# MULTISTAGE AMPLIFIERS
## PART I - BJT AMPLIFIERS

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- Many applications cannot be handled with single-transistor amplifiers in order to meet the specification of a given amplification factor, input resistance and output resistance

- As a solution – transistor amplifier circuits can be connected in series or cascaded amplifiers

# Introduction

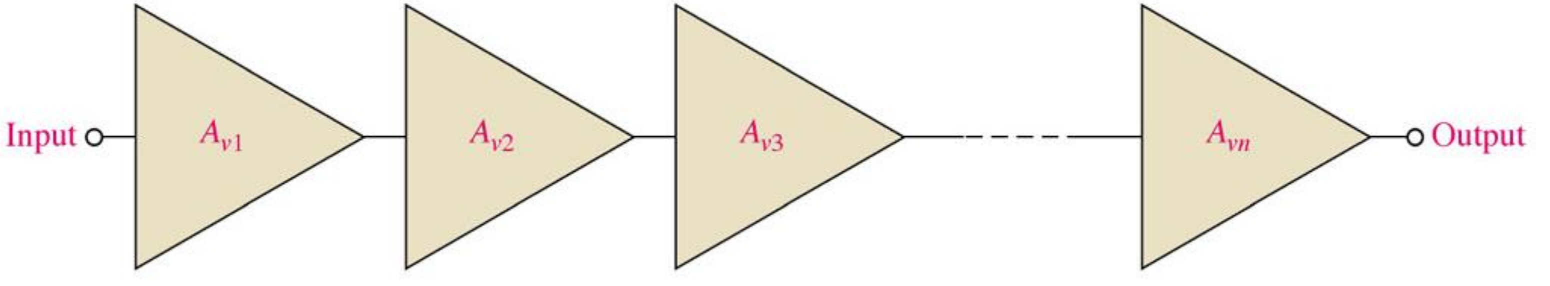
- **Typical spec for a general purpose operational amplifier**
  - **– Input resistance ~ 1MΩ**
  - **– Output resistance ~ 100Ω**
  - **– Voltage gain ~ 100,000**
- **No single transistor amplifier can satisfy the spec**
- **As a solution – multiple transistor amplifier circuits can be connected in series to meet the spec**
- **Usually**
  - **– An input stage to provide required input resistance**
  - **– A middle stage(s) to provide gain**
  - **– An output stage to provide required output resistance**
- **It is important to note that the input resistance of the next stage becomes the load of the previous stage**

# Multistage Amplifiers

Multi-stage amplifiers are amplifier circuits cascaded to increased gain. We can express gain in decibels(dB).

Two or more amplifiers can be connected to increase the gain of an ac signal. The overall gain can be calculated by simply multiplying each gain together.

$$A_{VTOT} = A_{v1} A_{v2} A_{v3} \ldots\ldots A_{VN}$$



# Multistage Amplifier Cutoff Frequencies and Bandwidth

- When amplifiers having equal cutoff frequencies are cascaded, the cutoff frequencies and bandwidth of the multistage circuit are found using

$$f_{C2(T)} = f_{C2}\sqrt{2^{1/n} - 1}$$

$$f_{C1(T)} = \frac{f_{C1}}{\sqrt{2^{1/n} - 1}}$$

$$\mathrm{BW} = f_{C2(T)} - f_{C1(T)}$$

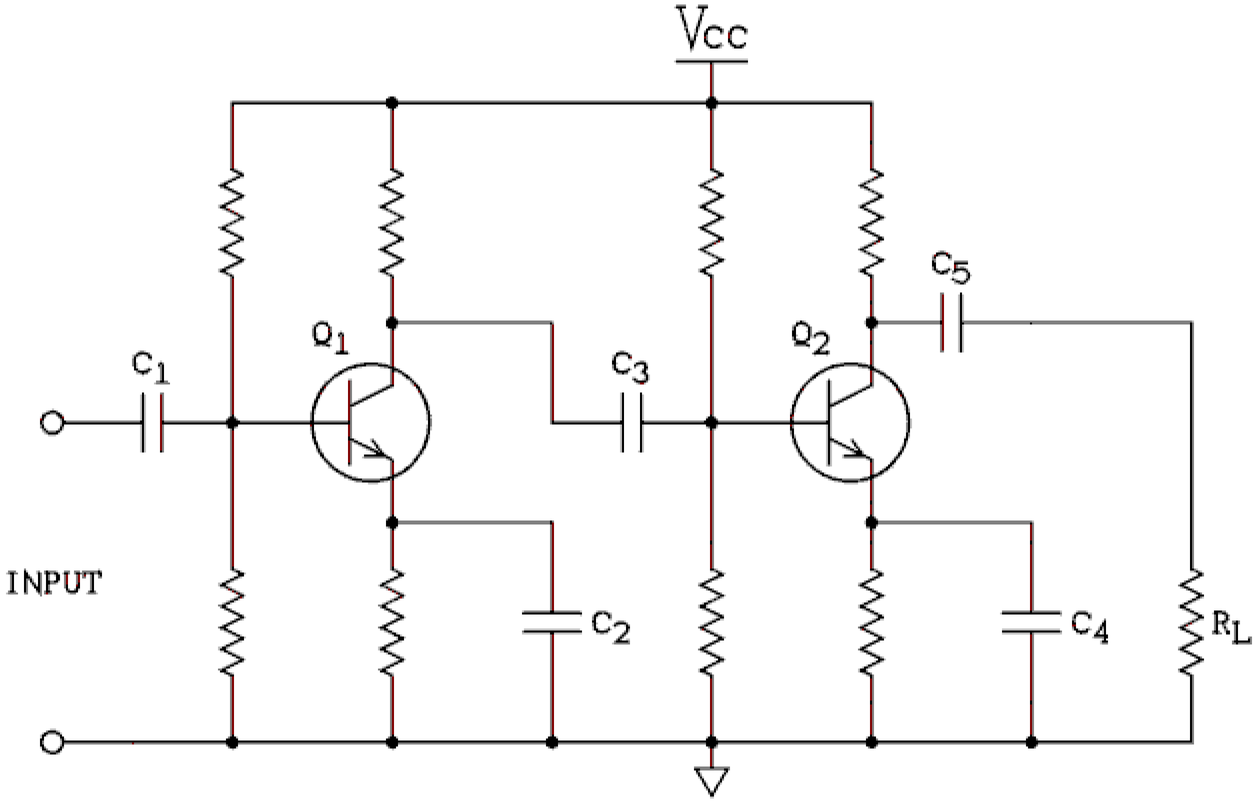
# Summary of Single stage BJT Amplifiers

| Characteristics of BJT Amplifiers | | | | | |
|---|---|---|---|---|---|
| | $R_{\text{in}}$ | $A_{vo}$ | $R_o$ | $A_v$ | $G_v$ |
| Common emitter | $(\beta+1)r_e$ | $-g_m R_C$ | $R_C$ | $-g_m(R_C \parallel R_L)$<br>$-\alpha \frac{R_C \parallel R_L}{r_e}$ | $-\beta \frac{R_C \parallel R_L}{R_{\text{sig}}+(\beta+1)r_e}$ |
| Common emitter with $R_e$ (Fig. 7.38) | $(\beta+1)(r_e+R_e)$ | $-\frac{g_m R_C}{1+g_m R_e}$ | $R_C$ | $\frac{-g_m(R_C \parallel R_L)}{1+g_m R_e}$<br>$-\alpha \frac{R_C \parallel R_L}{r_e+R_e}$ | $-\beta \frac{R_C \parallel R_L}{R_{\text{sig}}+(\beta+1)(r_e+R_e)}$ |
| Common base | $r_e$ | $g_m R_C$ | $R_C$ | $g_m(R_C \parallel R_L)$<br>$\alpha \frac{R_C \parallel R_L}{r_e}$ | $\alpha \frac{R_C \parallel R_L}{R_{\text{sig}}+r_e}$ |
| Emitter follower | $(\beta+1)(r_e+R_L)$ | 1 | $r_e$ | $\frac{R_L}{R_L+r_e}$ | $\frac{R_L}{R_L+r_e+R_{\text{sig}}/(\beta+1)}$<br>$G_{vo}=1$<br>$R_{\text{out}}=r_e+\frac{R_{\text{sig}}}{\beta+1}$ |

Setting $\beta=\infty$ $(\alpha=1)$ and replacing $r_e$ with $1/g_m$, $R_C$ with $R_D$, and $R_e$ with $R_s$ results in the corresponding formulas for MOSFET amplifiers.

