed in terms of y, (7) too can be expressed in terms of y as that enables the determination of equilibrium level of y by equating the RHS of (10) and (7), of which the latter may be expressed as follows:

$$U_x\left(\frac{M-y}{p_x}, y\right) = (U_{xx} - p_x U_{xy})(\frac{M-y}{p_x}) + U_x(0, M) \tag{14}$$

From the RHS of (10) and (14) as expressed above yields

$$\frac{1}{p_x}[(U_{xx} - p_x U_{xy})\left(\frac{M-y}{p_x}\right) + U_x(0.M)] = (U_{yy} - \frac{U_{xy}}{p_x})y + U_y(\frac{M}{p_x}, 0)$$

This yields the solution for y* as

$$y *= \frac{M\left(-\frac{U_{xx}}{p_x^2} + \frac{U_{xy}}{p_x}\right) + [U_y\left(\frac{M}{p_x}, 0\right) - \frac{U_x(0,M)}{p_x}]}{-\frac{U_{xx}}{p_x^2} + U_{xy}\left(\frac{2}{p_x}\right) - U_{yy}}$$

Note that $y *> 0$ under the stated sign of U_{ii} $(i = x, y)$ and U_{xy} iff

$$M\left(-\frac{U_{xx}}{p_x^2} + \frac{U_{xy}}{p_x}\right) + [U_y\left(\frac{M}{p_x}, 0\right) - \frac{U_x(0, M)}{p_x}] > 0$$

$$\Rightarrow [U_y\left(\frac{M}{p_x}, 0\right) - \frac{U_x(0,M)}{p_x}] > M\left(\frac{U_{xx}}{p_x^2} - \frac{U_{xy}}{p_x}\right) \tag{15}$$

Note that an interior solution is obtained iff $y *> 0$ and $x *> 0$. This implies $x^*\epsilon(0, \frac{M}{p_x}) \Leftrightarrow y^*\epsilon(0, M)$. Combining the inequalities (13) and (15) both $y *> 0$ and $x *> 0$ hold if we have the following condition for an interior solution

$$M\left(\frac{U_{xy}}{p_x} - U_{yy}\right) > \left[U_y\left(\frac{M}{p_x}, 0\right) - \frac{U_x(0,M)}{p_x}\right] > M\left(\frac{U_{xx}}{p_x^2} - \frac{U_{xy}}{p_x}\right) \tag{16}$$

Condition (16) is necessary as well as sufficient condition for both $x *> 0$ and $y *> 0$ simultaneously under the assumed sign of U_{ii} $(i = x, y)$ and U_{xy}. The lower bound of the term $\left[U_y\left(\frac{M}{p_x}, 0\right) - \frac{U_x(0,M)}{p_x}\right]$, as indicated by the above inequality, is negative and its upper bound is positive under the stated sign of U_{ii} $(i = x, y)$ and U_{xy}. Hence, there is enough scope for the inequality to hold good. For example, if $U_y\left(\frac{M}{p_x},0\right)-\frac{U_x(0,M)}{p_x}$ *is positive* but small then this inequality should hold for large enough magnitudes of U_{xy} ,U_{yy} and M.

Concrete illustrations of consumer equilibrium using BCMUS corresponding to specific functional forms

Now, we shall illustrate consumer equilibrium with a couple of examples using BCMUS. For clarity of exposition the prices of both the commodities are assumed to be one in all the examples unless otherwise stated. At first a Cobb-Douglas type utility function $U=x^{\frac{1}{2}}y^{\frac{1}{2}}$ is looked into. The conditions $\frac{\partial(MU_x)}{\partial x}=-\frac{1}{4}x^{-\frac{3}{2}}\{\frac{M}{p_y}-\left(\frac{p_x}{p_y}\right)x\}^{\frac{1}{2}}-\frac{1}{4}x^{-\frac{1}{2}}\left\{\frac{M}{p_y}-\left(\frac{p_x}{p_y}\right)x\right\}^{-\frac{1}{2}}(\frac{p_x}{p_y})<0$ and $\frac{\partial(MU_y)}{\partial y}=-\frac{1}{4}y^{-\frac{3}{2}}\{\frac{M}{p_x}-\left(\frac{p_y}{p_x}\right)y\}^{\frac{1}{2}}-\frac{1}{4}y^{-\frac{1}{2}}\left\{\frac{M}{p_x}-\left(\frac{p_y}{p_x}\right)y\right\}^{-\frac{1}{2}}(\frac{p_y}{p_x})<0$ implies that the BCMUS of x and the BCMUS of y are unambiguously negatively sloped. Thus, the two downward sloping BCMUSs intersect each other at $(x^*,y^*)=\left(\frac{M}{2},\frac{M}{2}\right)$ and consumer equilibrium is achieved corresponding to the intersection point as depicted in Fig. 19. It can be verified diagrammatically as well that any deviation from (x^*,y^*) in either direction results in decrease in total utility.

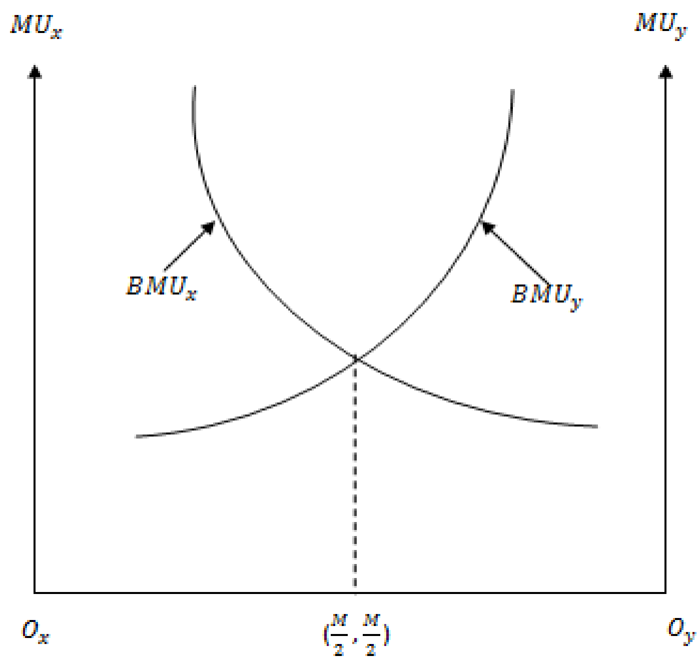

Figure 19 : Consumer equilibrium if the utility function is $U=x^{\frac{1}{2}}y^{\frac{1}{2}}$ and $p_x=p_y=1$.

Next, we shall delve into a few examples where the utility functions are positive monotonic transformations of the utility function $U=x^{\frac{1}{2}}y^{\frac{1}{2}}$. The utility function $U=xy$ exhibits diminishing marginal utility. The BCMUS of both x and y are unambiguously negatively sloped since $\frac{\partial(MU_x)}{\partial x} = -\frac{p_x}{p_y} < 0$ and $\frac{\partial(MU_y)}{\partial y} = -(\frac{p_y}{p_x}) < 0$. Consumer equilibrium is achieved corresponding to the intersection point of the two BCMUSs where $MU_x = MU_y$ i.e. $\left(\frac{M}{p_y}\right) - \left(\frac{p_x}{p_y}\right)x = \left(\frac{M}{p_x}\right) - \left(\frac{p_y}{p_x}\right)y$. If $p_x = p_y = 1$ then x = y $= \frac{M}{2}$ and $\frac{\partial(MU_x)}{\partial x} = \frac{\partial(MU_y)}{\partial y} = -1$. Utility maximization occurs corresponding to the allocation $(x^*, y^*) = (\frac{M}{2}, \frac{M}{2})$.

