

M.C.S.E. FIRST YEAR FIRST SEMESTER EXAMINATION-2025
THEORY OF COMPUTING

Time: 3 hours

Full Marks: 100

Answer any five questions.

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1. Taking language $L = \{a^k b^k \mid k \geq 0\}$ as an example and three machines, M_1 , M_2 , and M_3 , it can be shown that the runtime depends on a computational model. Machines, M_1 , M_2 , and M_3 are described as
- 1-tape Turing Machine M_1 : $O(n^2)$
 - 1-tape Turing Machine M_2 : $O(n \log n)$
 - 2-tape Turing Machine M_3 : $O(n)$
- Design, explain the acceptability of the language L , and derive time complexity for machines M_1 , M_2 , and M_3 .
- 6 + 8 + 6**
2. a) Show that every nondeterministic Turing machine has an equivalent deterministic Turing machine.
 b) Show that a k -tape Turing machine can be simulated in a 1-tape Turing machine with polynomial overhead.
 c) Show that simulating a nondeterministic Turing machine on a 1-tape deterministic Turing machine requires exponential overhead.
- 6 + 7 + 7**
3. Design a Turing Machine for the following language. Validate the construction by taking at least one non-trivial valid and another non-trivial invalid string
 $L = \{0^{2^n} \mid n \geq 0\}$
- 20**
4. Design a Turing Machine for the following language. Validate the construction by taking at least one non-trivial valid and another non-trivial invalid string.
 $L = \{www^R \mid w \in \{a, b\}^+\}$
- 20**
5. a) Prove that the family of recursive languages is closed under intersection.
 b) Suppose a Turing machine prints out a list of words in L in increasing order (using dictionary order). Show that L is decidable
 c) Prove that the problem of determining whether two Turing machines accept the same language is undecidable.
 d) Prove that the family of recursively enumerable languages is closed under union
- 4 × 5**
6. a) State and prove Savitch's theorem concerning space complexity
 b) "Increase in the number of tapes in a Turing machine does not asymptotically increase the overall space." -- Explain
- 15 + 5**
7. a) Let $A_{TM} = \{(M, w) \mid \text{the Turing machine } M \text{ accepts input } w\}$. Prove that A_{TM} is undecidable.
 b) Let $HALT_{TM} = \{(M, w) \mid \text{the Turing machine } M \text{ halts on input } w\}$. Prove that $HALT_{TM}$ is undecidable.
 c) Design a linear bounded automaton for the language $L = \{a^p \mid p \text{ is prime}\}$
- 7 + 7 + 6**
8. a) Show that the class P is closed with respect to union and complement.
 b) Establish $P \subseteq PSPACE \subseteq EXPTIME$
- 10 + 10**