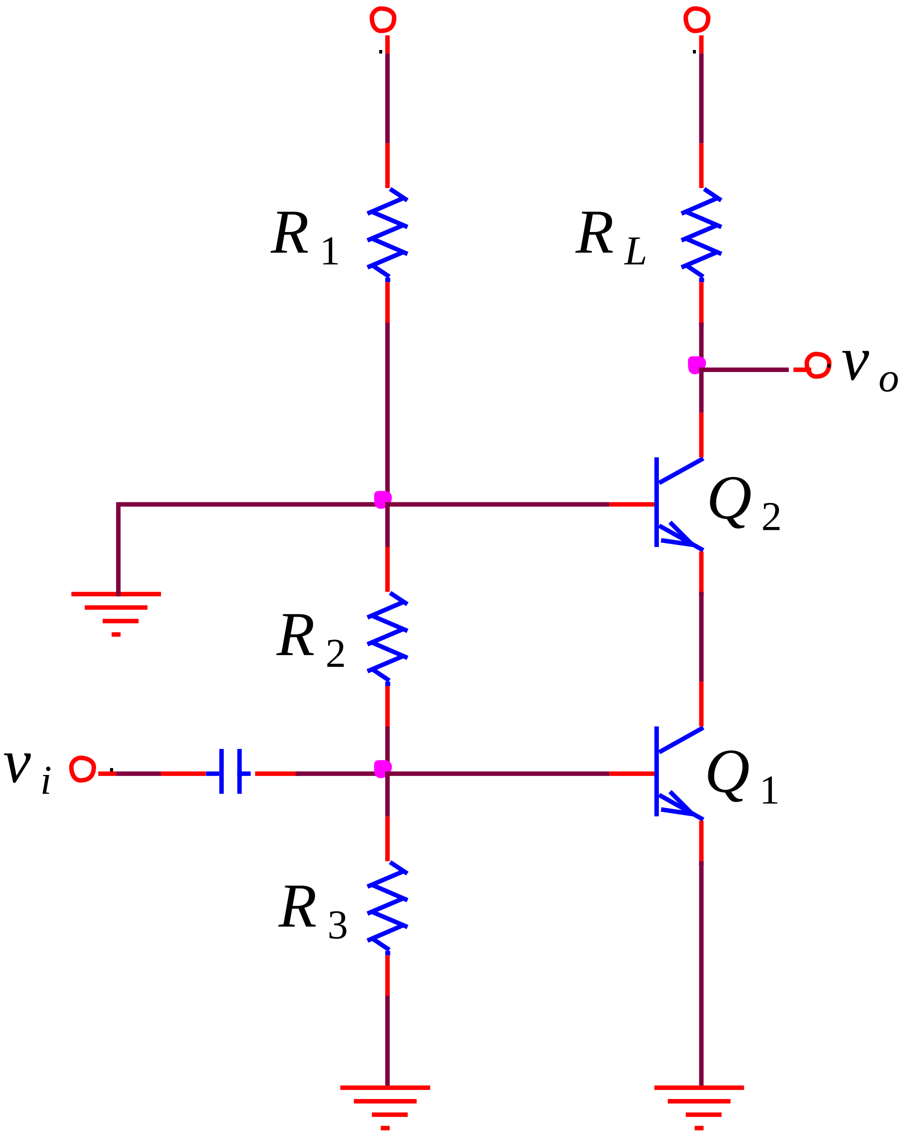
# Multistage amplifier configuration

Cascade RC coupling



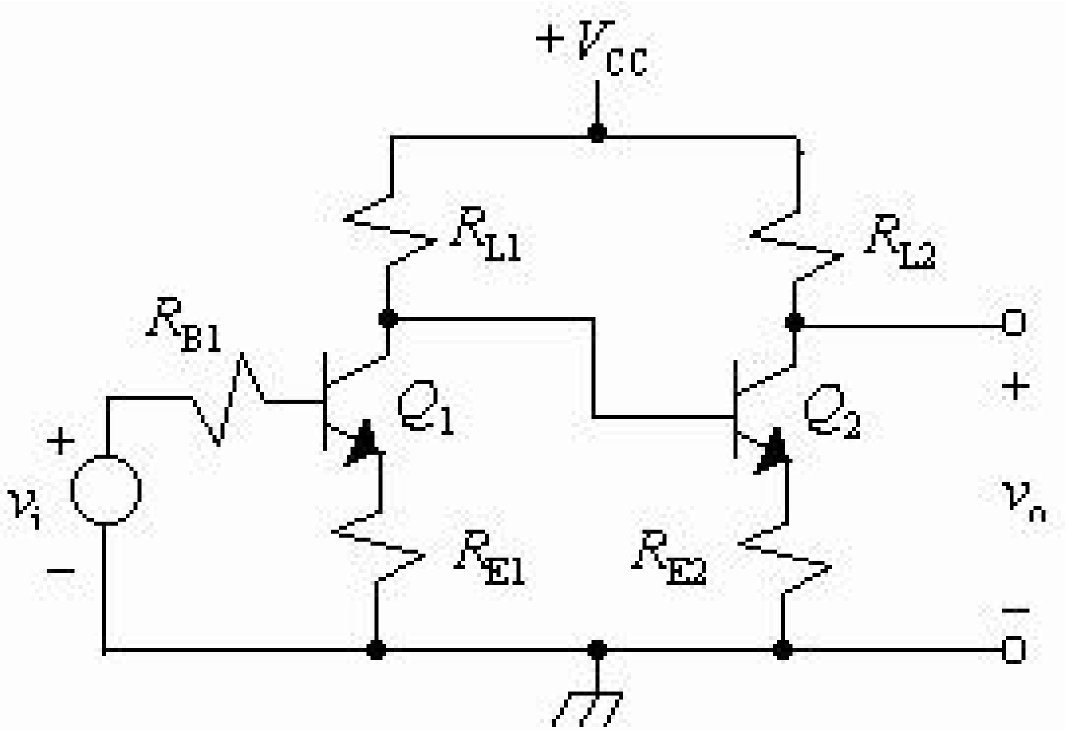
# Multistage amplifier configuration

Cascode



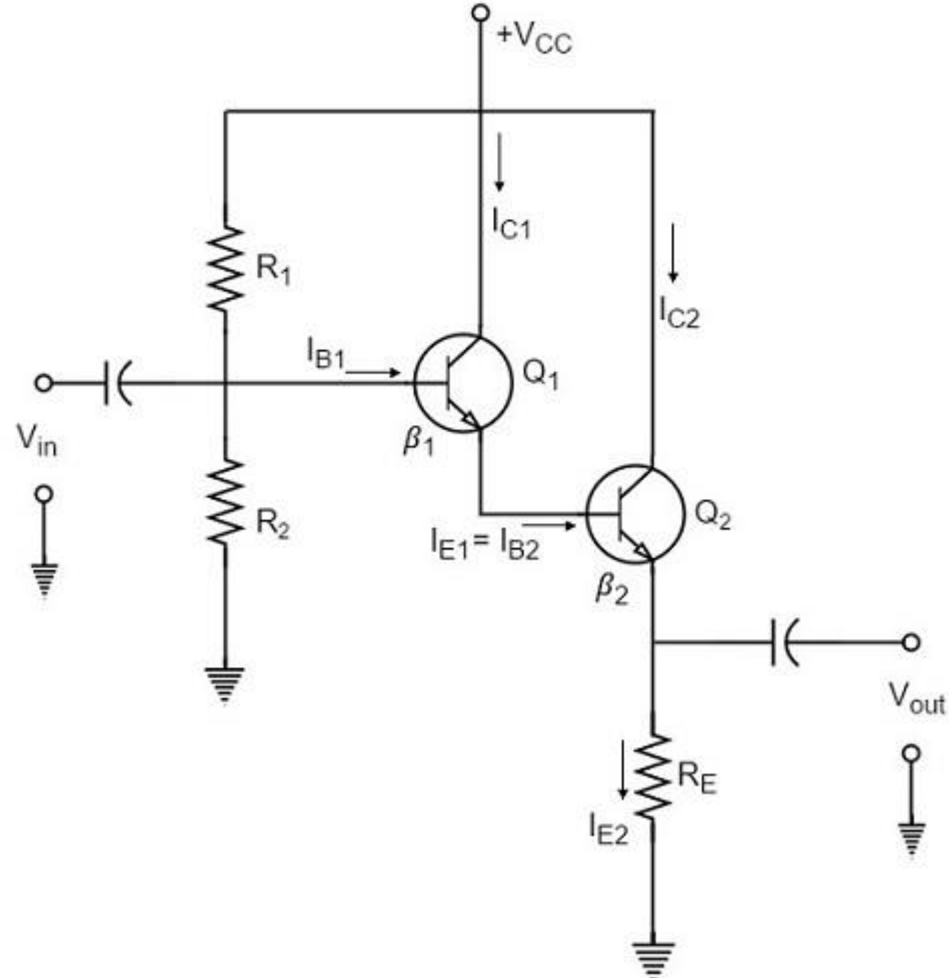
# Multistage amplifier configuration

Cascade Direct coupling



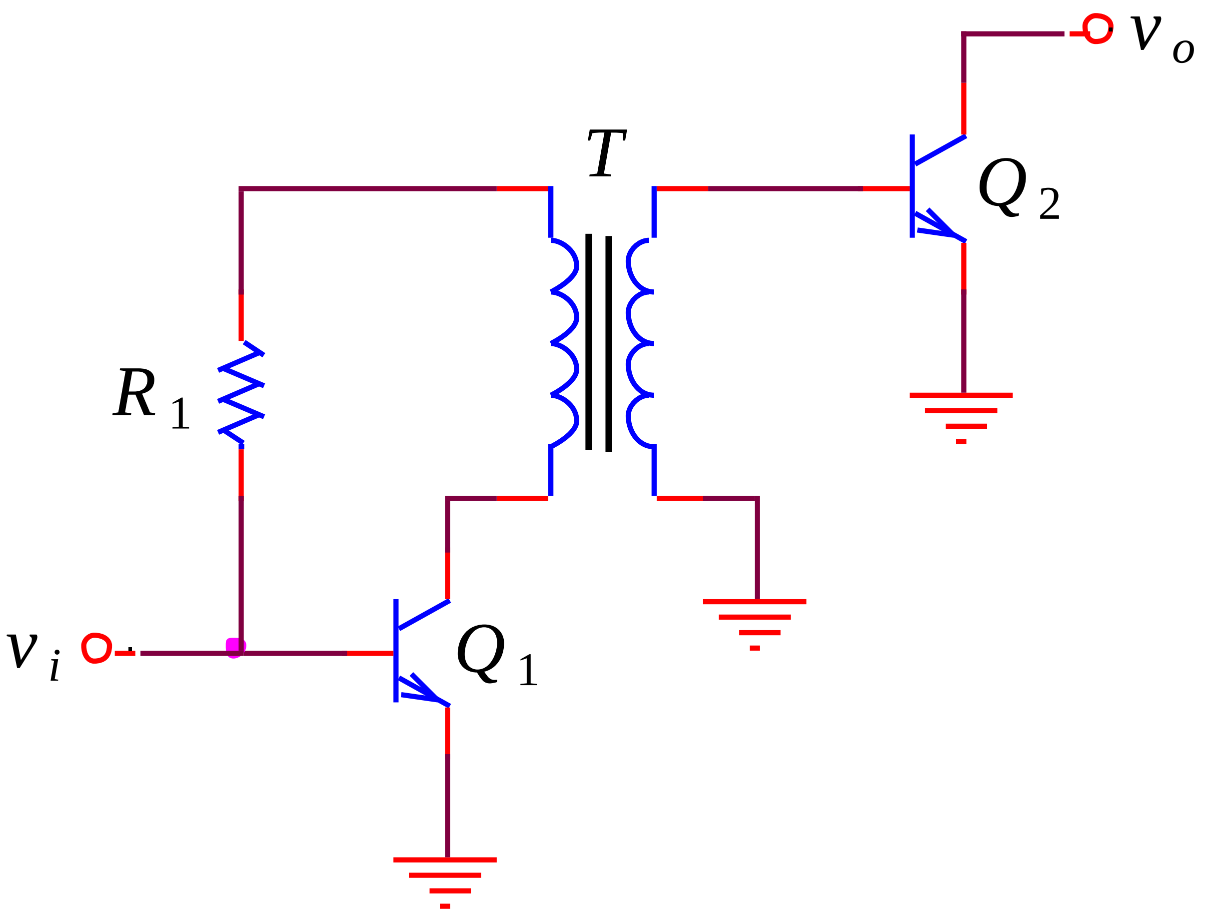
# Multistage amplifier configuration

Darlington



# Multistage amplifier configuration

Transformer coupling



# i) Cascade connection

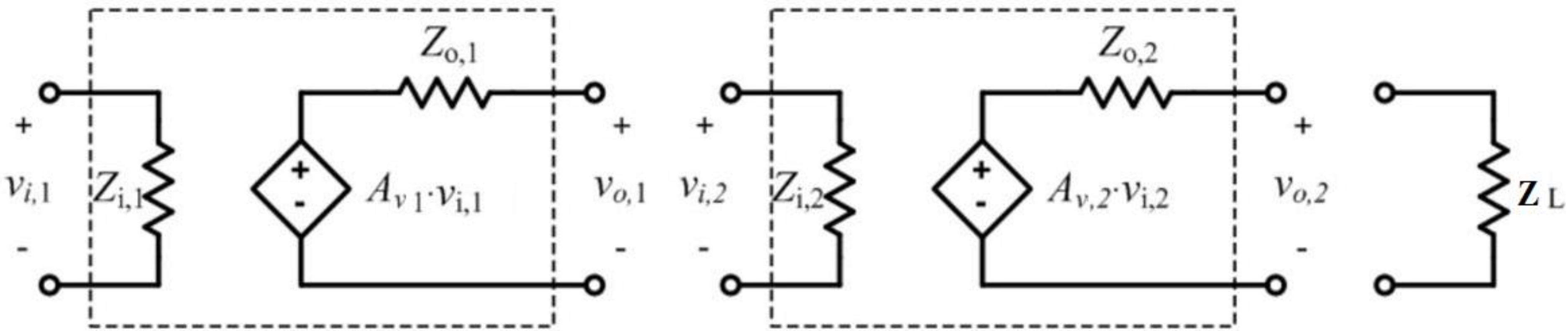
- The most widely used method

- Coupling a signal from one stage to the another stage and block dc voltage from one stage to the another stage

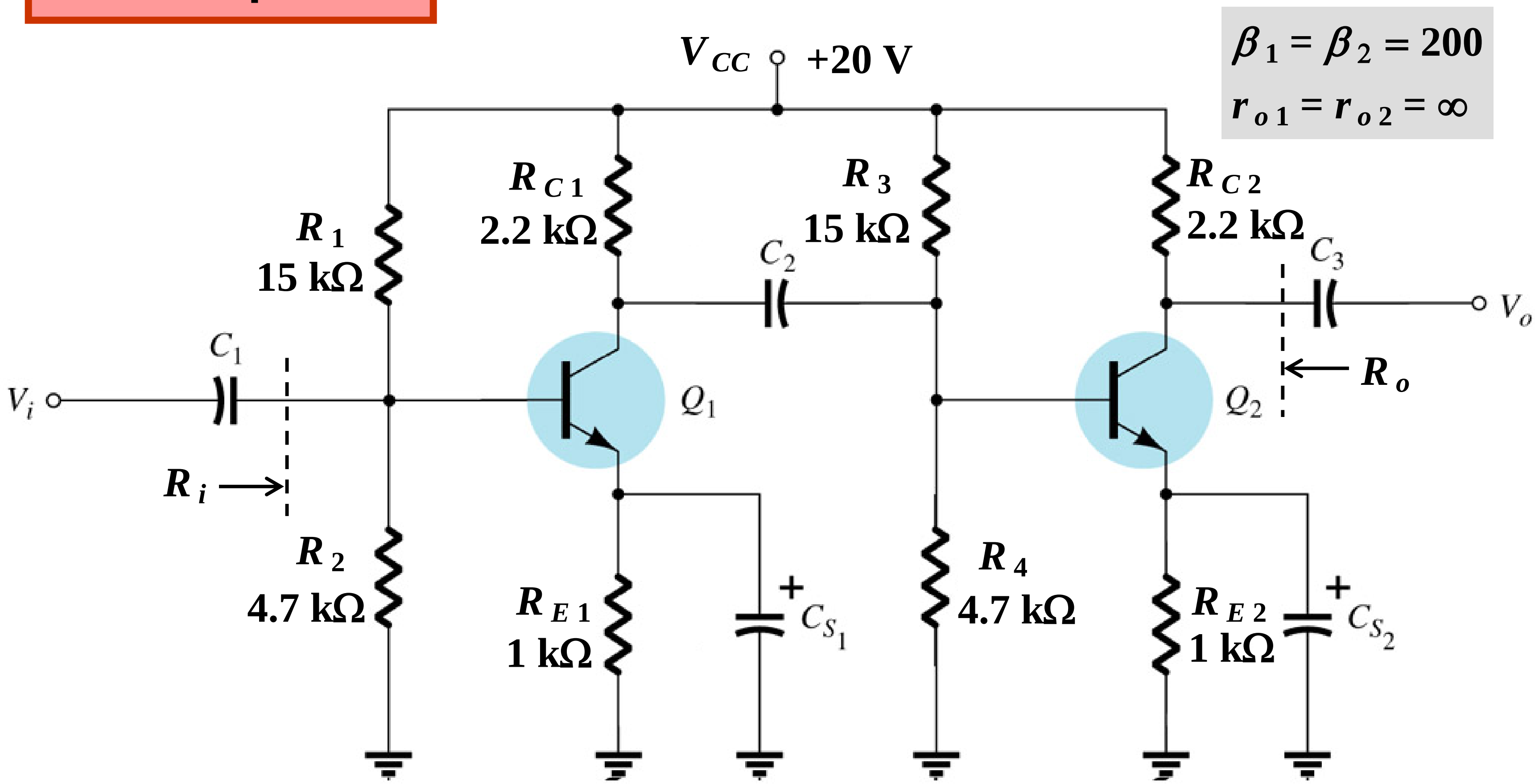
- The signal developed across the collector resistor of each stage is coupled into the base of the next stage

- The overall gain = product of gains of the individual stages (ideally)


Figure 1:- Block Diagram of Two Stage Cascade Amplifiier



# Example 1

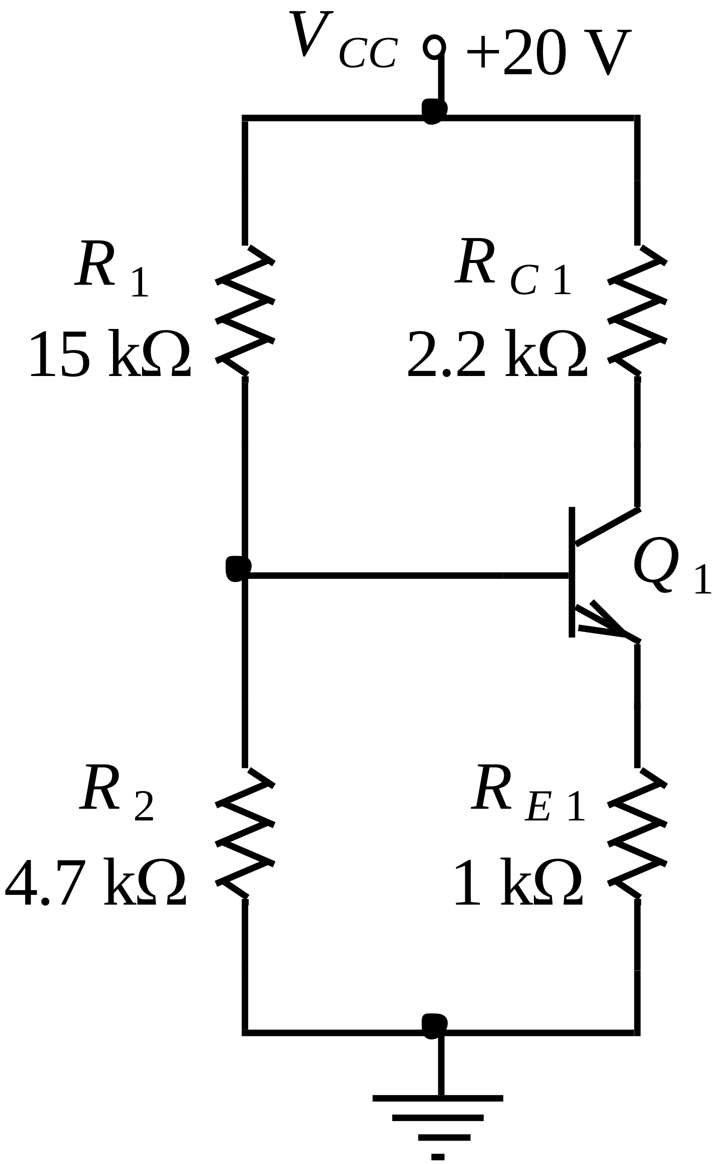



Draw the AC equivalent circuit and calculate $A_v$, $R_i$ and $R_o$.

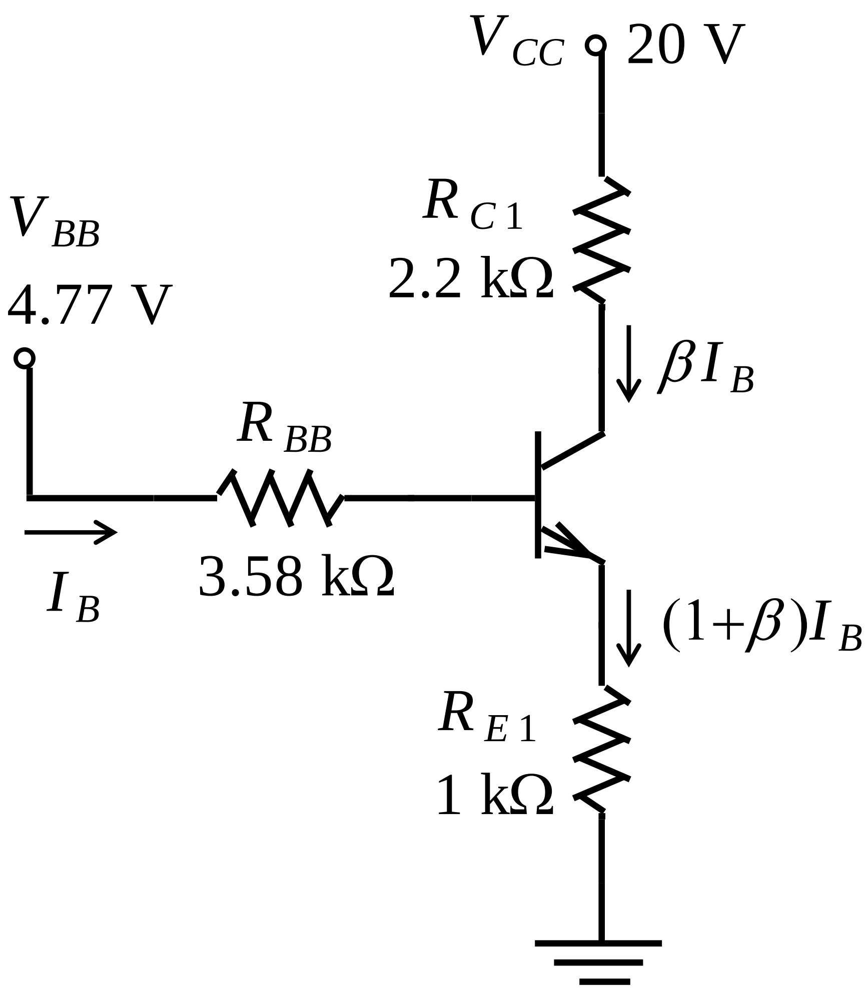
## Solution

DC analysis

The circuit under DC condition (stage 1 and stage 2 are identical)