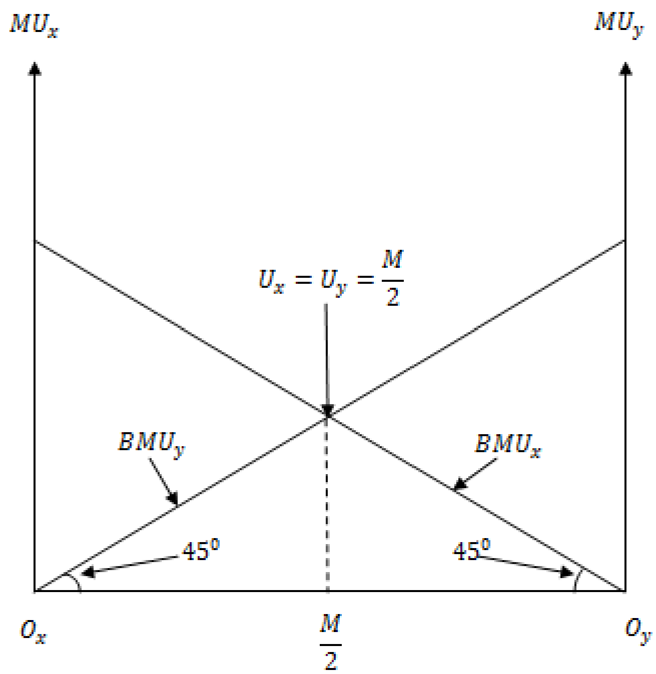

Figure 20 : Consumer equilibrium if the utility function is $U = xy$ and $p_x = p_y = 1$.

The utility function $U = x^2y^2$ exhibits increasing marginal utility. The BCMUSs of x and y are given by the following equations:

$BMU_x = 2x\left(\frac{M-p_x x}{p_y}\right)^2$ and $BMU_y = 2y\left(\frac{M-p_y y}{p_x}\right)^2$

BMU_x will take value zero and $2M\left(\frac{M-p_x M}{p_y}\right)^2$ when $x = 0$ and $x = M$ respectively. Similarly, BMU_y will take value zero and $2M\left(\frac{M-p_y M}{p_x}\right)^2$ when $y = 0$ and $y = M$ respectively. It can be shown that

$\frac{\delta(MU_x)}{\delta x} = 2y\left[3y - 2\frac{M}{p_y}\right] = 2\left(\frac{M-p_x x}{p_y}\right)\left[\frac{M-3p_x x}{p_y}\right]$

For $\frac{\delta(MU_x)}{\delta x}$ to be negative, M $< 3p_x x \; and \; M > p_x x \Rightarrow \frac{M}{3p_x} < x < \frac{M}{p_x}$. Therefore, the BCMUS of x is downward sloping in the range $\frac{M}{3p_x} < x < \frac{M}{p_x}$. Similarly, the BCMUS of y is downward sloping in the range $\frac{M}{3p_y} < y < \frac{M}{p_y}$. Stable consumer equilibrium can be achieved if both the BCMUSs intersect each other at their downward sloping stretches. To get the stationary point of the BCMUS of x i.e. the point at which the BCMUS of x changes its curvature, $\frac{\delta(MU_x)}{\delta x}$ has to be set equal to zero which implies x $= \frac{M}{3p_x}$ and $x = \frac{M}{p_x}$ .The value of $\frac{\partial^2(MU_x)}{\partial x^2} \, at \; x = \frac{M}{3p_x} \, is - \frac{4Mp_x}{p_y^2} < 0$ which satisfies the second order condition for unconstrained maximization. Hence, the BCMUS of x corresponding to this type of utility function is inverted U-shaped and reaches its maximum value corresponding to x $= \frac{M}{3p_x}$. Similarly, the BCMUS of y in this case is also inverted U-shaped and reaches its maximum value corresponding to $= \frac{M}{3p_y}$. Now, for the sake of illustration we consider the following two sub-cases.

*Sub-case 1: Suppose both x and y have price equalling unity. Thus the budget constraint of a consumer reduces to M = x+y. In this sub-case, under the stated assumptions the BCMUS of x and y would boil down to

$BMU_x = 2x(M-x)^2$and $BMU_y = 2y(M-y)^2$

The values of BMU_x at $x = 0$and at $x = M$are zero. The values of BMU_y at $y = 0$and at $y = M$are also zero since the two BCMUSs are symmetric in nature in this case. The BCMUS of x and BCMUS of y are downward sloping in the ranges $\frac{M}{3} < x < M$ and $\frac{M}{3} < y < M$ respectively. The BCMUS of x takes the value zero at$x = 0$, then increases as the value of x increases, reaches its maximum value $\left(\frac{2M}{3}\right)^3$ when $x = \frac{M}{3}$ and again falls to zero when $x = M$. Therefore, in the pseudo Edgeworth box kind of diagram the BCMUS of x passes through both the origin of x and the origin of y. The BCMUS of y also follows the same properties. The height of BMU_x at $x = \frac{M}{3}$ is $(\frac{2M}{3})^3$ which is same as the height of BMU_y at $y = \frac{M}{3}$ and $x = \frac{2M}{3}$ since BCMUSs of both x and y are symmetric. The two BCMUSs intersect each other at $(x^*, y^*) = \left(\frac{M}{2}, \frac{M}{2}\right)$ and total utility of a consumer is maximized corresponding to that allocation. Since, the two BCMUSs intersect each other at their downward sloping parts there is a consumer equilibrium at this point.

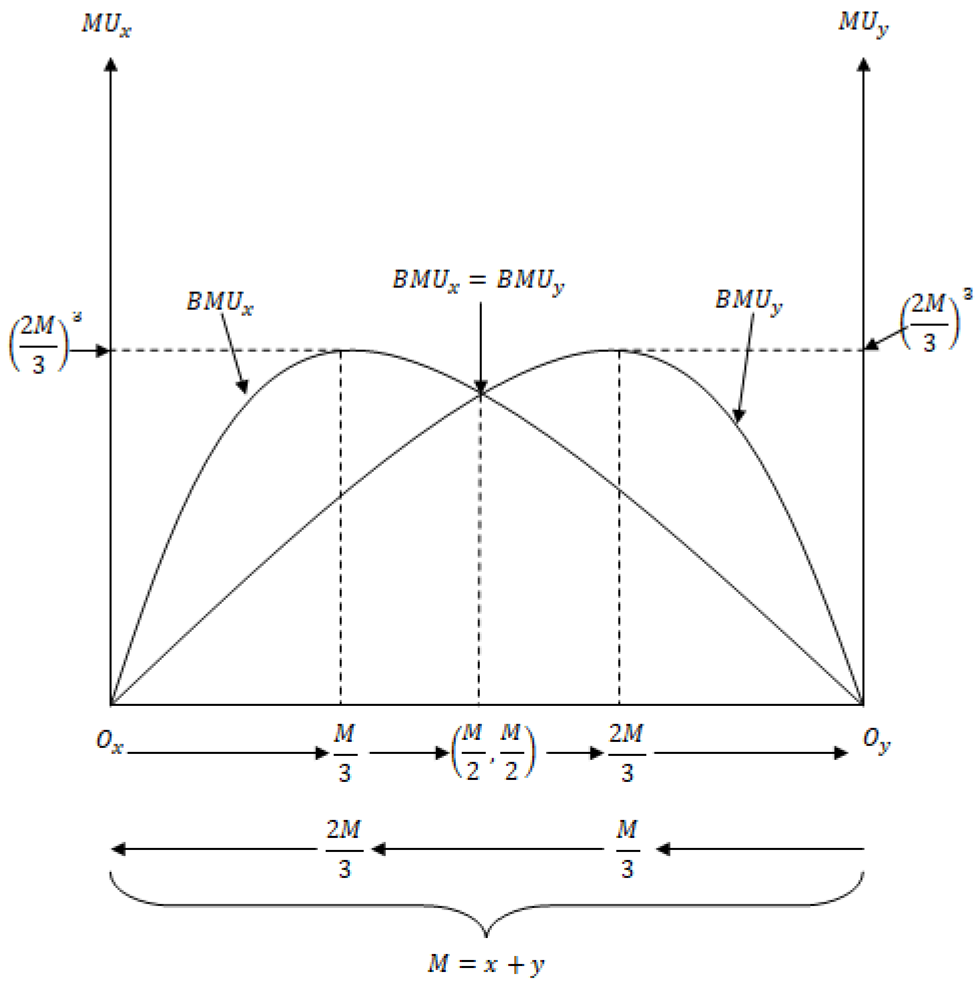

Figure 21: Consumer equilibrium corresponding to the inverted U-shaped BCMUSs of x and y if utility function is characterised by $U = x^2y^2$ where $p_x = p_y = 1$.

