

Time: Three Hours

Full Marks 100

Answer Question No. 1 and any four from the rest.

1. Answer any four of the following questions: 20
- (a) Insert the given values in the order in initially empty AVL tree: 34, 21, 27, 14, 10, 24, 43, and 6. What is the value at the root of the tree.
- (b) Construct a B-tree of the order of 3 with the following values: 34, 21, 27, 14, 10, 15, 18, 24, 43, and 6.
- (c) Answer the following questions:
- (i) Red-Black trees are an example of a self-balancing binary search tree. That is, they reorganize themselves when you insert nodes, so no global rebalancing is ever necessary. – true or false?
 - (ii) Nodes in a red-black tree are assigned a color once and may never be recolored. – true or false?
 - (iii) When a node is first inserted in a red-black tree, it is placed according to the insert procedure in a binary search tree. What color is this newly inserted node (initially) if it is NOT the root?
 - (iv) When you insert a node in a Red-Black tree, what do you do if the parent of the newly inserted node is red, and the sibling of the parent is black? Show with an example.
- (d) What are the collision resolution techniques in hashing. Briefly explain linear probing and quadratic probing.
- (e) Briefly explain the BSP cost model.
- (f) Write a randomized algorithm for hiring problem. What is the expected total fee one needs to pay to the employment agency with a randomized hiring problem?
2. (a) Explain the terminologies – speedup, cost and work in the context of parallel algorithm analysis.
- (b) Describe a PRAM model for parallel computation.
“PRAM is not a realistic abstract machine model” – Explain the statement.
- (c) How is a BSP abstract machine model different from PRAM abstract machine model.
- (c) Write a PRAM algorithm for a graph coloring problem. What is the time complexity of the algorithm?
- 3+6+4+7=20
3. (a) If decision problem A can be polynomially reduced to decision problem B, then explain the following statement.
“if decision algorithm for B is polynomial, so does A, i.e. A is no harder than B (or B is no easier than A) and if A is NPC, so does B. i.e. if A is hard, so does B”.
- (b) How do you prove that a problem is in NP? How do you prove that a problem is in NPC?
- (b) Prove that CNF-SAT is in NP. Next prove that CNF-SAT is NP-hard.
- 6+4+10=20

4. (a) Why do we need approximation algorithms? Define approximation ratio for a maximization problem and a minimization problem. Define PTAS and FPTAS.
 (b) Discuss the exact algorithm for subset sum problem. Comment on its complexity. Discuss how you can provide an approximate solution for the problem.
 (c) Consider a list containing all digits of your date of birth (in dd-mm-yyyy format) in sorted order. Ignore duplicate digits, if any. Using the approximation algorithm for subset sum problem, show how the lists of integers are generated in each step. Show only the first five steps. Consider the value of δ for the Trim function is 0.1.
 $(2+2+3)+(4+3)+6=20$

5. (a) Prove the duality theorem in linear programming.
 (b) If a problem is feasible, what will be the status of its dual? Also if a problem is infeasible, what are the possible cases for its dual?
 (c) Formulate a linear programming problem for the following (express in its standard form):
 A cake company produces two types of fruit cakes – A and B. Both the cakes require flour and butter. To manufacture each unit of A and B, following quantities are required: Each unit of A requires 2 units of flour and 3 units of butter. Each unit of B requires 1 unit of flour and 2 units of butter. The company kitchen buys a total of 5 units of flour and 12 units of butter every day. On each sale, the company makes a profit of Rs 6 per unit A sold and Rs 5 per unit B sold. How many units of A and B should the company produce every day to maximize its profit?

Write a dual minimization problem of the same.

- (d) Represent the above problem graphically and show the optimal solution.
 $5+3+(5+2)+5=20$

6. (a) Draw the KMP flowchart with fail indexes for the pattern "ababaca".
 (b) Given the string "bacbabababacaca", use the above flowchart and KMPScan algorithm to trace the pattern "ababaca". Show every step while tracing the pattern. How many comparisons are made by the algorithm?
 (c) Using Boyer-Moore first heuristic, find how many comparisons are required to locate the pattern "ababaca" in the string "bacbabababacaca".
 $6+7+2+5=20$

7. (a) "For some problems, the best known polynomial-time approximation algorithms have approximation ratios that grow as functions of the input size n." Give an example of such a problem. Prove that the approximation algorithm has an approximation ratio that grows as function of the input size.
 (b) Why are randomized algorithms important? What do you mean by 'Monte Carlo' algorithms? Give an example of Monte Carlo algorithm.
 $16+4=20$