Applying Thevenin’s theorem, the circuit becomes;




$$g_m = \frac{1}{r_e}$$

$$r_e = \frac{V_t}{I_E}, \quad r_\pi = \beta r_e$$

$$I_{BQ1} = I_{BQ2} = 19.89\ \mu\text{A}$$

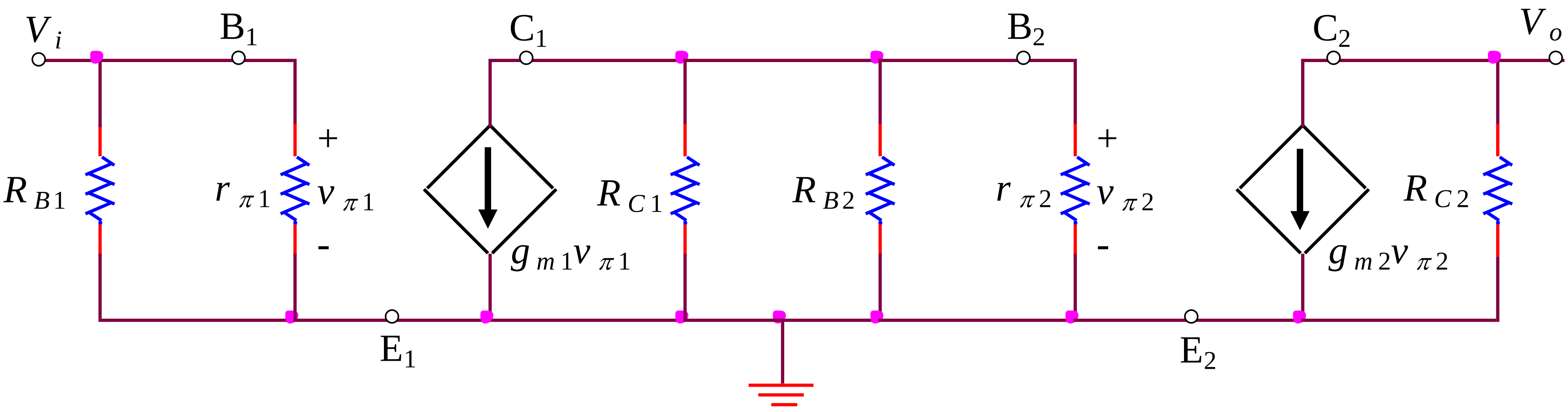
$$I_{CQ1} = I_{CQ2} = 3.979\ \text{mA}$$

$$r_{\pi 1} = r_{\pi 2} = 1.307\ \text{k}\Omega$$

$$g_{m1} = g_{m2} = 0.153\ \text{S}$$

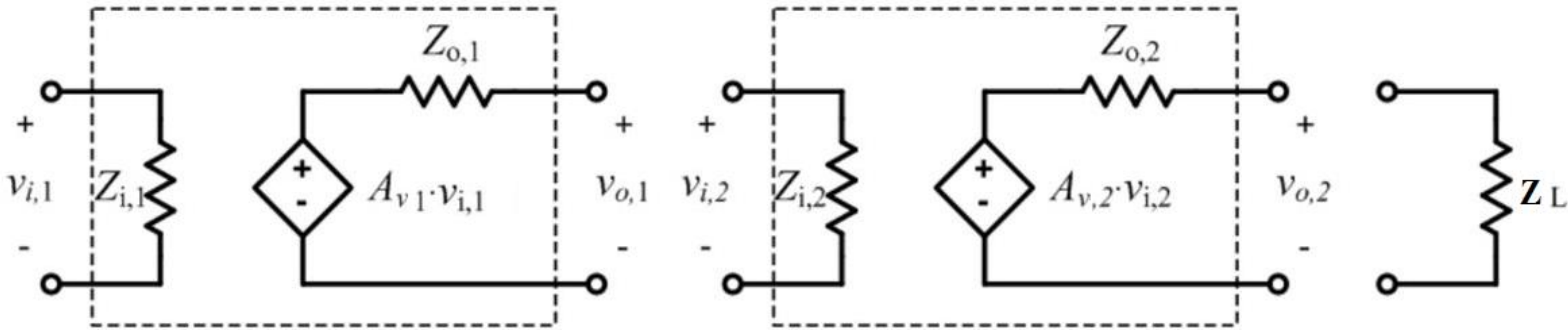
## AC analysis

The small-signal equivalent circuit (mid-band);




$$R_{B1} = R_1 // R_2$$

$$R_{B2} = R_3 // R_4$$



$Z_{o,1}$
$Z_{o,2}$
+
$v_{i,1}$
-
$Z_{i,1}$
$A_{v\,1} \cdot v_{i,1}$
+
$v_{o,1}$
-
+
$v_{i,2}$
-
$Z_{i,2}$
$A_{v,2} \cdot v_{i,2}$
+
$v_{o,2}$
-
Z L

$$V_o = -g_{m2} v_{\pi 2} R_{C2}$$

$$A_2 = \frac{V_o}{v_{\pi 2}} = -g_{m2} R_{C2}$$

$$v_{\pi 2} = -g_{m1} v_{\pi 1} \left( R_{C1} // R_{B2} // r_{\pi 2} \right)$$

$$= -g_{m1} V_i \left( R_{C1} // R_{B2} // r_{\pi 2} \right) \qquad \left[ v_{\pi 1} = V_i \right]$$

$$A_1 = \frac{v_{\pi 2}}{V_i} = -g_{m1} \left( R_{C1} // R_{B2} // r_{\pi 2} \right)$$

The small-signal voltage gain;

$$A = A_1 A_2 = g_{m1} g_{m2} R_{C2} \left( R_{C1} // R_{B2} // r_{\pi 2} \right)$$

Substituting values;

$$R_{B1} = R_{B2} = R_3 // R_4 = 15 // 4.7 = 3.579 \text{ k}\Omega$$

$$R_{C1} // R_{B2} // r_{\pi 2} = 2.2 // 3.579 // 1.307 = 667 \ \Omega$$

$$A = 0.153 \times 0.153 \times 2200 \times 667 = 34350 \ \text{V/V}$$

The input resistance;

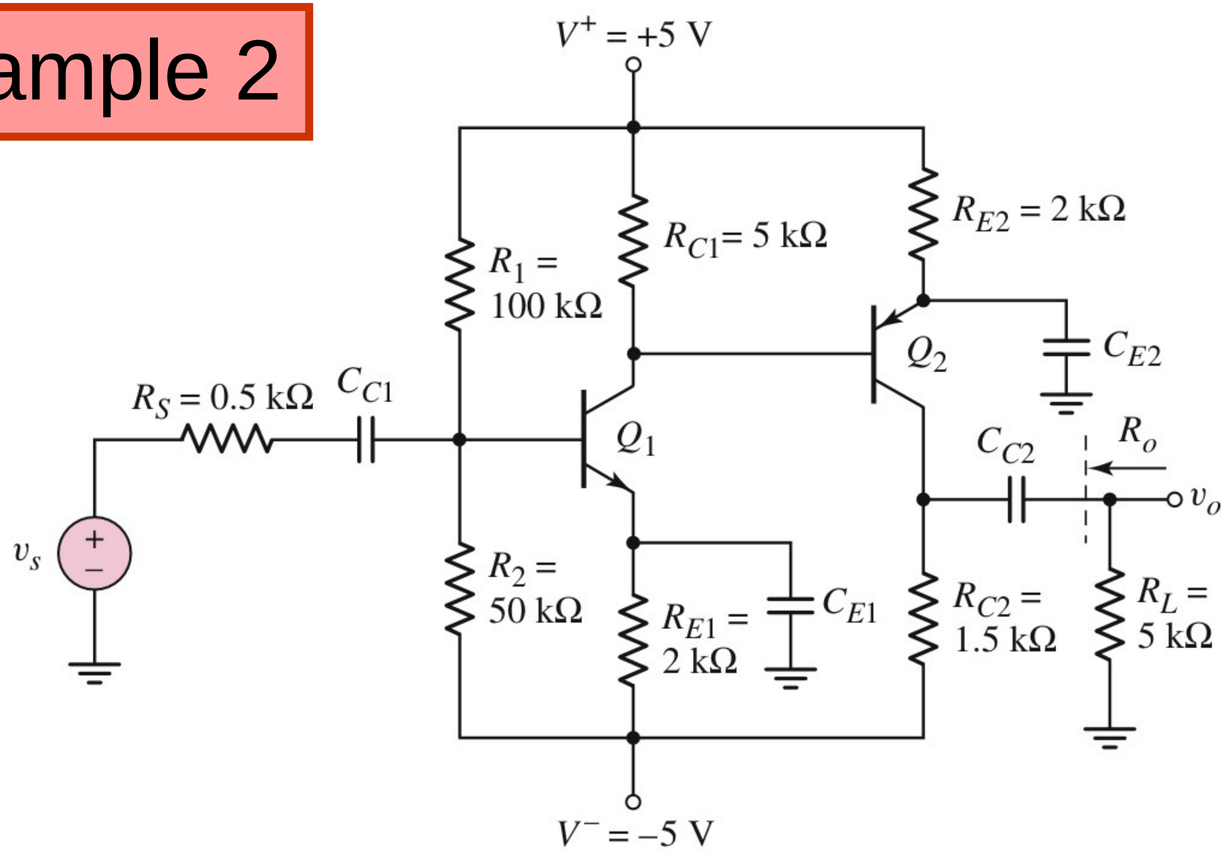
$$R_{in} = R_{B1} // r_{\pi 1} = 3.579 // 1.307 = 0.957 \text{ k}\Omega$$

The output resistance;

$$R_o = R_{C2} = 2.2 \text{ k}\Omega$$

Example:
R1=47 k, R2=10k Rc=4.7k, RE=1k, VCC=10V
VB=1.7V, IE=1mA,
re=25 $\Omega$, Av=-Rc/re=-188, Avtot=12400

# Example 2

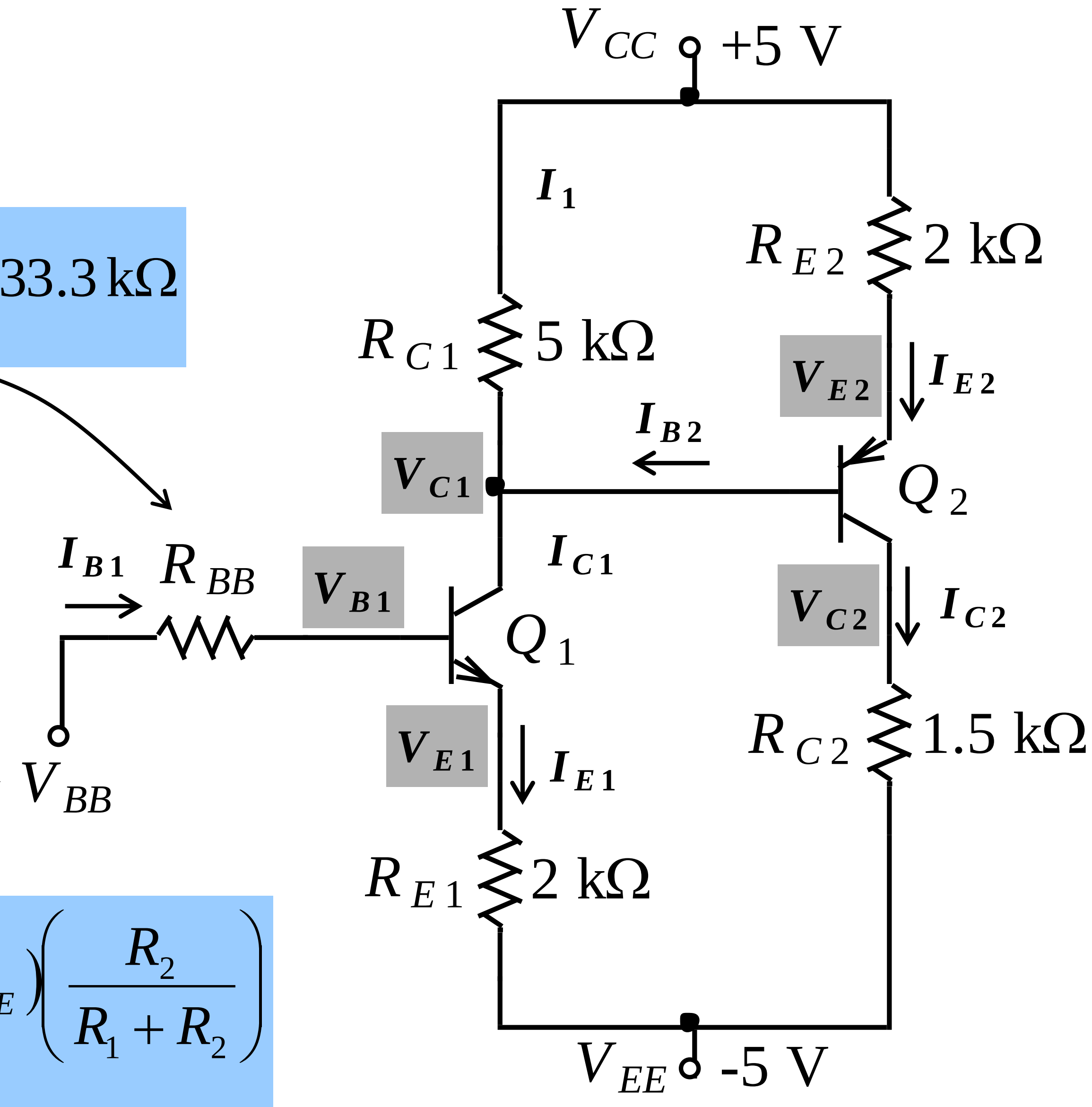



Assuming $\beta_1 = 170$, $\beta_2 = 150$ and $V_{BE(ON)} = 0.7$ V, calculate the voltage gain $A_v$ where;

$$A_v = \frac{v_o}{v_s}$$

## DC analysis

$$R_{BB} = \frac{R_1 R_2}{R_1 + R_2} = 33.3\,\text{k}\Omega$$




$$V_{BB} = V_{EE} + \left(V_{CC} - V_{EE}\right)\left(\frac{R_2}{R_1 + R_2}\right)$$

$$= -1.667V$$

The base-emitter loop of $Q_1$

$$V_{BB} + R_{BB} I_{B1} + V_{BE1} + R_{E1} I_{E1} + V_{EE} = 0$$

$$R_{BB} I_{B1} + (\beta_1 + 1) R_{E1} I_{B1} = V_{BB} - V_{EE} - V_{BE1}$$

Substituting values;

$$33.3 \times 10^3 I_{B1} + (170 + 1) 2 \times 10^3 I_{B1} = -1.667 + 5 - 0.7$$

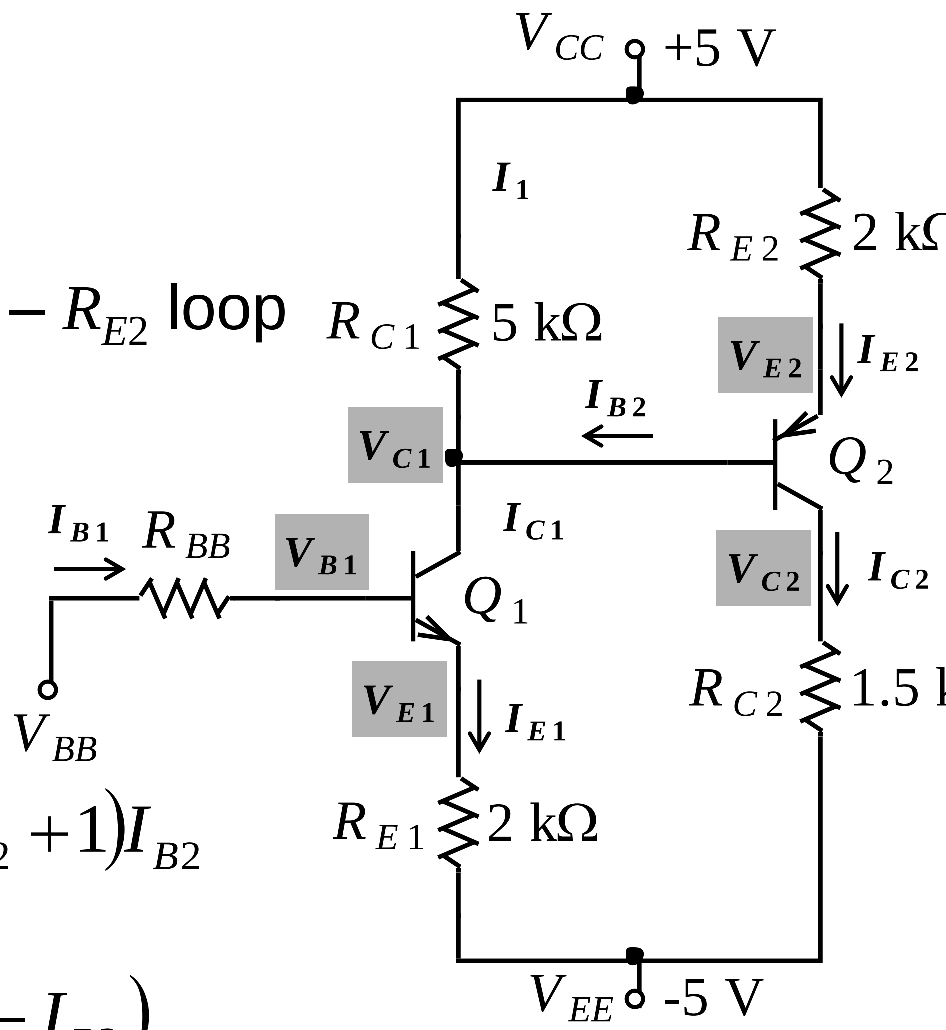
$$I_{B1} = 7\ \mu\text{A} \qquad I_{C1} = \beta_1 I_{B1} = 1.19\ \text{mA}$$