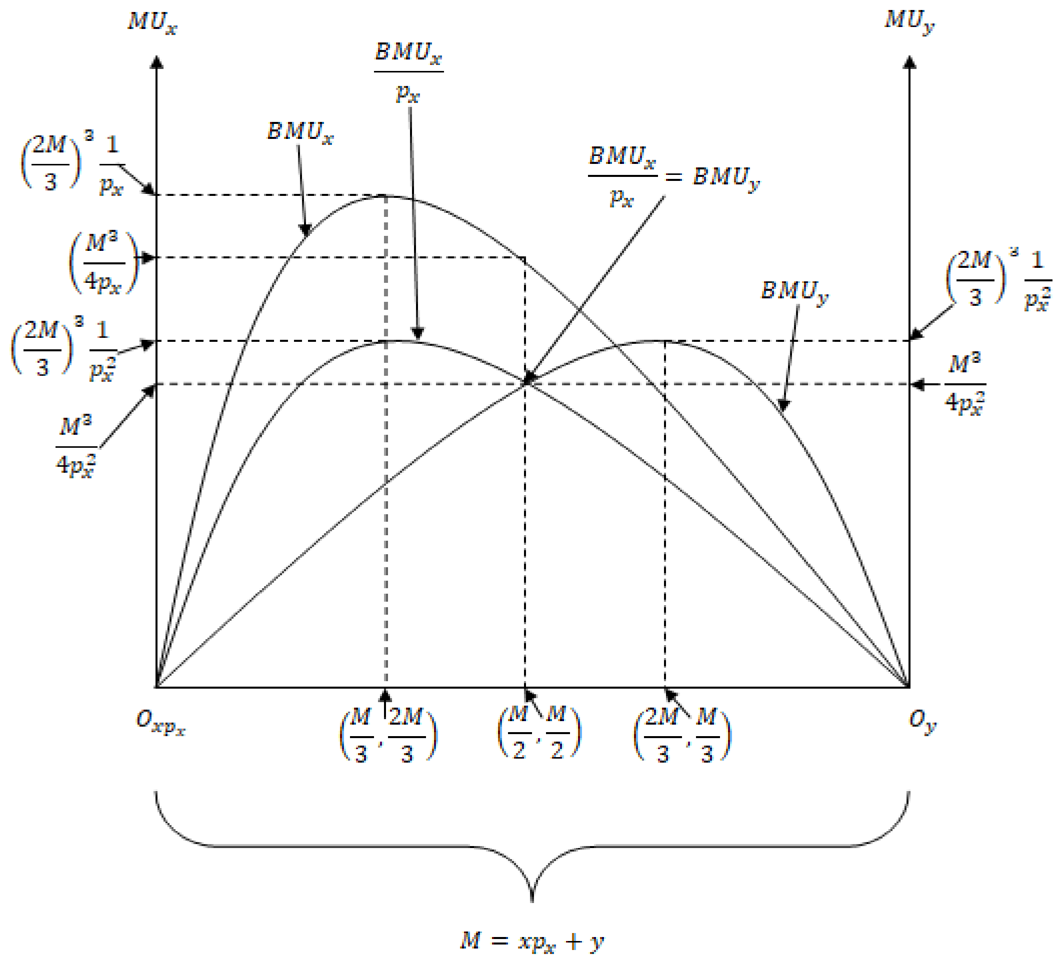

Figure 22 : Consumer equilibrium corresponding to the inverted U-shaped BCMUSs of x and y if utility function is characterised by U=x^2y^2 where $p_x \neq p_y = 1$.

*Sub-case 2: In this sub-case we shall consider y is a numeraire commodity but x is not. So, $p_x \neq p_y = 1$ and p_x can take any positive magnitude. The budget constraint of a consumer reduces to $M = p_x x + y$. The BCMUSs of x and y in this sub-case boil down to $BMU_x = 2x(M - p_x x)^2$ and $BMU_y = 2y\frac{(M-y)^2}{p_x^2}$. But the BCMUSs of x and y are not symmetric unlike the previous sub-case. The BCMUS of x will be higher than the BCMUS of y since the height of the peak of BCMUS of x at $x = \frac{M}{3p_x}$ is $\left(\frac{2M}{3}\right)^3 \frac{1}{p_x}$ whereas the height of the peak of BCMUS of y at $y = \frac{M}{3}$ is $\left(\frac{2M}{3}\right)^3 \frac{1}{p_x^2}$ as depicted in Fig. 22. But what is interesting is that unlike the previous Edgeworth box kind of diagrams, in the above diagram the leftward origin is $p_x x$ instead of x. So, we are measuring expenditure on x instead of x rightward along the horizontal axis. Now, $\frac{\partial(BMU_x)}{\partial(xp_x)} = \frac{1}{p_x}\frac{\partial(BMU_x)}{\partial x}$ means the slope of the BCMUS of x with respect to the expenditure on x is nothing but the slope of the BCMUS of x with respect to x multiplied by a positive constant, and hence both will take the same sign. $\frac{\partial\left(\frac{BMU\ x}{p_x}\right)}{\partial(xp_x)} = \frac{1}{p_x^2}\frac{\partial(BMU_x)}{\partial x}$ implies that the sign of the slope of the $\frac{BMU_x}{p_x}$ with respect to the expenditure on x would be the same as the sign of the slope of the BCMUS of x. The $\frac{BMU_x}{p_x}$ schedule has two stationary points at $x = \frac{M}{p_x}\ or\ at\ xp_x = M$ and at $x = \frac{M}{3p_x}$ or at $xp_x = \frac{M}{3}$ since $\frac{\partial\left(\frac{BMU\ x}{p_x}\right)}{\partial(xp_x)} = \frac{2}{p_x^2}(M - p_x x)(M - 3xp_x) = 0$ solves $x = \frac{M}{p_x}$ and $= \frac{M}{3p_x}$. The second order conditions $\frac{\partial^2\left(\frac{BMU\ x}{p_x}\right)}{\partial(xp_x)^2}\left(at\ x = \frac{M}{p_x}\right) = \frac{4M}{p_x^2} > 0$ and $\frac{\partial^2\left(\frac{BMU\ x}{p_x}\right)}{\partial(xp_x)^2}\left(at\ x = \frac{M}{3p_x}\right) = -\frac{4M}{p_x^2} < 0$ ensures that the $\frac{BMU_x}{p_x}$ schedule reaches its maximum value at $x = \frac{M}{3p_x}$ and reaches its minimum value at $= \frac{M}{p_x}$. The $\frac{BMU_x}{p_x}$ schedule takes the value zero at $xp_x = 0$ i.e. starts from the origin and then increases as the value of xp_x increases. At $xp_x = \frac{M}{3}$ it reaches its maximum value $\left(\frac{2M}{3}\right)^3\frac{1}{p_x^2}$ and then falls till it reaches its minimum value zero at $xp_x = M.$

However, the equilibrium occurs corresponding to the point of intersection of the $\frac{BMU_x}{p_x}$ and BMU_y schedules. To be more specific, equilibrium occurs at $(x^*p_x, y^*) = \left(\frac{M}{2}, \frac{M}{2}\right)$ or at $(x^*, y^*) = \left(\frac{M}{2p_x}, \frac{M}{2}\right)$. It can be verified that any deviation from (x^*p_x, y^*) will result in decrease in total utility. Since the two schedules intersect each other at their negatively sloping parts, the intersection point corresponds to a consumer equilibrium. The reader can also verify that in this sub-case $U_x(x, M - xp_x)\left(at\ x = \frac{M}{3}\right) = \frac{2}{3}M^3\left(1 - \frac{1}{3}p_x\right)^2\ or\ \frac{2}{3}M^3\left(\frac{1}{3}p_x - 1\right)^2$. If one puts $p_x = 1$ in the above result he/she ends up with $U_x(x, M - x)\left(at\ x = \frac{M}{3}\right) = \left(\frac{2M}{3}\right)^3$,which is perfectly compatible with the result obtained in sub-case 1. This is expected as sub-case 1 is obtained from sub-case 2 by fixing the prices of x and y.

Note that the demand functions derived from both cardinal and ordinal theories are obviously identical . The only difference is that we get the results in this thesis using the newly formulated tools of cardinal utility. This has an advantage as under the assumption of cardinal utility we cannot only identify the quantities demanded using the Pseudo Edgeworth apparatus but also measure the utility accruing to the consumer.

In our daily life, we often encounter some goods which are perfect substitutes or perfect complements in nature. If two goods x and y are perfect substitutes with the same price (e.g. red pencils and black pencils) then the utility function of a consumer is given by $U = ax + by$. If $a = b = 1$ then, $MU_x = MU_y = 1$ and $\frac{\partial(MU_x)}{\partial x} = \frac{\partial(MU_y)}{\partial y} = 0$ which implies that both the BCMUSs of x and y are horizontal. Under this case, the two BCMUSs coincide with each other at $MU_x = MU_y = 1$ as depicted in Fig. 23 below. Total utility is maximized at any (x^*, y^*)

where $x^* \in [0, M]$ and $y^* \in [0, M]$. Therefore, consumer equilibrium is achieved at any (x^*, y^*) where $x^* \in [0, M]$ and $y^* \in [0, M]$. However, when one allows for the price ratio to differ from 1 then the schedule of budget constrained marginal utility of one good divided by price is below or above that of the other, making a corner solution the only equilibrium.

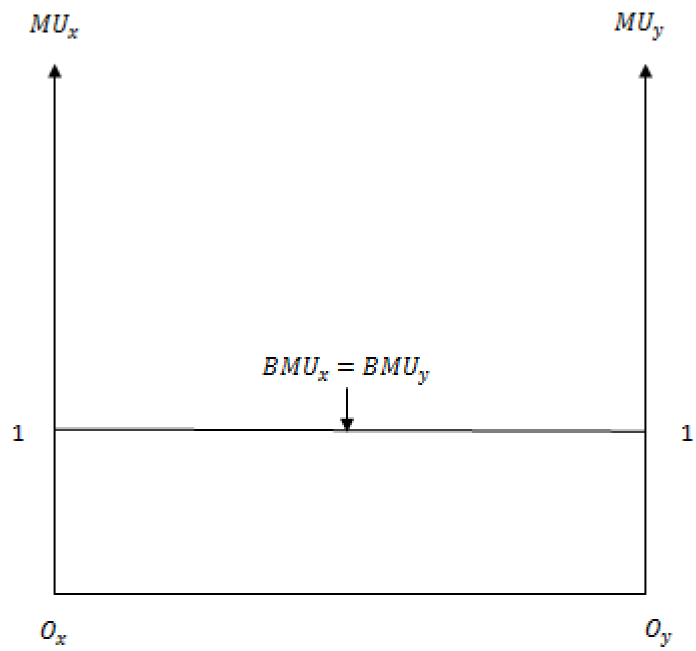

Figure 23 : Consumer equilibrium if $U = x + y$ with price of x as well as y equalling unity.

If the two goods x and y are perfect complements e.g. left pair of shoes and right pair of shoes, then the Leontief type utility function of a consumer is U = Min (x,y). The indifference curves are L shaped as depicted in Fig. 24 below and $MU_x = MU_y = 0$ everywhere along the budget line except at the intersection of the budget line with the 45^0 Line from the origin, where it is undefined. Given this fact, the BCMUSs cannot be used directly in this case. Instead, we just rely on the logical deduction that a magnitude of $x = y$, such that the budget constraint is satisfied, yields an equilibrium. If this equality is not fulfilled the excess of one magnitude over the other will clearly not be associated with any addition to utility. Hence, income spent on this excess can be reallocated to equate x and y and generate additional utility. However, any reallocation from the consumption bundle on the budget line such that $x = y$ will similarly result in a decline in utility.

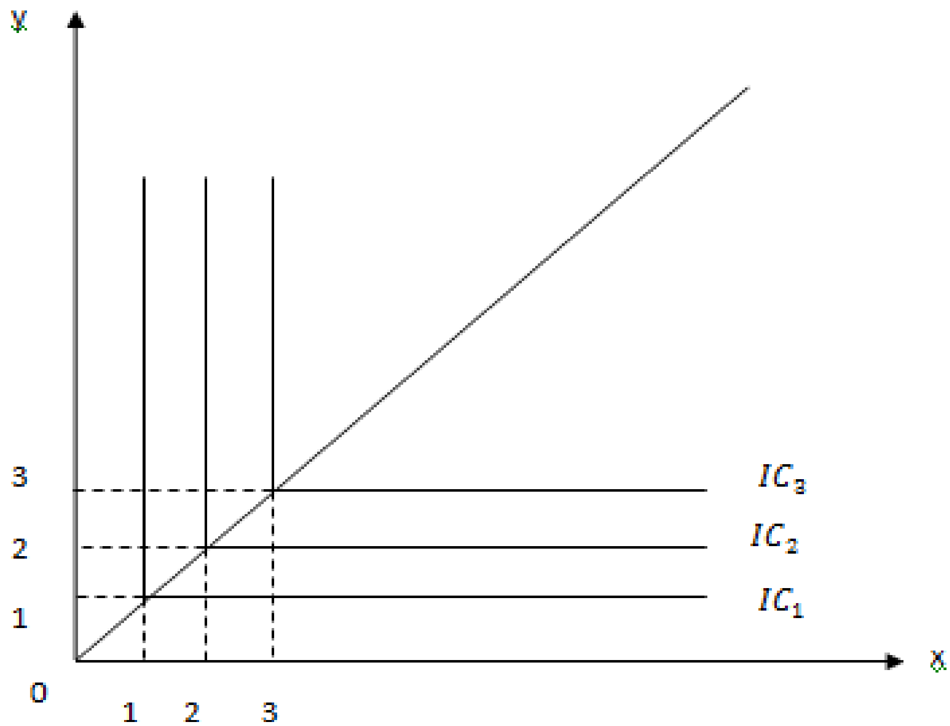

Figure 24 : L shaped indifference curves of perfect complements.

Chapter 6:

Conclusion

In standard microeconomics textbooks, consumer behaviour is generally discussed on the basis of ordinal utility theory. Indifference curves, budget lines and their properties etc. are discussed and finally it is inferred that consumer equilibrium occurs where the slope of the indifference curve equals the slope of the budget constraint, given convexity of indifference curves. But, ordinal utility theory is only based on a ranking of bundles and therefore the utility function corresponding to any given ranking is not unique. This implies that measurement of utility in equilibrium is not possible in a meaningful manner under the ordinal utility approach.

For a long period of time it was thought that utility could not be measured in the sense that we could not quantitatively and uniquely identify the increase in satisfaction to an individual in going from one situation to the other. Yet recent developments have brought back a resurgence of interest in 'cardinal utility' and made meaningful quantification a possibility. Given that such quantification is possible, this thesis tries to answer the question as to whether all consumer behaviour, as captured through demand curves, is possible to analyze using new tools derived from the concept of marginal utility schedules used by practitioners of early cardinal utility theory. If that is the case then cardinal utility theory would become quite self-sufficient in the sense that it could be used to derive demand functions as well as measure utility in consumer equilibrium.

In this thesis we have tried to explore consumer behavior using new tools similar to concepts in early cardinal utility theory such as 'marginal utility schedules': the "budget constrained marginal utility schedule (BCMUS)" (and the 'schedule of budget constrained utility normalized by price'). We show that these schedules alone help us to identify equilibrium. Moreover, if the utility function is cardinal and gives us a unique measure of satisfaction, we would be able to also measure the utility accruing to the consumer through our apparatus, which consists of the BCMUSs in a pseudo-Edgeworth box. Thus, an important start has been made in developing a complete theory of consumer behaviour and welfare using cardinal utility functions.