$$I_{E1} = (\beta_1 + 1) I_{B1} = 1.197\ \text{mA}$$

$R_{BB}I_{B1} = -1.9\ \text{V}$

$_{E1}I_{E1} + V_{EE} = -2.606\ \text{V}$

e $R_{C1}$ – collector of $Q_1$ – base of $Q_2$ – $R_{E2}$ loop




$$R_{E2}I_{E2} + V_{EB2} = R_{C1}I_1$$

$I_1 = I_{C1} - I_{B2}$ and $I_{E2} = (\beta_2 + 1)I_{B2}$

$$(\beta_2 + 1)R_{E2}I_{B2} + V_{EB2} = R_{C1}(I_{C1} - I_{B2})$$

Substituting values;

$$(150+1)2\times10^{3}I_{B2}+0.7=5\times10^{3}\left(1.19\times10^{-3}-I_{B2}\right)$$

$$I_{B2}=17\ \mu\text{A}$$

$$I_{C2}=\beta_{2}I_{B2}=2.565\ \text{mA}$$

$$I_{E2}=(\beta_{2}+1)I_{B2}=2.582\ \text{mA}$$

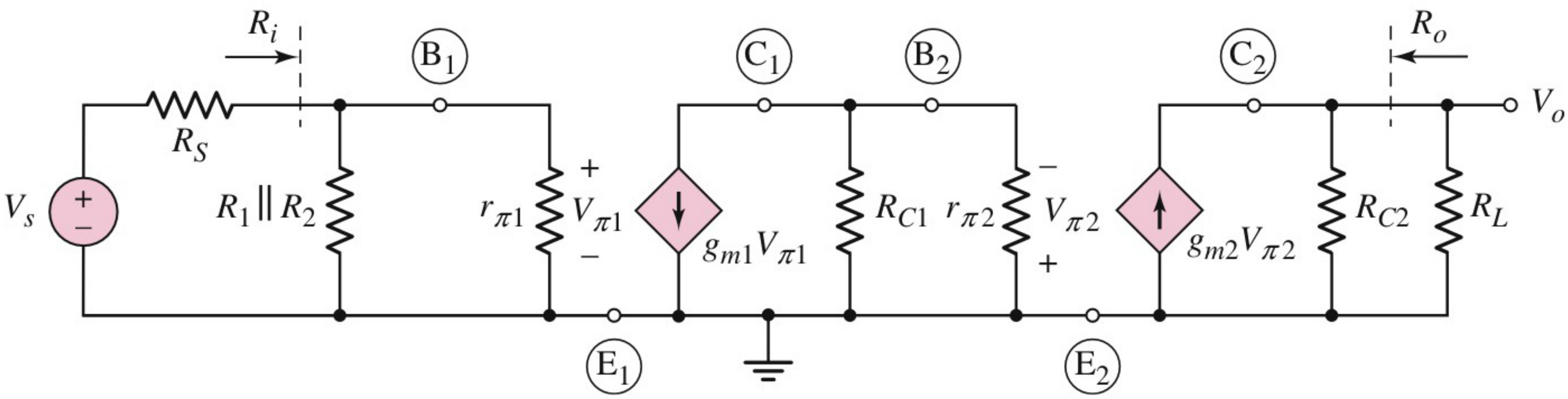
$$I_{1}=I_{C1}-I_{B2}=1.19-0.017=1.173\ \text{mA}$$

$$V_{C1} = V_{CC} - R_{C1} I_1 = 5 - 5 \times 1.173 = -0.865 \text{ V}$$

$$V_{E2} = V_{CC} - R_{E2} I_{E2} = 5 - 2 \times 2.582 = -0.164 \text{ V}$$

$$V_{C2} = R_{C2} I_{C2} + V_{EE} = 1.5 \times 2.565 - 5 = -1.175 \text{ V}$$

# AC analysis




$$A_1 \equiv \frac{-V_{\pi 2}}{V_s}$$

$$A_2 \equiv \frac{V_o}{-V_{\pi 2}}$$

$$V_{\pi 2} = g_{m1} V_{\pi 1} \left( R_{C1} // r_{\pi 2} \right) \Longrightarrow \frac{-V_{\pi 2}}{V_{\pi 1}} = -g_{m1} \left( R_{C1} // r_{\pi 2} \right)$$

$$V_{\pi 1} = V_s \left( \frac{R_{in}}{R_S + R_{in}} \right) \Longrightarrow \frac{V_{\pi 1}}{V_s} = \frac{R_{in}}{R_S + R_{in}}$$

$$A_1 = \frac{-V_{\pi 2}}{V_s} = \frac{-V_{\pi 2}}{V_{\pi 1}} \times \frac{V_{\pi 1}}{V_s} = -g_{m1} \left( R_{C1} // r_{\pi 2} \right) \left( \frac{R_{in}}{R_S + R_{in}} \right)$$

$$V_o = g_{m2} V_{\pi 2} \left( R_{C2} // R_L \right) \Longrightarrow A_2 = \frac{V_o}{-V_{\pi 2}} = -g_{m2} \left( R_{C2} // R_L \right)$$

$$A = A_1 A_2 = -g_{m1}\left(R_{C1} // r_{\pi 2}\right)\left(\frac{R_{in}}{R_S + R_{in}}\right)\left[-g_{m2}\left(R_{C2} // R_L\right)\right]$$

$$A = g_{m1} g_{m2}\left(\frac{R_{in}}{R_S + R_{in}}\right)\left(R_{C1} // r_{\pi 2}\right)\left(R_{C2} // R_L\right)$$

Substituting values;

$$g_{m1} = \frac{I_{E1}}{V_T} = \frac{1.197}{26} = 45.77 \text{ mA/V}$$

$$r_{\pi 1} = \beta_1 \frac{V_T}{I_{C1}} = 170 \times \frac{26}{1.19} = 3714\ \Omega$$

$$g_{m2} = \frac{I_{E2}}{V_T} = \frac{2.565}{26} = 98.65 \text{ mA/V}$$

$$r_{\pi 2} = \beta_2 \frac{V_T}{I_{C2}} = 150 \times \frac{26}{2.565} = 1520\ \Omega$$

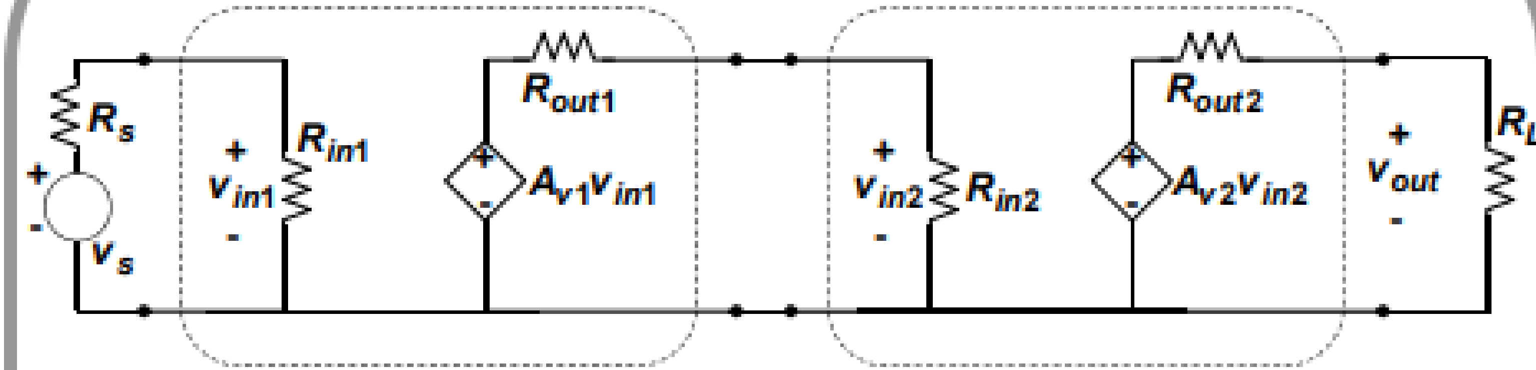
$$R_{in} = R_1 // R_2 // r_{\pi 1} = 100 // 50 // 3.714 = 3.342 \text{ k}\Omega$$

$$A = g_{m1} g_{m2} \left( \frac{R_{in}}{R_S + R_{in}} \right) \left( R_{C1} // r_{\pi 2} \right) \left( R_{C2} // R_L \right)$$

$$= 45.77 \times 98.65 \left( \frac{3.342}{0.5 + 3.342} \right) (5 // 1.52)(1.5 // 5)$$

$$A = 5286$$

# A Cascade of Two Amplifiers with a Source Resistor




Voltage gain:

$$v_{out} = A_{v2} v_{in2} \frac{R_L}{R_{out2} + R_L} = A_{v2} A_{v1} v_{in1} \frac{R_{in2}}{R_{out1} + R_{in2}} \frac{R_L}{R_{out2} + R_L}$$

$$= A_{v2} A_{v1} v_{in1} \frac{R_{in2}}{R_{out1} + R_{in2}} \frac{R_L}{R_{out2} + R_L}$$

$$= A_{v2} A_{v1} v_s \frac{R_{in1}}{R_s + R_{in1}} \frac{R_{in2}}{R_{out1} + R_{in2}} \frac{R_L}{R_{out2} + R_L}$$

$$\Rightarrow \frac{v_{out}}{v_s} = A_{v2} A_{v1} \frac{R_{in1}}{R_s + R_{in1}} \frac{R_{in2}}{R_{out1} + R_{in2}} \frac{R_L}{R_{out2} + R_L}$$

Input voltage divider — Inter-stage voltage divider — Output voltage divider

For the CS FET stages: $R_{in1} = R_{in2} = \infty$

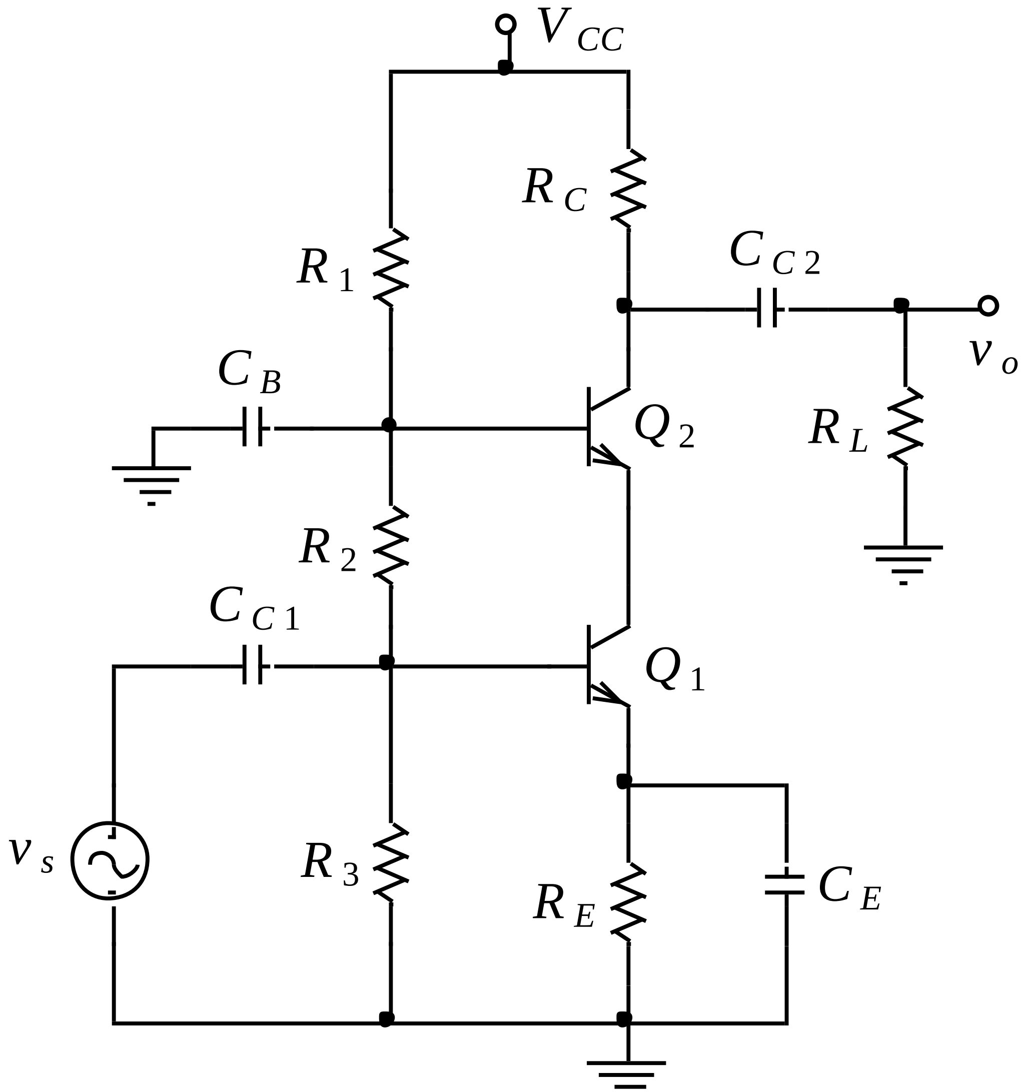
## ii) Cascode connection

- A cascode connection has one transistor on top of (in series with) another- current through two stages are equal

- The i/p applied to a C-E amp. ($Q_1$) whose output is used to drive a C-B amp. ($Q_2$)

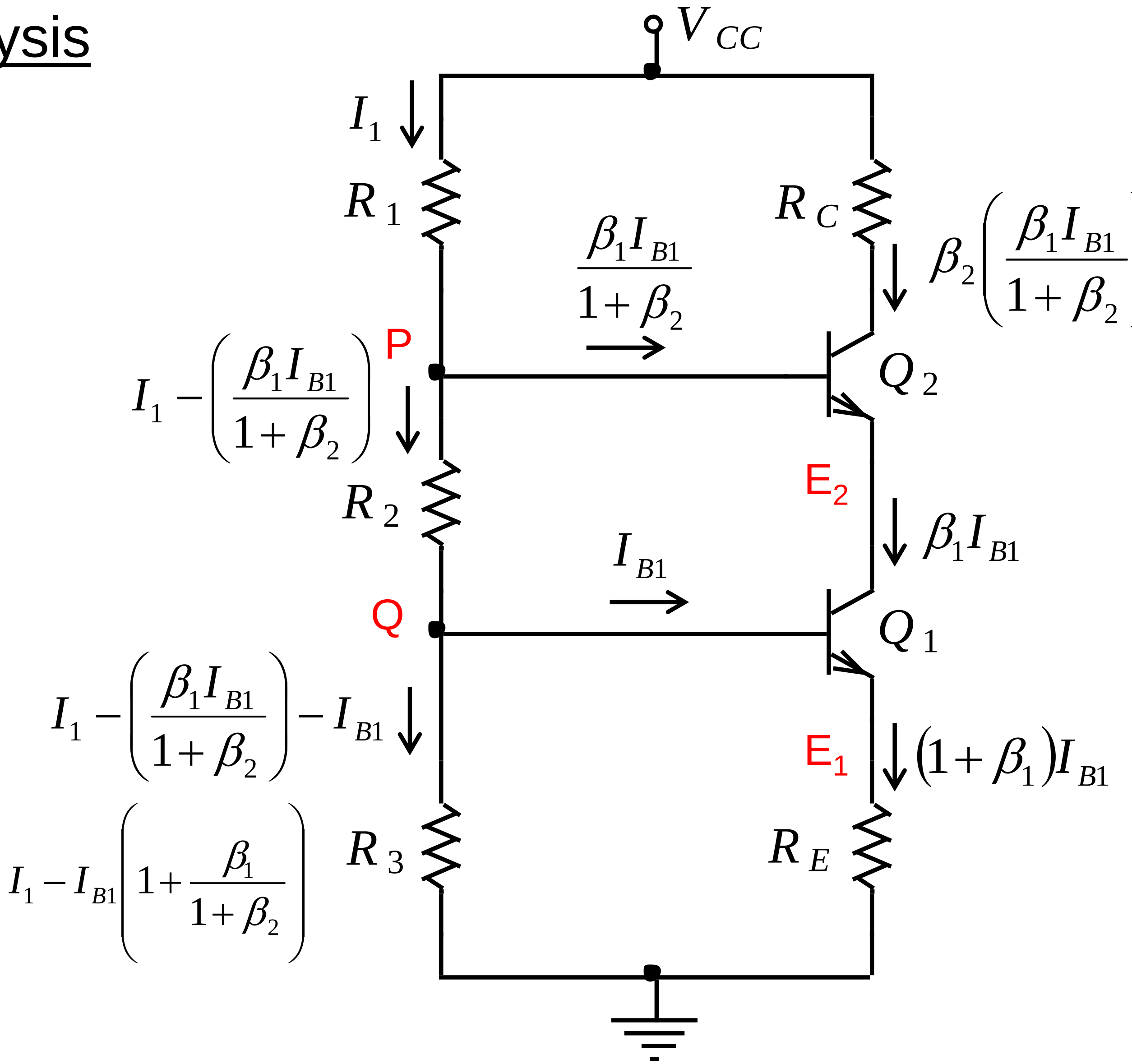
- The o/p signal current of $Q_1$ is the i/p signal of $Q_2$