Appendix

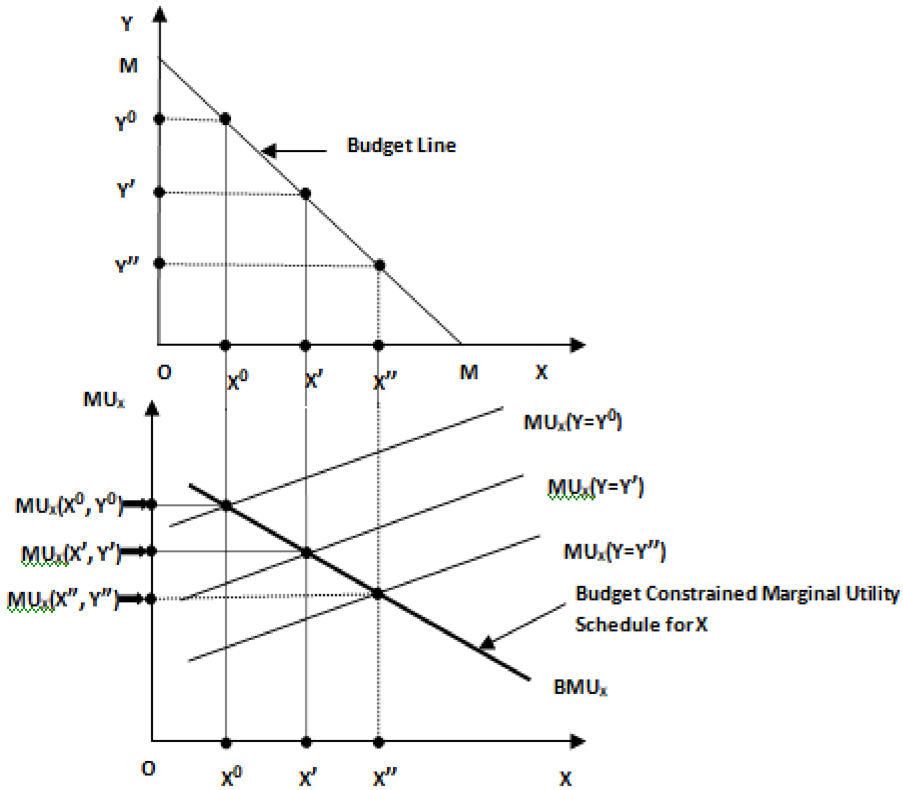

Figure 25 : BCMUS of X in case of net decrease in MU for all possible increases in value of X moving down the budget line under the assumptions of (a) utility functions exhibiting increasing marginal utility, (b) positive complementarity in respect to utility between the two commodities.

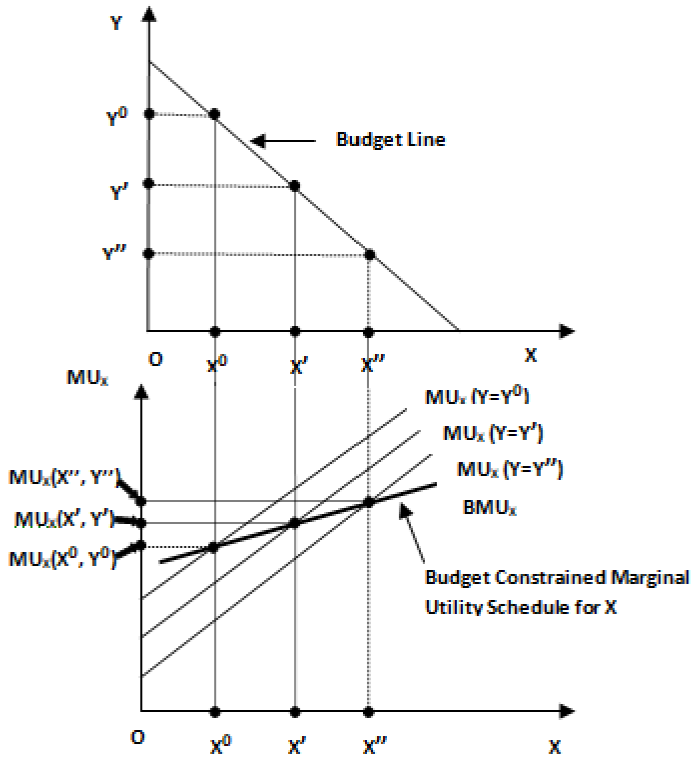

Figure 26 : BCMUS of X in case of net increase in MU for all possible increases in value of X moving down the budget line under the assumptions of (a) utility functions exhibiting increasing marginal utility, (b) positive complementarity in regard to utility between the two commodities.

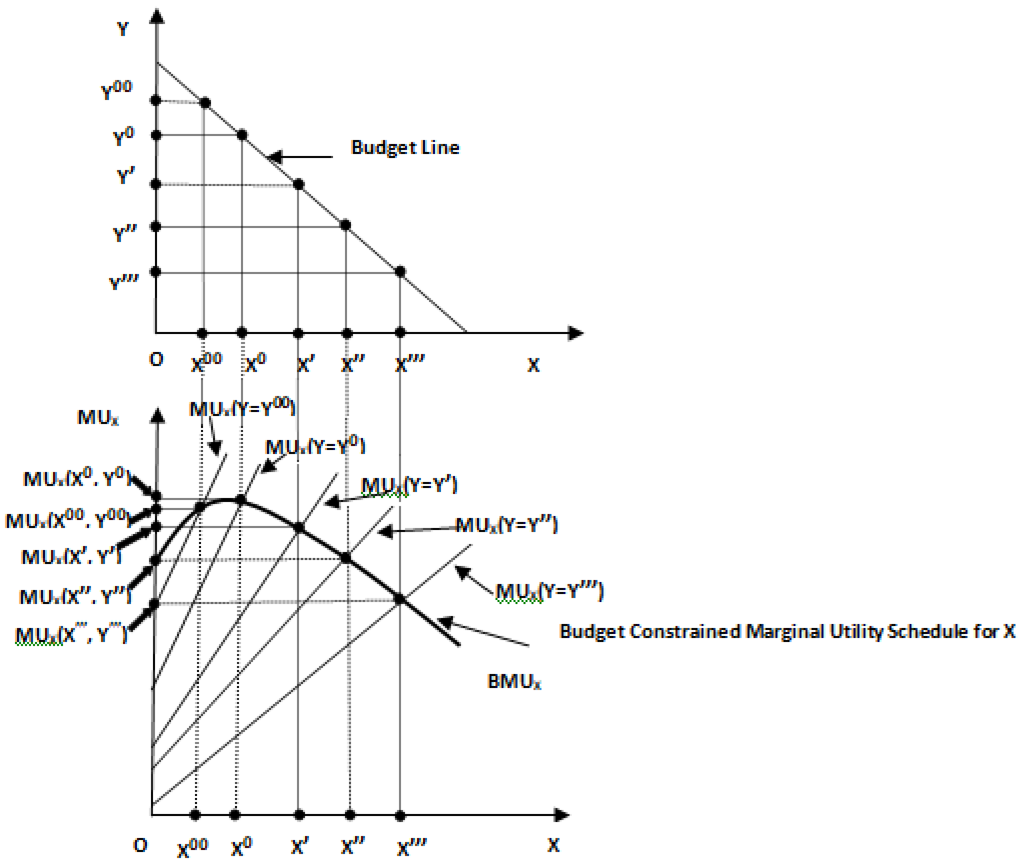

Figure 27: Inverted U-shaped BCMUS of X when upward sloping marginal utility schedule of X is concave and downward shifts in the marginal utility schedule corresponding to decreases in Y are small for high values of Y but become progressively larger as Y decreases under the assumptions of (a) increasing marginal utility and (b) positive complementarity in regard to utility between the two commodities.