- The advantage: provides a high i/p impedance with high voltage gain to ensure the i/p Miller capacitance is at a min. with the C-B stage providing good high freq. operation




Cascode amplifier

# DC analysis


$V_{CC}$
$I_1$
$R_1$
$R_C$
$\frac{\beta_1 I_{B1}}{1+\beta_2}$
$\beta_2\left(\frac{\beta_1 I_{B1}}{1+\beta_2}\right)$
P
$I_1-\left(\frac{\beta_1 I_{B1}}{1+\beta_2}\right)$
$Q_2$
$E_2$
$R_2$
$\beta_1 I_{B1}$
$I_{B1}$
Q
$Q_1$
$I_1-\left(\frac{\beta_1 I_{B1}}{1+\beta_2}\right)-I_{B1}$
$E_1$
$(1+\beta_1)I_{B1}$
$I_1-I_{B1}\left(1+\frac{\beta_1}{1+\beta_2}\right)$
$R_3$
$R_E$

The equations are (assuming $V_{BE}$ = 0.7 V for both BJT's);

Applying KVL in the loop VCC, P, Q, Gnd

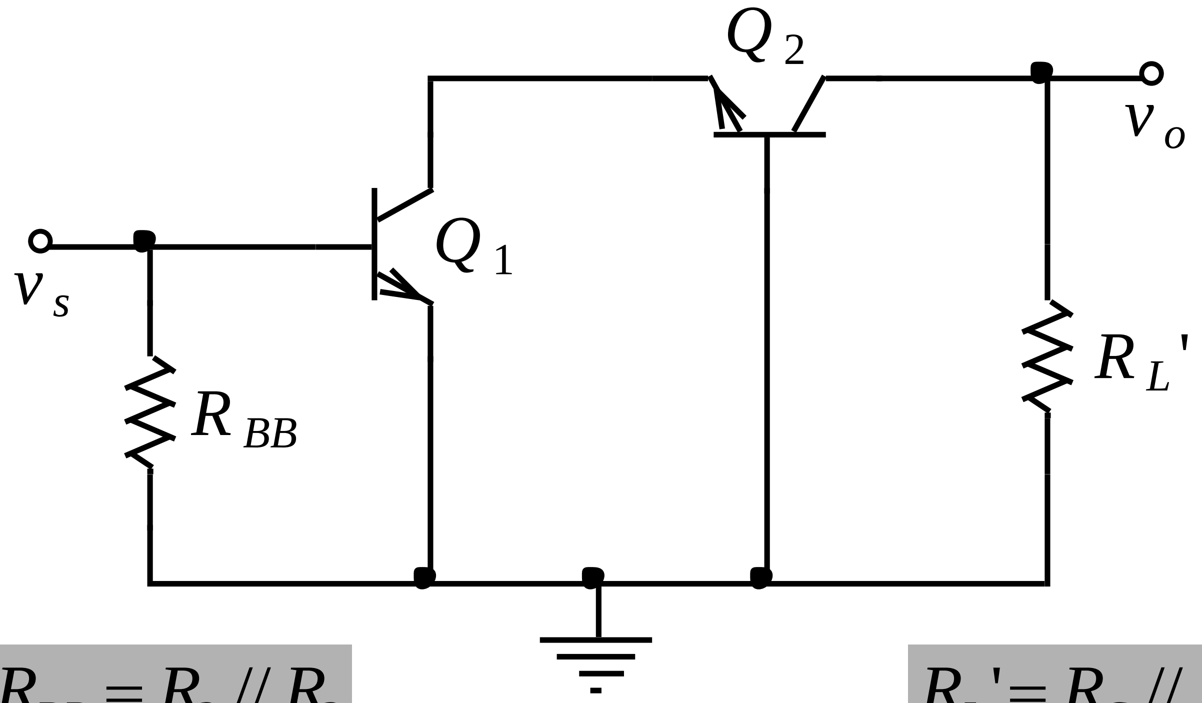
$$R_1 I_1 + R_2\left(I_1 - \frac{\beta_1 I_{B1}}{\beta_2 + 1}\right) + R_3\left[I_1 - \left(1 + \frac{\beta_1}{\beta_2 + 1}\right) I_{B1}\right] = V_{CC}$$

Applying KVL in the loop Q, $E_1$, Gnd

$$R_3\left[I_1 - \left(1 + \frac{\beta_1}{\beta_2 + 1}\right) I_{B1}\right] = V_{BE1} + R_E(\beta_1 + 1) I_{B1}$$

The above equations may solved for the two unknown currents namely $I_1$ and $I_{B1}$.

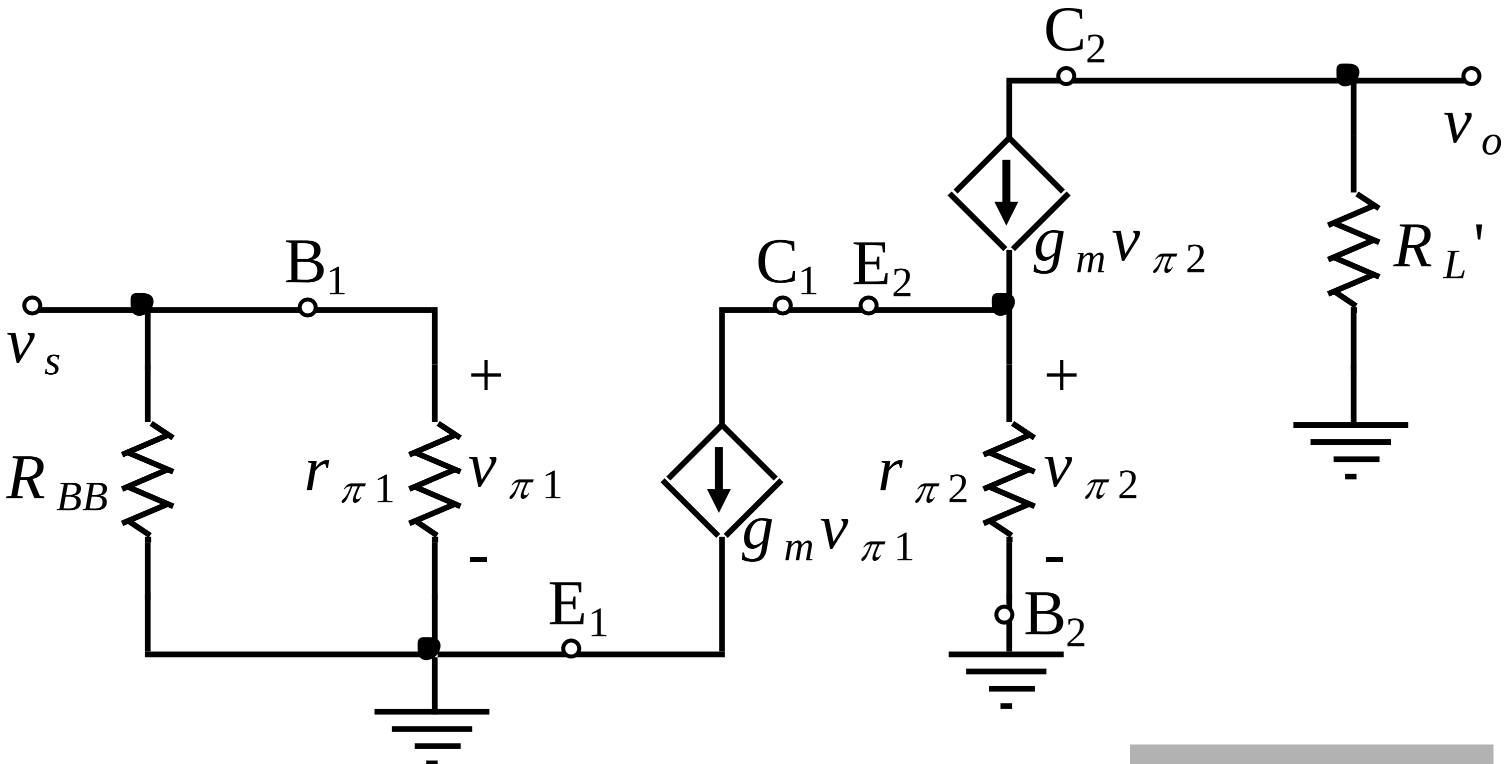
## AC analysis




$R_{BB} = R_2 \,//\, R_3$

$R_L' = R_C \,//\, R_L$

The equivalent circuit under AC condition




$$R_{BB} = R_2 \,//\, R_3$$

$$R_L' = R_C \,//\, R_L$$

The ac equivalent circuit using hybrid-π model for BJT

$$v_o = -g_{m2} v_{\pi 2} R_L' \qquad (1)$$

At node $E_2$;

$$g_{m1} v_{\pi 1} = \frac{v_{\pi 2}}{r_{\pi 2}} + g_{m2} v_{\pi 2} \qquad \text{Or;} \qquad v_{\pi 2} = \frac{g_{m1} r_{\pi 2} v_{\pi 1}}{1 + g_{m2} r_{\pi 2}}$$

Substituting in (1);

$$v_o = -\left( \frac{g_{m1} g_{m2} r_{\pi 2}}{1 + g_{m2} r_{\pi 2}} \right) R_L' v_{\pi 1} = -g_{m1} \left( \frac{\beta_2}{1 + \beta_2} \right) R_L' v_s$$

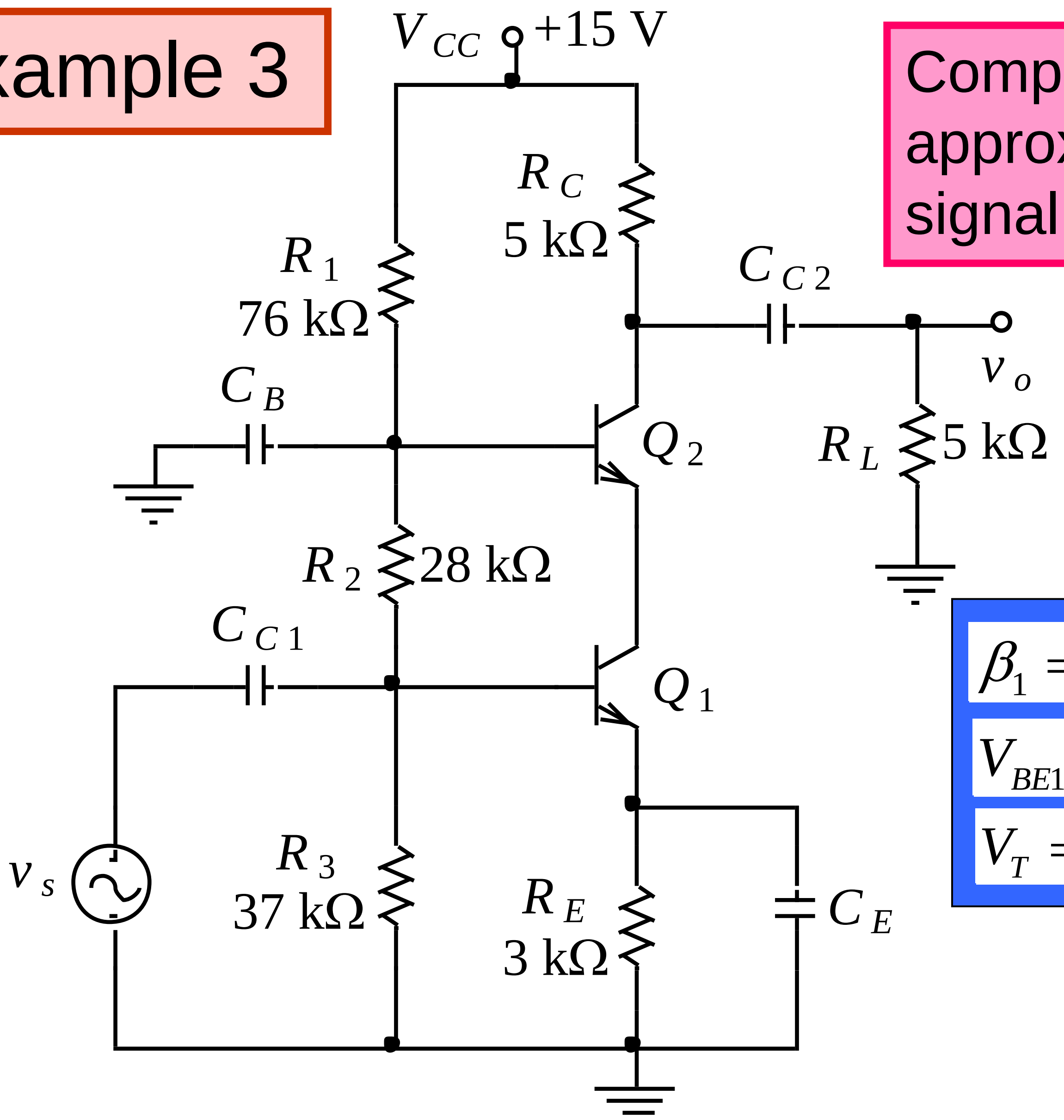
The small-signal voltage gain;