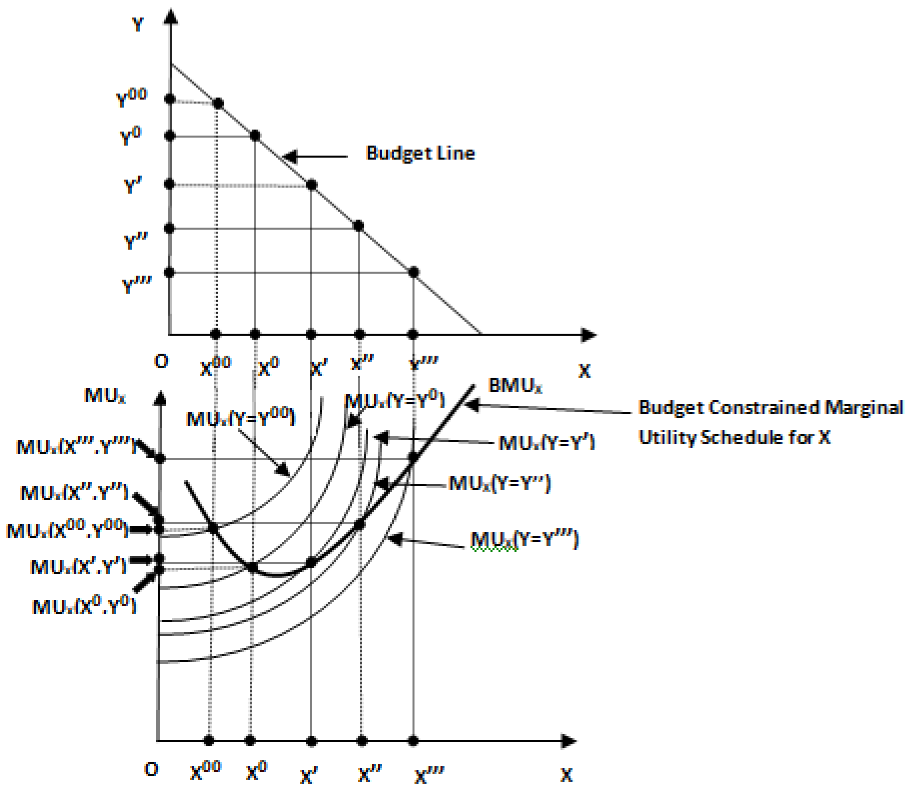

Figure 28 : U-shaped BCMUS of X in case of strictly convex marginal utility schedule of X coupled with downward shifts in the schedule becoming smaller progressively with decreases in Y under the assumptions of (a) increasing marginal utility and (b) positive complementarity in regard to utility between the two commodities.

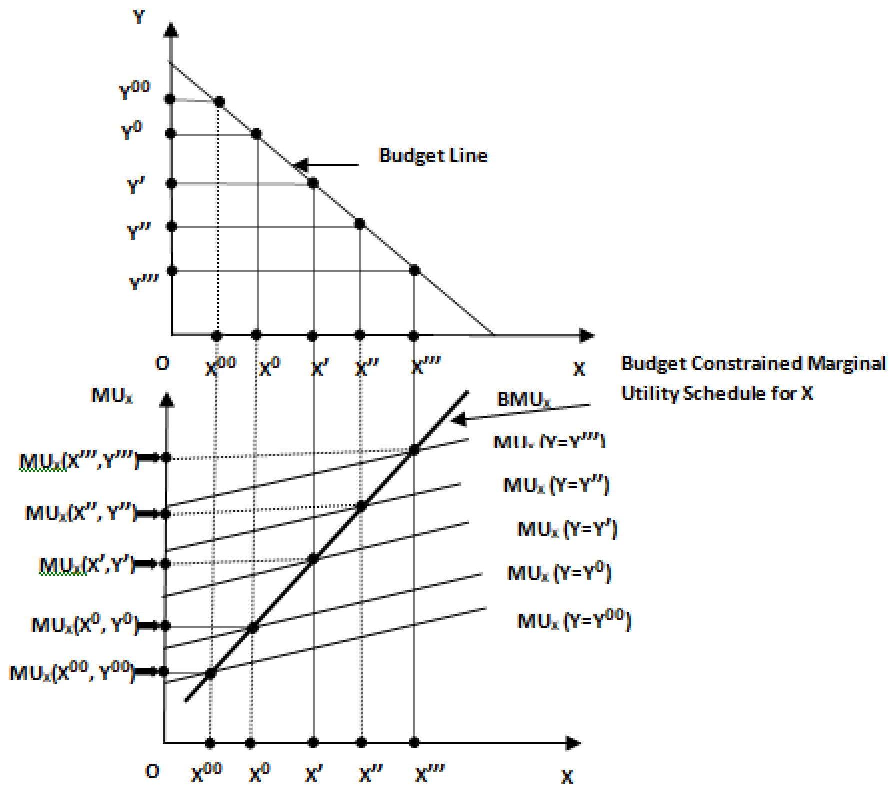

Figure 29: BCMUS of X in case of increasing marginal utility accompanied by negative complementarity.

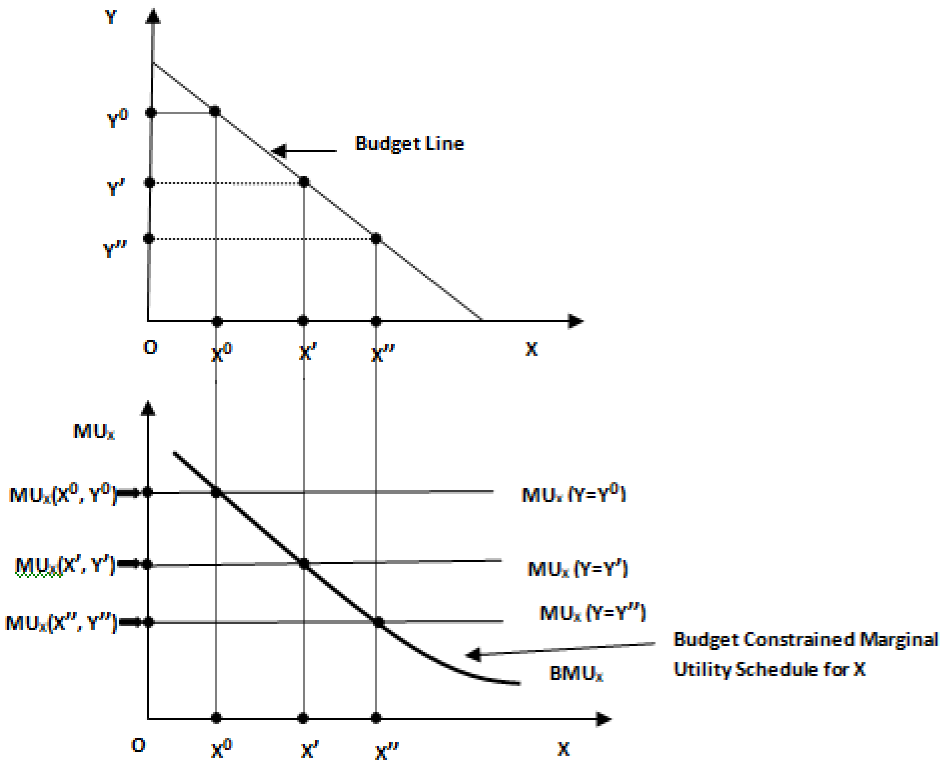

Figure 30 : BCMUS of X in case of constant marginal utility accompanied by positive complementarity.

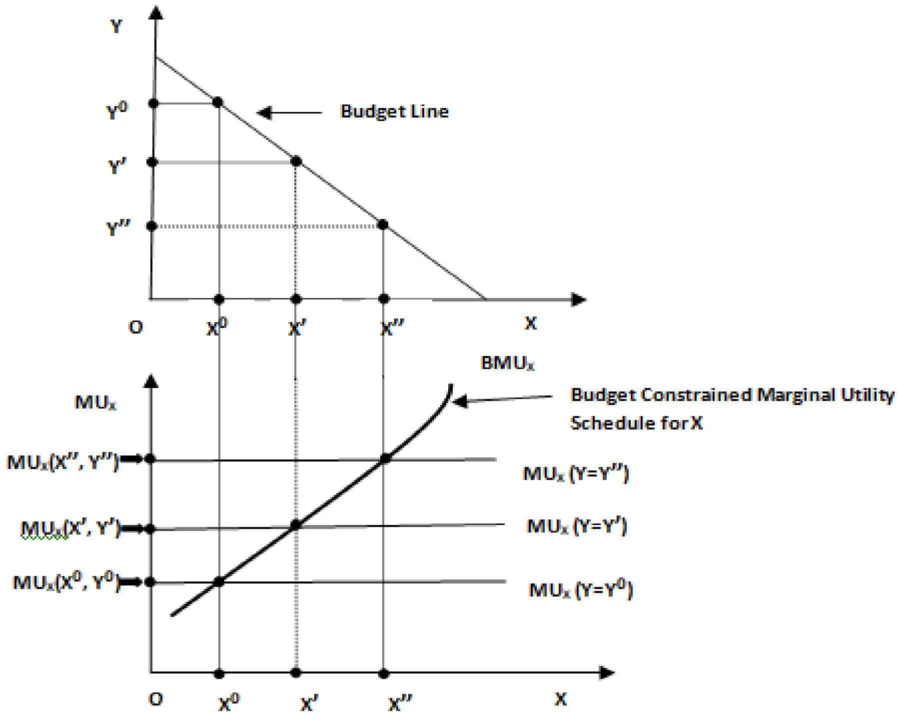

Figure 31 : BCMUS of X in case of constant marginal utility accompanied by negative complementarity.

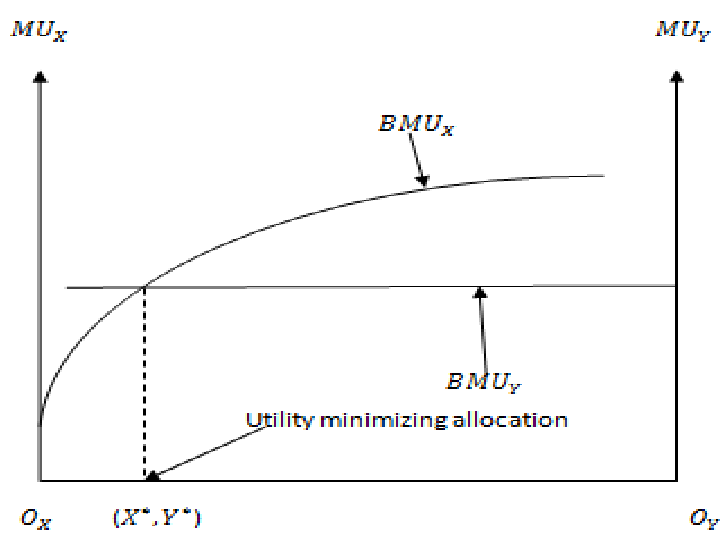

Figure 32: Consumer equilibrium in case of horizontal BCMUS of Y accompanied by positively sloped BCMUS of X.

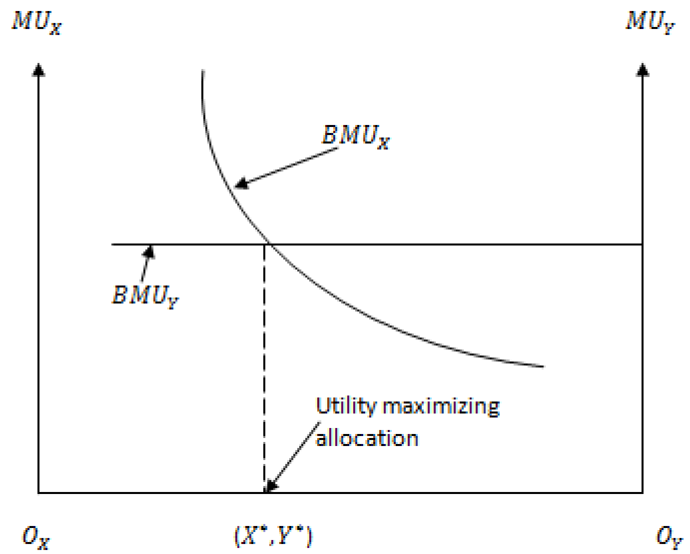

Figure 33 : Consumer equilibrium in case of horizontal BCMUS of Y accompanied by negatively sloped BCMUS of X.

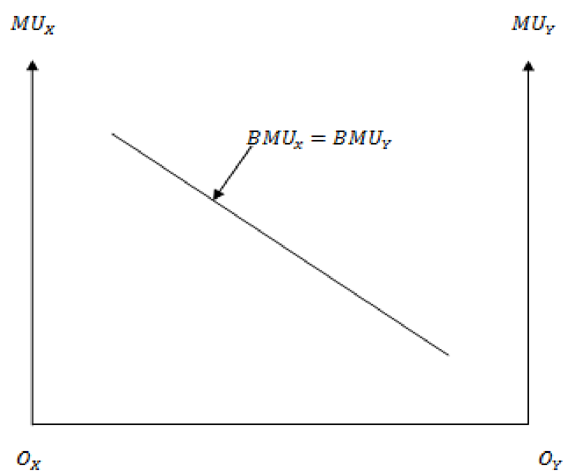

Figure 34 : Coincidence of negatively sloped BCMUS of X and positively sloped BCMUS of Y.

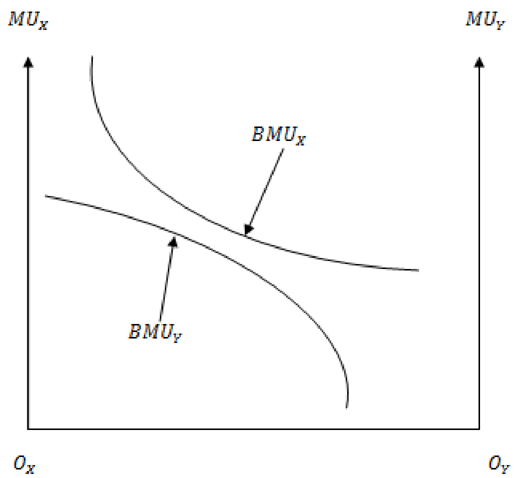

Figure 35 : Non-intersection of negatively sloped BCMUS of X and positively sloped BCMUS of Y where BCMUS of X is convex to O_X and BCMUS of Y is concave to O_X.

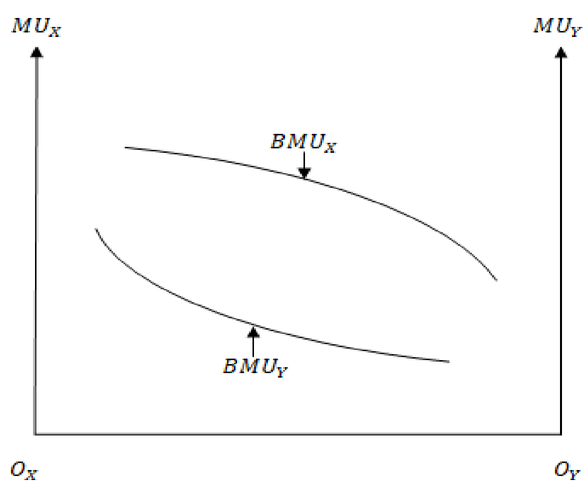

Figure 36 : Non-intersection of negatively sloped BCMUS of X and positively sloped BCMUS of Y where BCMUS of X is concave to O_X and BCMUS of Y is convex to O_X.

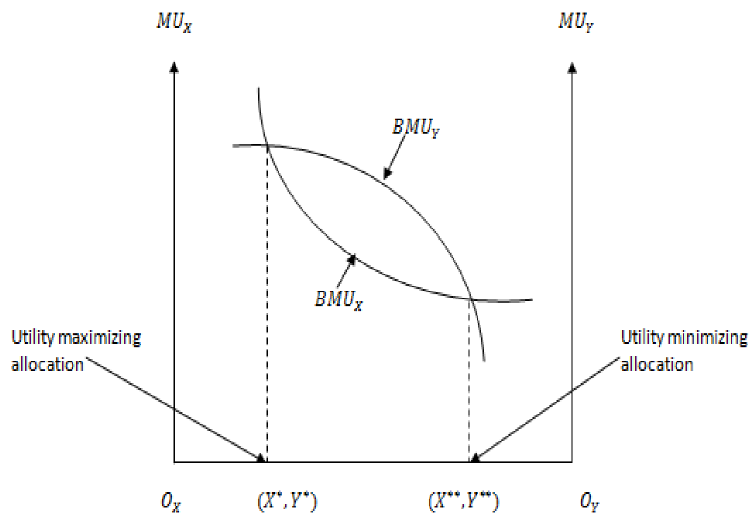

Figure 37 : Negatively sloped and convex to O_X BCMUS of X accompanied with positively sloped and concave to O_X BCMUS of Y and related consumer equilibrium.