$$A_v = \frac{v_o}{v_s} = -g_{m1}\left(\frac{\beta_2}{1+\beta_2}\right)R_L'$$

When $\beta_2 >> 1$

$$A_v \cong -g_{m1}R_L'$$

# Example 3




Compute the approximate small-signal voltage gain

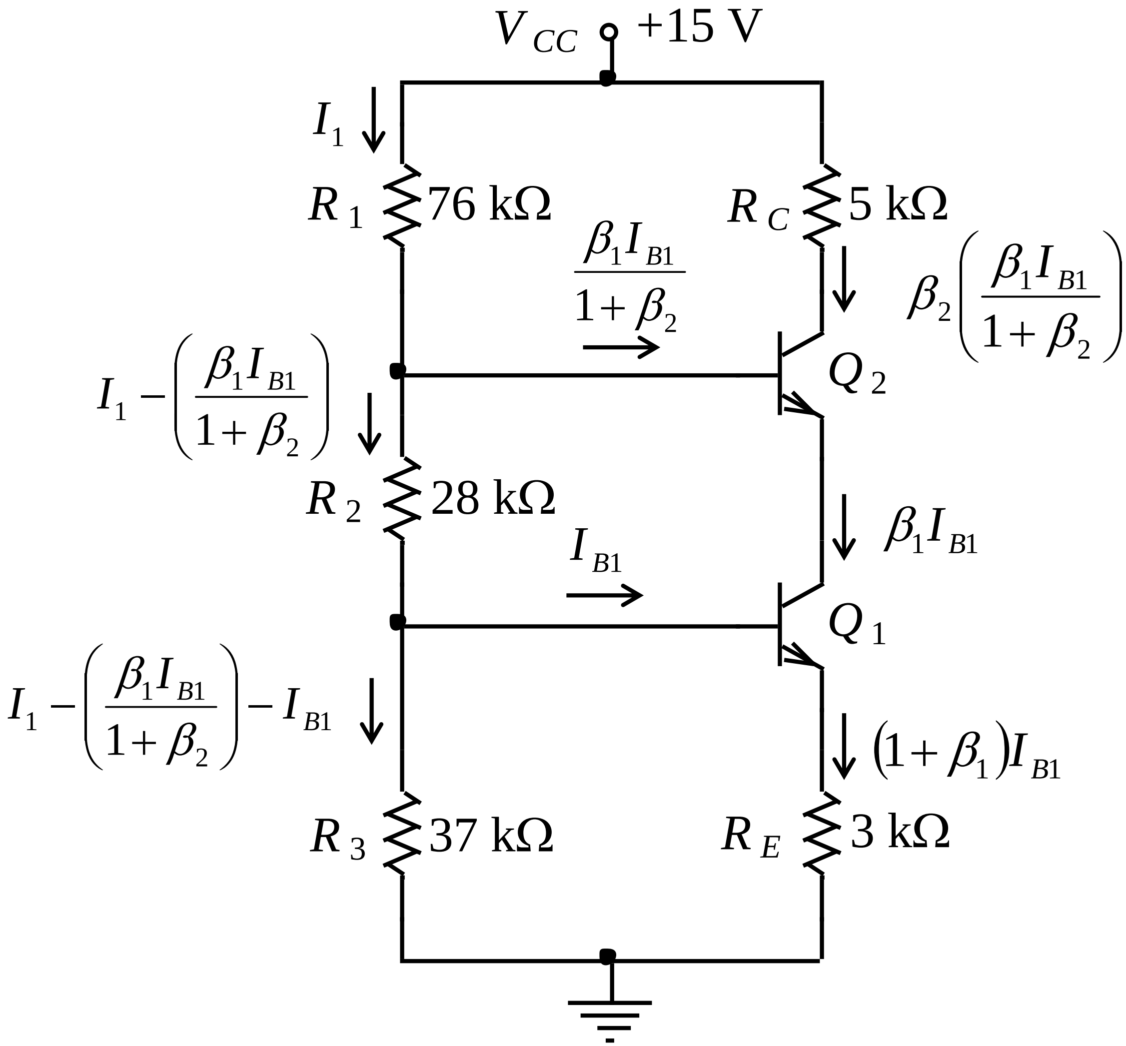
$\beta_1 = \beta_2 = 150$

$V_{BE1} = V_{BE2} = 0.7\text{ V}$

$V_T = 26\text{ mV}$

**SOLUTION**

DC analysis




The circuit under DC condition

$$R_1 I_1 + R_2\left(I_1 - \frac{\beta_1 I_{B1}}{\beta_2 + 1}\right) + R_3\left(I_1 - \frac{\beta_1 I_{B1}}{\beta_2 + 1} - I_{B1}\right) = V_{CC}$$

Substituting values;

$$76\text{k} I_1 + 28\text{k}\left(I_1 - \frac{150 I_{B1}}{150 + 1}\right) + 37\text{k}\left(I_1 - \frac{150 I_{B1}}{150 + 1} - I_{B1}\right) = 15$$

$$141 I_1 - 101.54 I_{B1} = 15 \times 10^{-3} \qquad (1)$$

$$R_3\left(I_1 - \frac{\beta_1 I_{B1}}{\beta_2 + 1} - I_{B1}\right) = 0.7 + R_E(\beta_1 + 1)I_{B1}$$

Or;

$$R_3\left(I_1 - \frac{\beta_1 I_{B1}}{\beta_2 + 1} - I_{B1}\right) - R_E(\beta_1 + 1)I_{B1} = 0.7$$

Substituting values;

$$37\left(I_1 - \frac{150 I_{B1}}{150 + 1} - I_{B1}\right) - 3(150 + 1)I_{B1} = 0.7 \times 10^{-3}$$

$$37 I_1 - 526.75 I_{B1} = 0.7 \times 10^{-3}$$

$$I_1 = 0.0189 \times 10^{-3} + 14.24 I_{B1} \qquad (2)$$

Substituting for $I_1$ in (1)

$$141\left(0.0189 \times 10^{-3} + 14.24 I_{B1}\right) - 101.54 I_{B1} = 15 \times 10^{-3}$$

$$I_{B1} = \frac{12.335 \times 10^{-3}}{1906.3} = 6.47\ \mu\text{A}$$

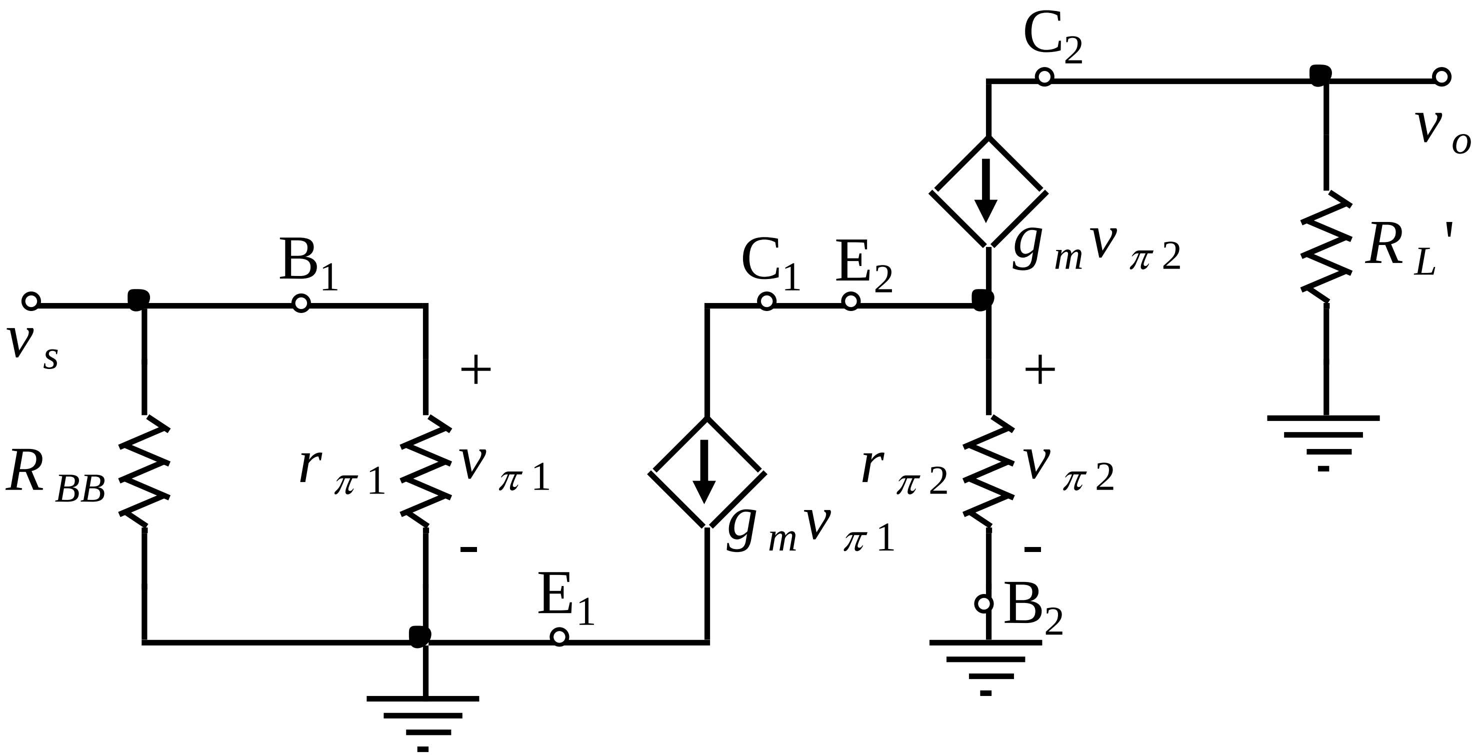
$$I_{CQ1} = \beta I_{B1} = 150 \times 6.47\ \mu\text{A} = 0.97\ \text{mA}$$

$$I_{E2} = I_{CQ1} = 0.97\ \text{mA}$$

$$I_{B2} = \frac{I_{E2}}{\beta + 1} = \frac{0.97}{151} = 6.42\ \mu\text{A}$$

$$I_{CQ2} = \beta I_{B2} = 150 \times 6.42\ \mu\text{A} = 0.964\ \text{mA}$$

# AC analysis




Small-signal equivalent circuit using hybrid-π model

$$v_o = -g_{m2} v_{\pi 2} R_L' \qquad (1)$$

At node $\mathrm{E}_2$; $$g_{m1} v_{\pi 1} = \frac{v_{\pi 2}}{r_{\pi 2}} + g_{m2} v_{\pi 2}$$

Hence; $$v_{\pi 2} = \frac{g_{m1} r_{\pi 2} v_{\pi 1}}{1 + r_{\pi 2} g_{m2}}$$

Substituting for $v_{\pi 2}$ in (1);

$$v_o = -\left( \frac{g_{m1} r_{\pi 2} g_{m2}}{1 + r_{\pi 2} g_{m2}} \right) R_L' v_{\pi 1}$$

Or; $$v_o = -g_{m1}\left(\frac{\beta_2}{1+\beta_2}\right)R_L' v_i$$

The voltage gain;

$$A_v \equiv \frac{v_o}{v_i} = -g_{m1}\left(\frac{\beta_2}{1+\beta_2}\right)R_L'$$

When $\beta_2 >> 1$;

$$A_v \approx -g_{m1}R_L'$$

Substituting values;

$$g_{m1} = \frac{I_{CQ1}}{V_T} = \frac{0.97}{26} = 37.3\,\text{mA/V}$$

$$R_L' = R_C \,//\, R_L = 5\text{k} \,//\, 5\text{k} = 2.5\,\text{k}\Omega$$

$$A_v \approx -0.0373(2500)$$

$$A_v = -93.25\ \text{V/V}$$

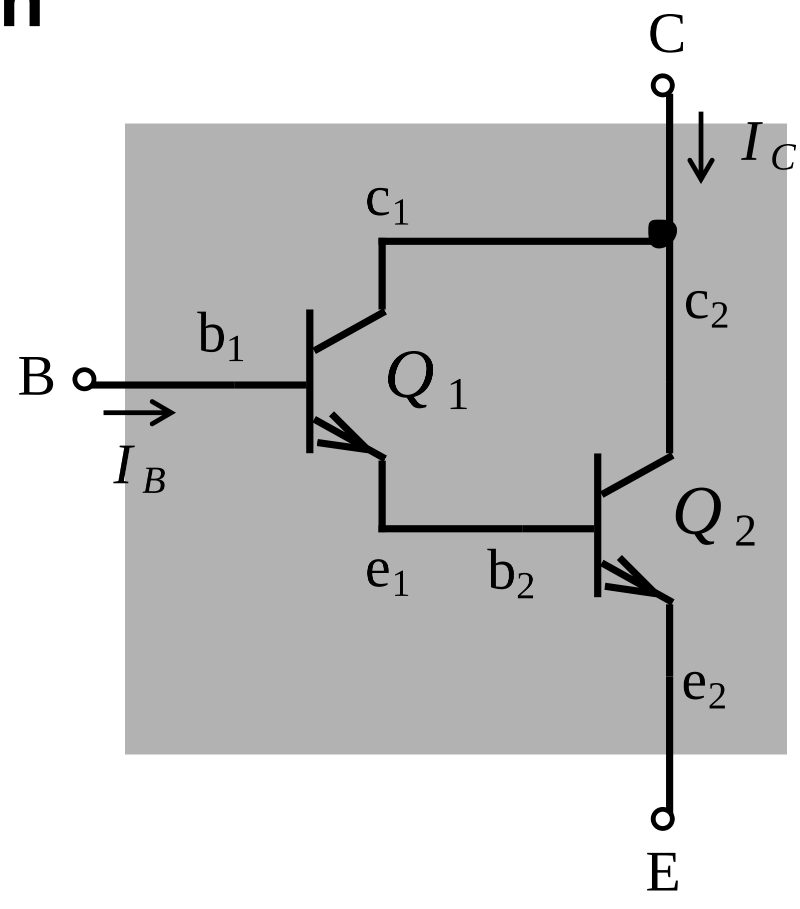
# iii) Darlington connection

Darlington pair

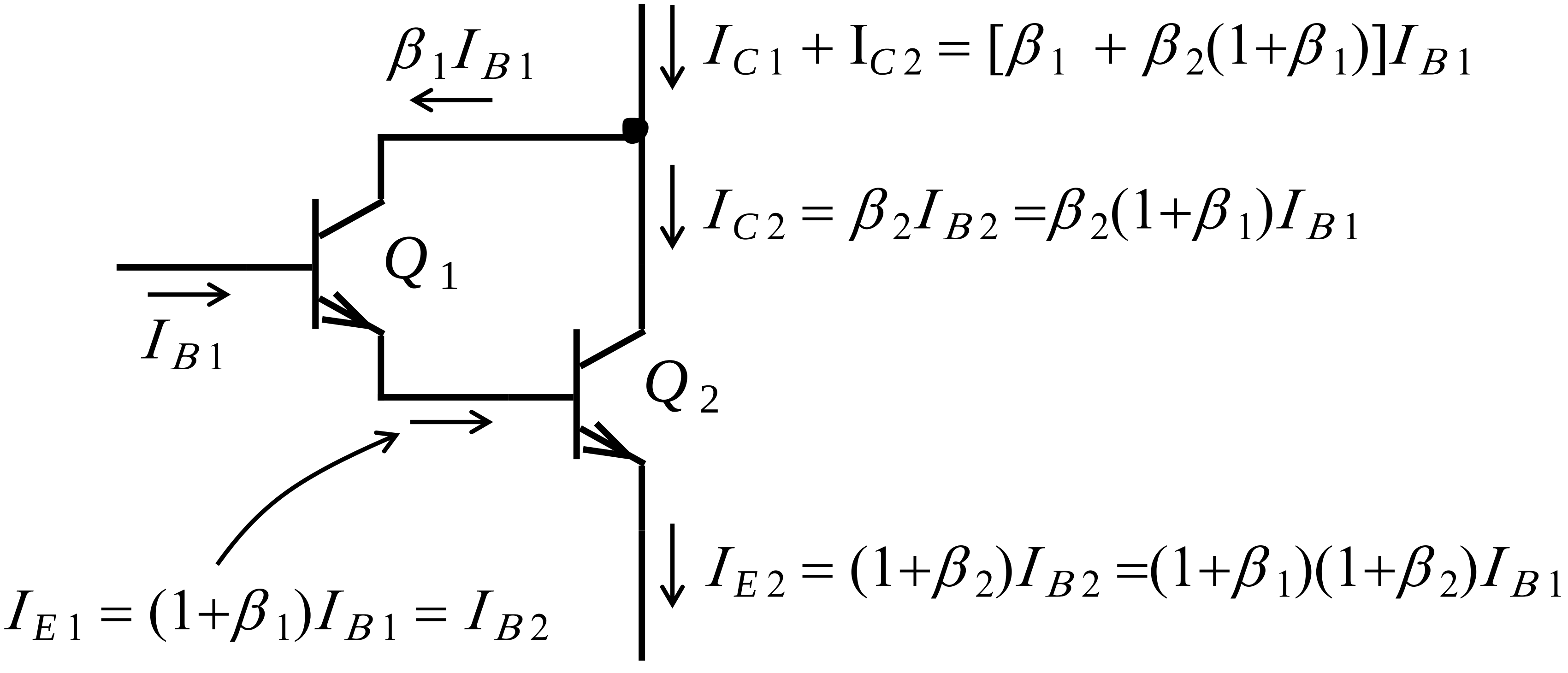
Internal connection;

- Collectors of $Q_1$ and $Q_2$;
- Emitter of $Q_1$ and base of $Q_2$.

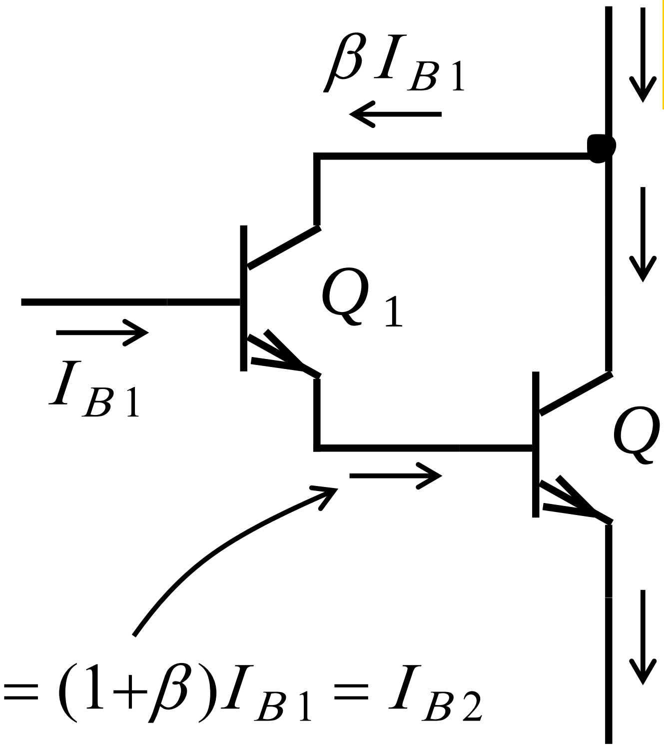
Provides high current gain : $I_C \cong \beta^2 I_B$



# Currents in darlington pair



If $\beta_1 = \beta_2 = \beta$ and assuming $\beta$ is large;



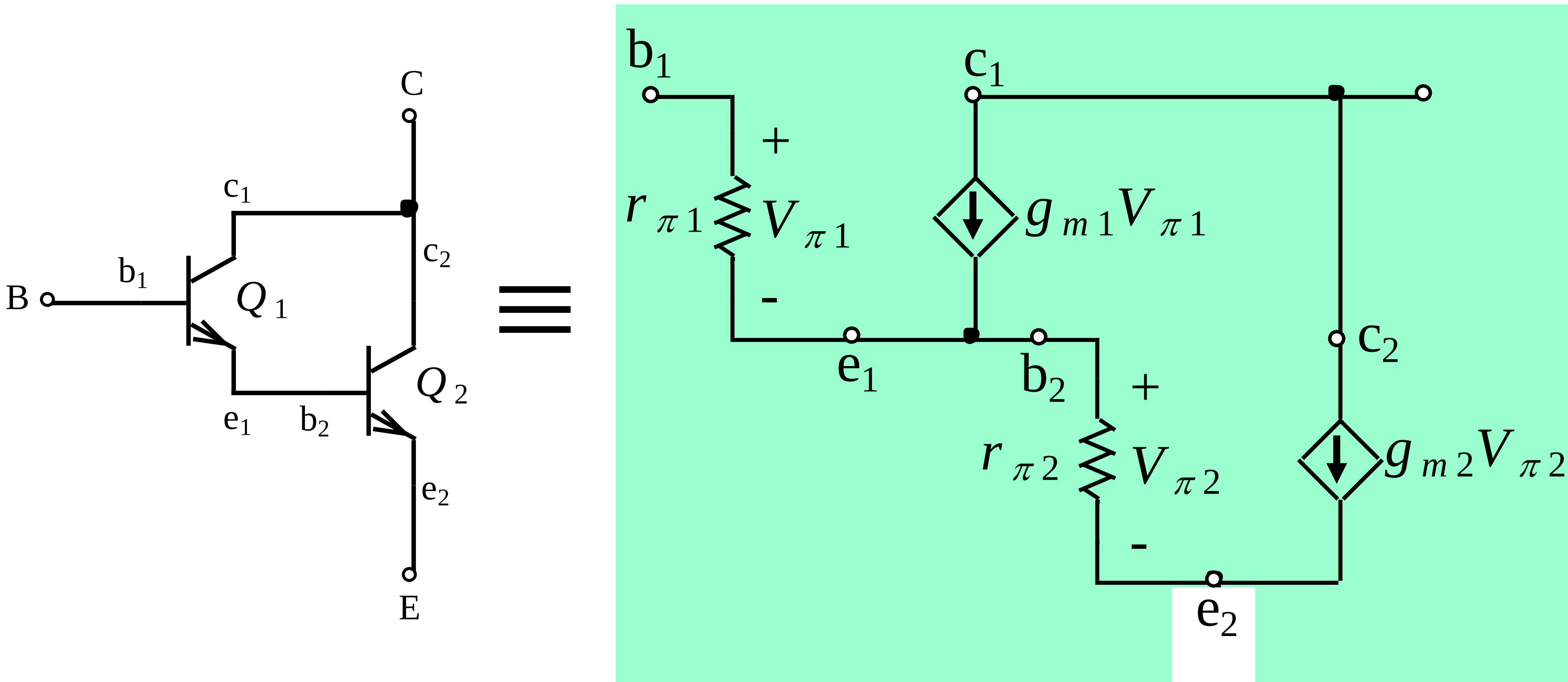

$$I_{C1} + I_{C2} \cong \beta^2 I_{B1}$$

$$I_{C2} = \beta I_{B2} = \beta(1+\beta) I_{B1}$$

$$I_{E2} = (1+\beta) I_{B2} = (1+\beta)^2 I_{B1}$$

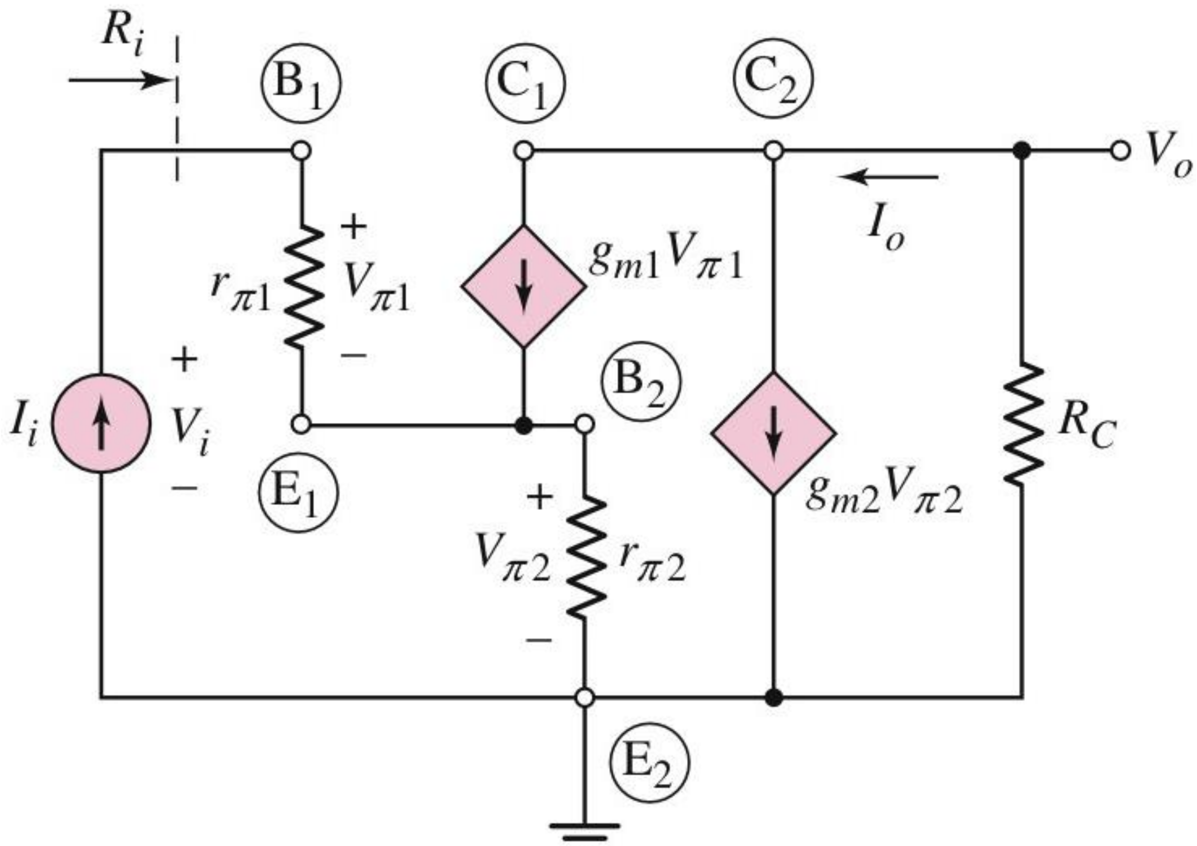
$$I_{E1} = (1+\beta) I_{B1} = I_{B2}$$

Hybrid-π model (assuming $r_{o1} = r_{o2} = \infty$);



Darlington configuration provides;

- Increased current;
- High input resistance.
- Low output impedance



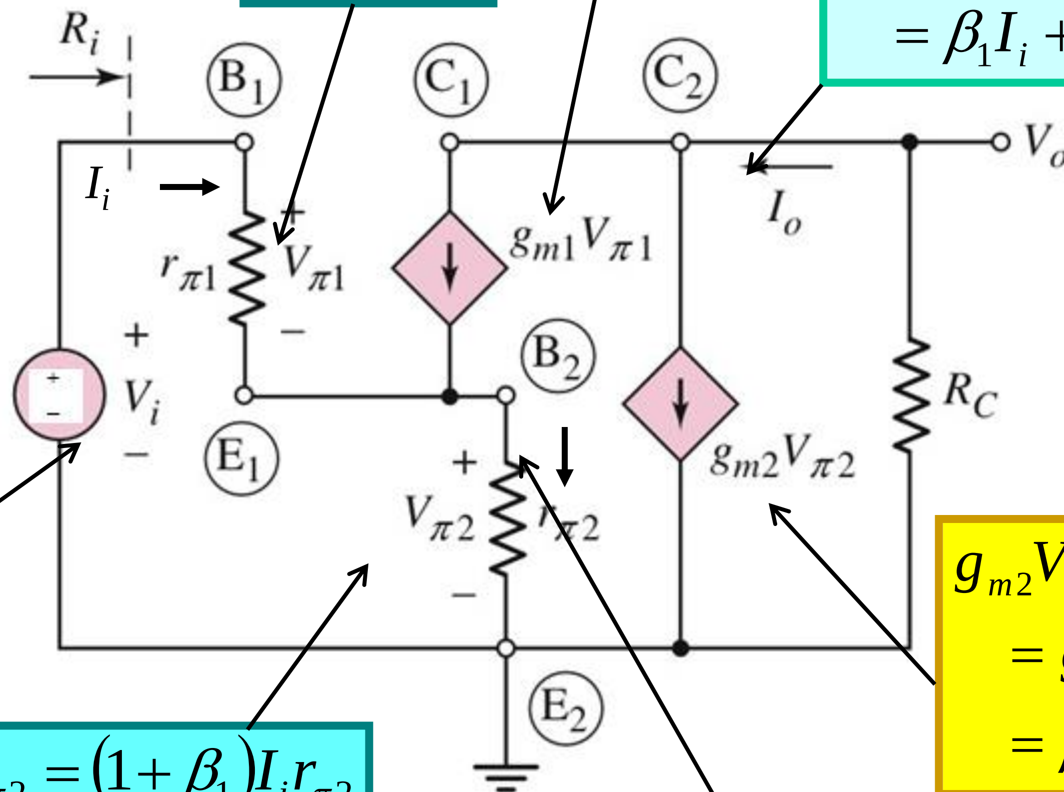

Small-signal equivalent circuit

Input voltage source is transformed into current source

$$g_{m1}V_{\pi 1} = g_{m1}r_{\pi 1}I_i = \beta_1 I_i$$

$$V_{\pi 1} = I_i r_{\pi 1}$$

$$I_o = g_{m1}V_{\pi 1} + g_{m2}V_{\pi 2} = \beta_1 I_i + \beta_2 (1+\beta_1) I_i$$




$$V_i = V_{\pi 1} + V_{\pi 2}$$

$$g_{m2}V_{\pi 2} = g_{m2}(1+\beta_1) I_i r_{\pi 2} = \beta_2 (1+\beta_1) I_i$$

$$V_{\pi 2} = (1+\beta_1) I_i r_{\pi 2}$$

$$I_{b2} = I_i + \beta_1 I_i = (1+\beta_1) I_i$$

$$V_{\pi 1} = I_i r_{\pi 1} \Longrightarrow g_{m1} V_{\pi 1} = g_{m1} r_{\pi 1} I_i = \beta_1 I_i$$

$$I_{b2} = I_i + \beta_1 I_i = (1 + \beta_1) I_i \Longrightarrow V_{\pi 2} = I_{b2} r_{\pi 2} = (1 + \beta_1) I_i r_{\pi 2}$$

$$g_{m2} V_{\pi 2} = g_{m2} (1 + \beta_1) I_i r_{\pi 2} = \beta_2 (1 + \beta_1) I_i$$

$$I_o = g_{m1} V_{\pi 1} + g_{m2} V_{\pi 2} = \beta_1 I_i + \beta_2 (1 + \beta_1) I_i$$

The current gain is;

$$A_i = \frac{I_o}{I_i} = \beta_1 + \beta_2 (1 + \beta_1) \cong \beta_1 \beta_2$$

$$V_i = V_{\pi 1} + V_{\pi 2} = I_i r_{\pi 1} + (1+\beta_1) I_i r_{\pi 2}$$

The input resistance is;

$$R_i = \frac{V_i}{I_i} = r_{\pi 1} + (1+\beta_1) r_{\pi 2}$$

**EXERCISE 1**

Show that the approximate expression for the input resistance of the darlington configuration above is;

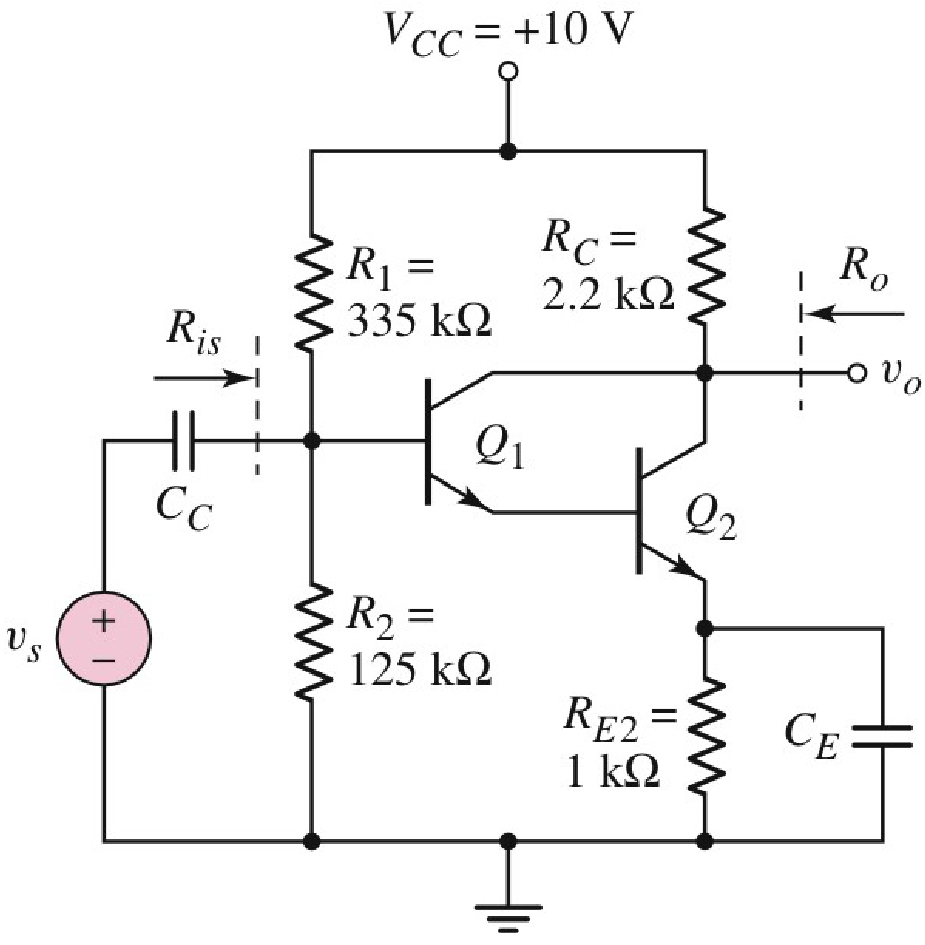
$$R_i \cong 2\beta_1 r_{\pi 2}$$

Hints: use the relationships:

$$r_\pi = \frac{\beta V_T}{I_{CQ}} \ \& \ I_{CQ1} \cong \frac{I_{CQ2}}{\beta_2}$$

$$\Rightarrow r_{\pi 1} = \beta_1 r_{\pi 2}$$

# Example 4




$$\beta_1 = \beta_2 = 100$$

$$\text{Assume } V_{A1} = V_{A2} = \infty$$

$$V_{BE1} = V_{BE2} = 0.7 \text{ V}$$

Determine the;