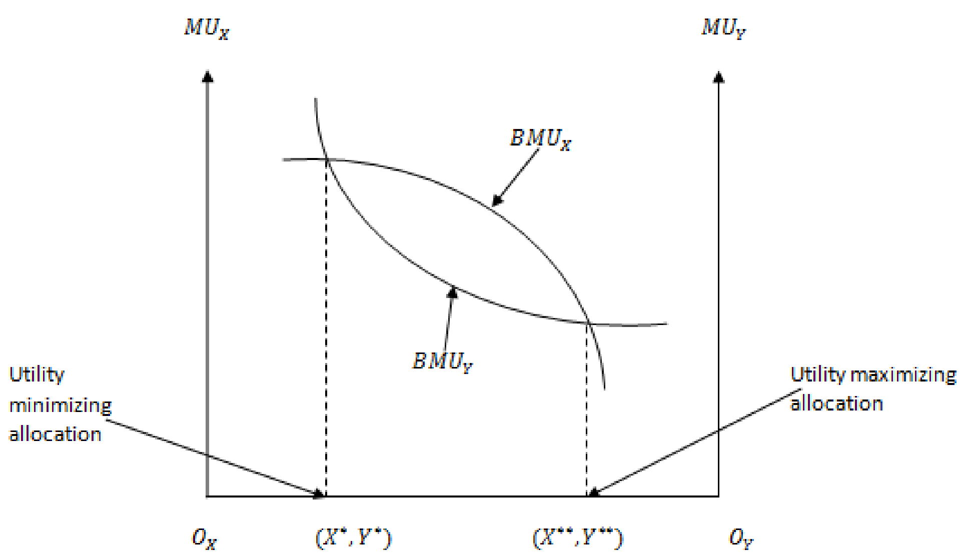

Figure 38 : Negatively sloped and concave to O_X BCMUS of X accompanied with positively sloped and convex to O_X BCMUS of Y and related consumer equilibrium.

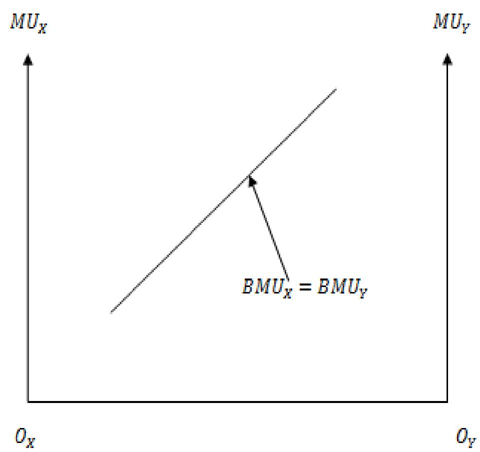

Figure 39 : Coincidence of positively sloped BCMUS of X and negatively sloped BCMUS of Y.

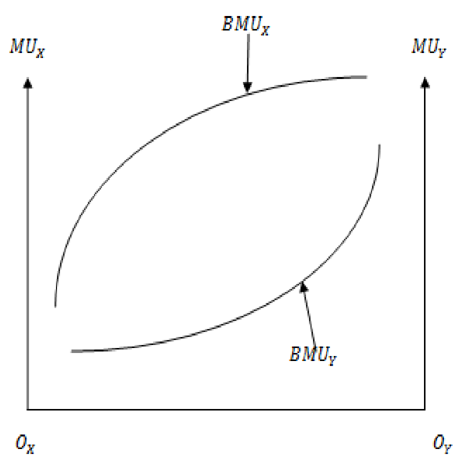

Figure 40 : Non-intersection of positively sloped BCMUS of X and negatively sloped BCMUS of Y where BCMUS of X is concave to O_Y and BCMUS of Y is convex to O_Y.

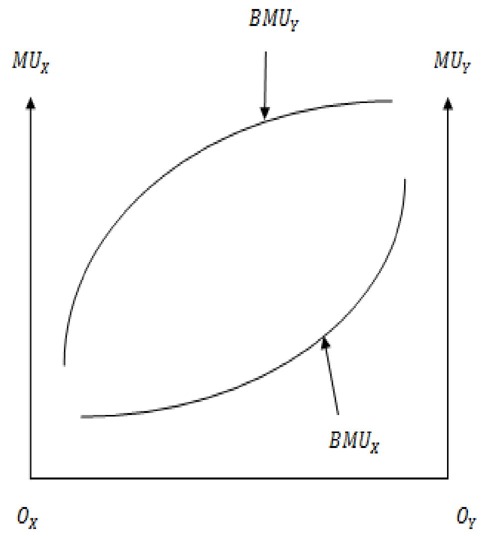

Figure 41 : Non-intersection of positively sloped BCMUS of X and negatively sloped BCMUS of Y where BCMUS of X is convex to O_Y and BCMUS of Y is concave to O_Y.

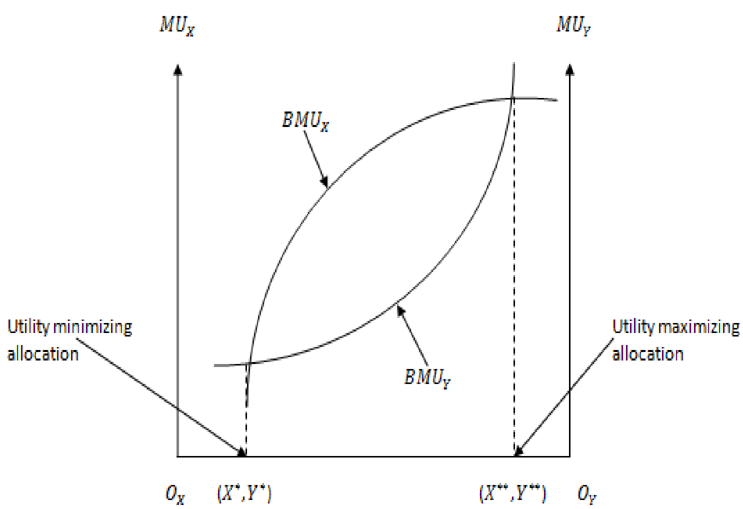

Figure 42 : Positively sloped and concave to O_Y BCMUS of X accompanied with negatively sloped and convex to O_Y BCMUS of Y and related consumer equilibrium.

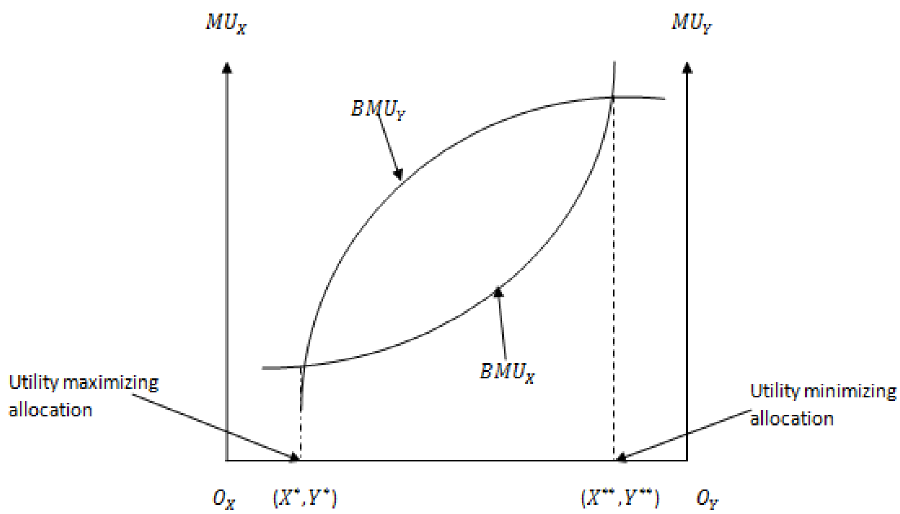

Figure 43 : Positively sloped and convex to O_Y BCMUS of X accompanied with negatively sloped and concave to O_Y BCMUS of Y and related consumer equilibrium.

Bibliography

1. Kahneman, D., Wakker, P. P., &Sarin, R. (1997). Back to Bentham? Explorations of experienced utility. *Quarterly Journal of Economics, 112,* 375–405.

2. Baum, T.(2008). A quest for inspiration in the liberal peace paradigm: back to Bentham?, *European Journal of International Relations, 2008,14*,431-453.

3. Kahneman, D. (1994). New challenges to the rationality assumption. *Journal of Institutional and Theoretical Economics, 150,* 18–36.

4. Kahneman, D. (2000a). Evaluation by moments: Past and future. In D. Kahneman & A. Tversky (Eds.), *Choices, values, and frames* (pp. 693–708). New York: Cambridge University Press.

5. Lipsey, R. (1987). *An Introduction to Positive Economics*, English Language Book Society / Weidenfeld & Nicolson.

6. Lipsey, R. & Chrystal, A. (2003). *Economics.(pp.)* Oxford: Oxford University Press.