(a) Q-point for $Q_1$ and $Q_2$;

(b) voltage gain $v_o/v_s$;

(c) input resistance $R_{is}$;

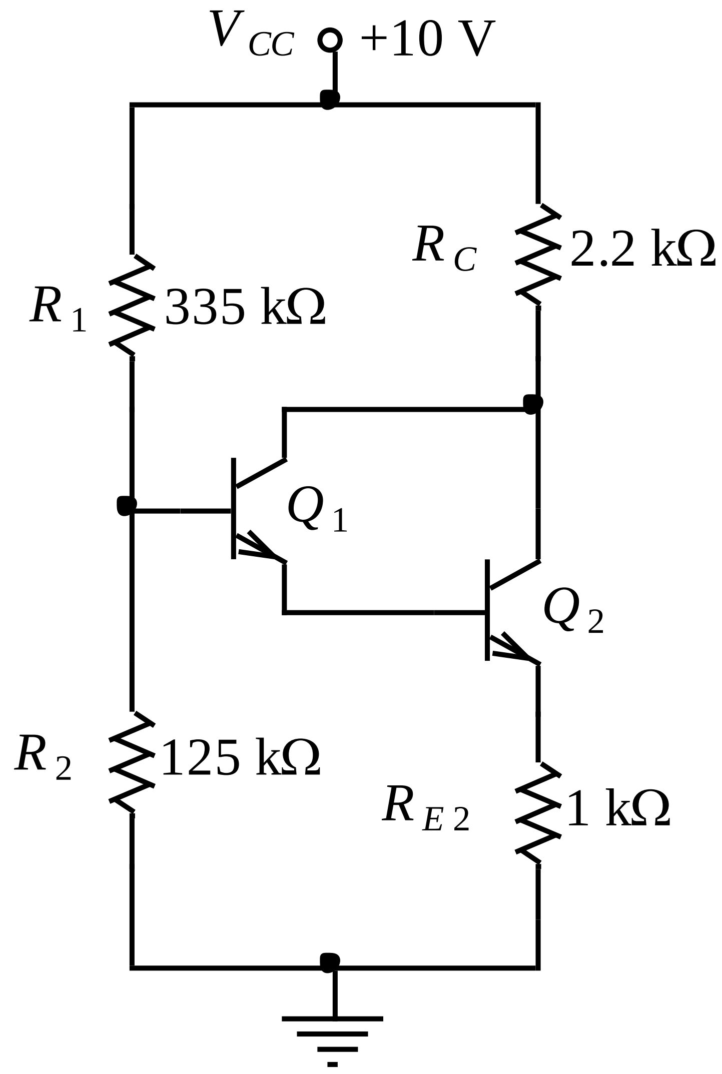
(d) output resistance $R_o$

## (a) Determination of Q-points

Using Thevenin's theorem;

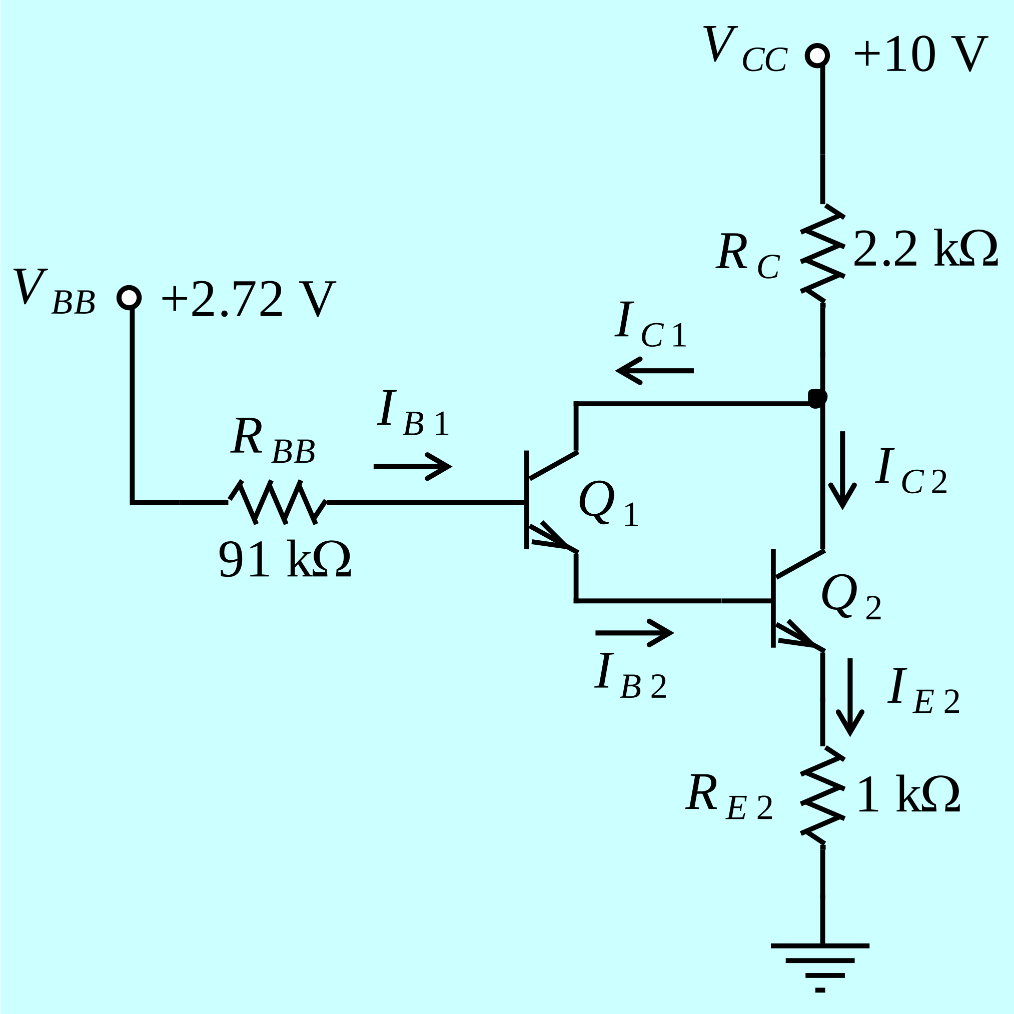
$$V_{BB} = V_{CC}\left(\frac{R_2}{R_1 + R_2}\right) = 2.72\text{ V}$$

$$R_{BB} = \frac{R_1 R_2}{R_1 + R_2} = 91\text{ k}\Omega$$




DC equivalent circuit

The circuit becomes;




$$\beta_1 = \beta_2 = 100$$
$$V_{A1} = V_{A2} = \infty$$
$$V_{BE1} = V_{BE2} = 0.7\text{ V}$$

$$I_{B2} = I_{E1} = (\beta + 1)I_{B1}$$

$$I_{E2} = (\beta + 1)I_{B2} = (\beta + 1)^2 I_{B1}$$

$$R_{BB} I_{B1} + 2V_{BE} + R_{E2}(\beta + 1)^2 I_{B1} = V_{BB}$$

Substituting values;

$$91\text{k} I_{B1} + 2 \times 0.7 + 1\text{k}(100 + 1)^2 I_{B1} = 2.72$$

$$I_{B1} = \frac{1.32}{10292} \times 10^{-3} = 0.128\ \mu\text{A}$$

$$I_{C1} = 12.8\ \mu\text{A} \qquad I_{E1} = I_{B2} = 12.93\ \mu\text{A}$$

$$I_{C2} = 1.293\ \text{mA} \qquad I_{E2} = 1.3\ \text{mA}$$

$$V_{E2} = 1.3 \times 1 = 1.3\ \text{V} \qquad V_{E1} = 1.3 + 0.7 = 2\ \text{V}$$

$$V_{C1} = V_{C2} = 10 - 2.2(1.293 + 0.0128) = 7.127\ \text{V}$$

$$V_{CE1} = V_{C1} - V_{E1} = 7.127 - 2 = 5.127 \text{ V}$$

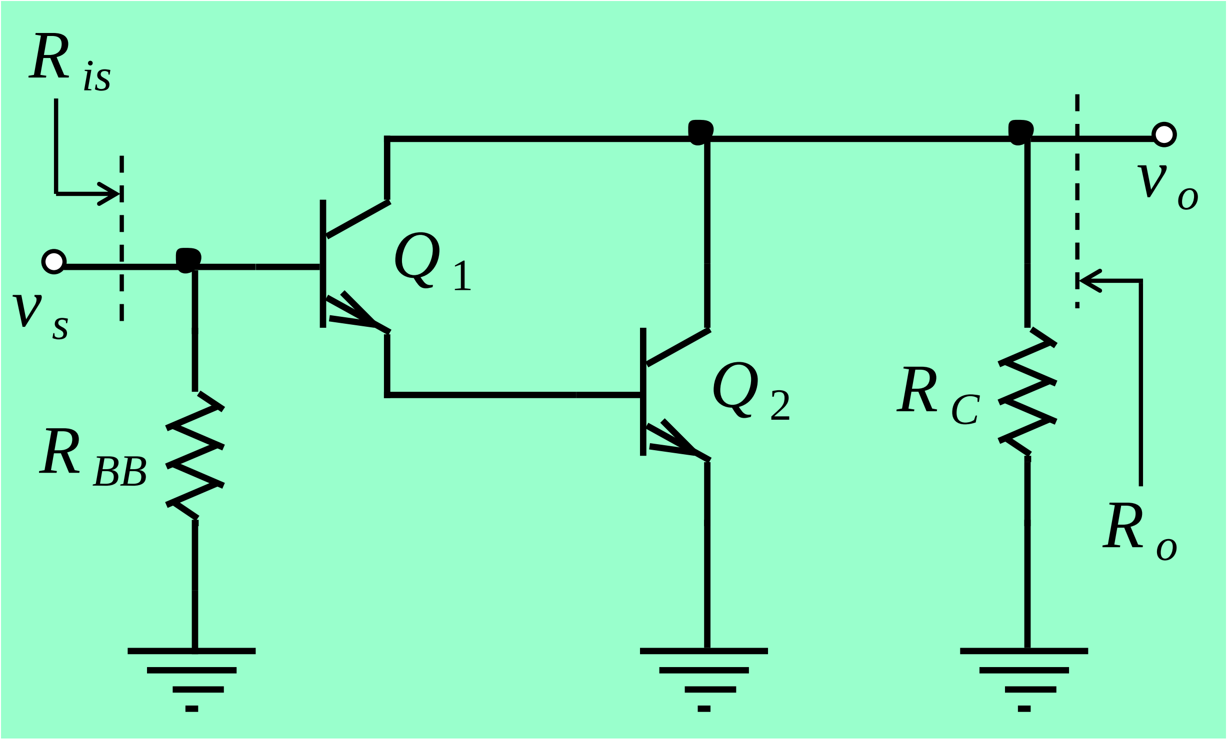
$$V_{CE2} = V_{C2} - V_{E2} = 7.127 - 1.3 = 5.827 \text{ V}$$

(a) The Q-points are;

$$Q_1 : \quad I_{CQ1} = 12.8\,\mu\text{A};\ V_{CEQ1} = 5.127 \text{ V}$$

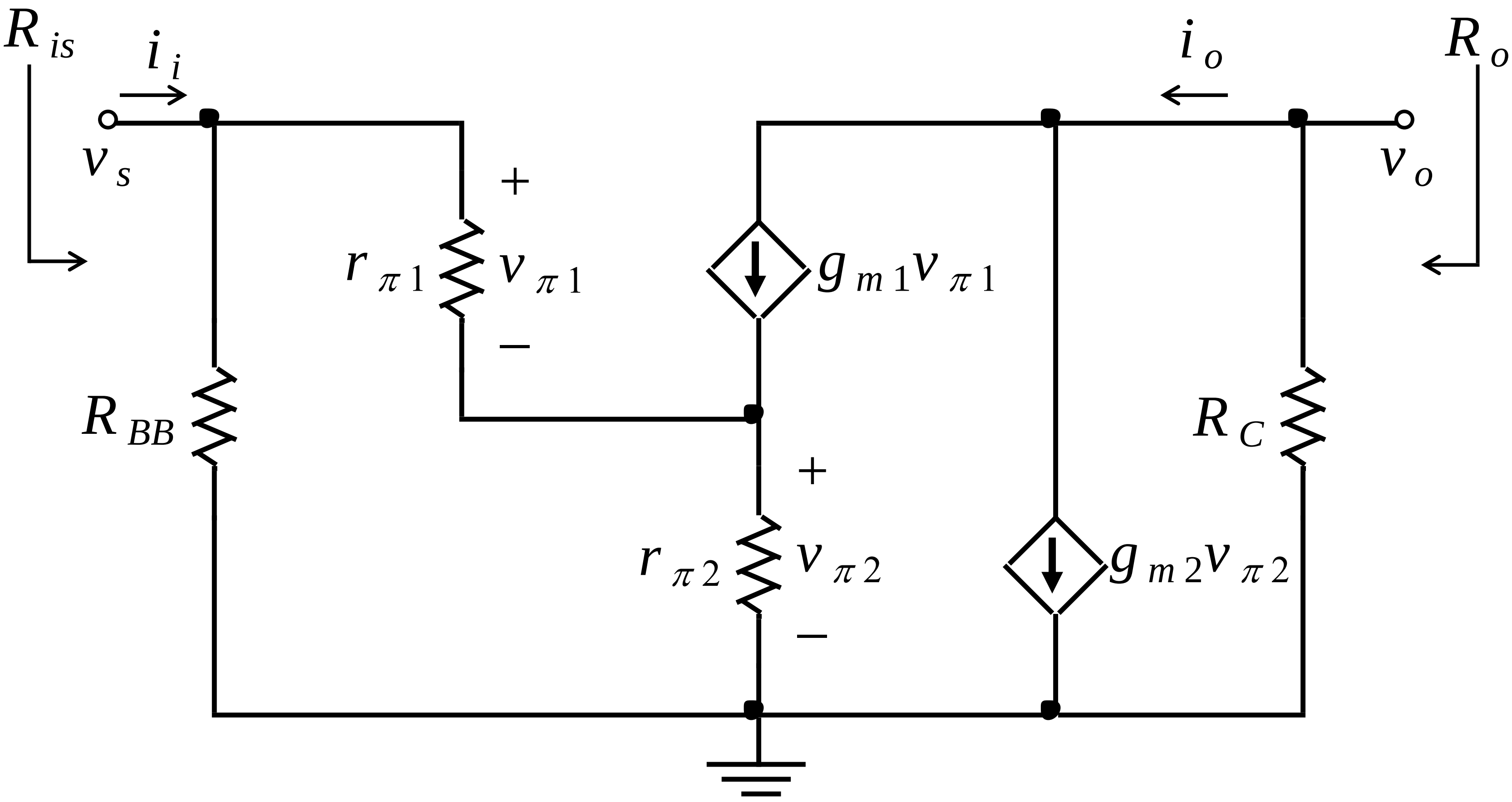
$$Q_2 : \quad I_{CQ2} = 1.293\,\text{mA};\ V_{CEQ2} = 5.827 \text{ V}$$

(b) The small-signal voltage gain (mid-band);




The equivalent circuit under AC condition

Using the hybrid-$\pi$ model of transistor, the equivalent circuit becomes;



$$g_{m1} = \frac{I_{CQ1}}{V_T} = \frac{12.8\ \mu\text{A}}{26\ \text{mV}} = 0.492\ \text{mA/V}$$

$$g_{m2} = \frac{I_{CQ2}}{V_T} = \frac{1.293\ \text{mA}}{26\ \text{mV}} = 49.73\ \text{mA/V}$$

$$r_{\pi 1} = \frac{\beta_1}{g_{m1}} = \frac{100}{0.492 \times 10^{-3}} = 203.25\ \text{k}\Omega$$

$$r_{\pi 2} = \frac{\beta_2}{g_{m2}} = \frac{100}{49.73 \times 10^{-3}} = 2\ \text{k}\Omega$$

$$v_o = -\left(g_{m1}V_{\pi 1} + g_{m2}V_{\pi 2}\right)R_C$$

$$V_{\pi 2} = \left(\frac{V_{\pi 1}}{r_{\pi 1}} + g_{m1}V_{\pi 1}\right)r_{\pi 2} = \left(\frac{r_{\pi 2}}{r_{\pi 1}} + g_{m1}r_{\pi 2}\right)V_{\pi 1}$$

$$V_{\pi 2} = \left(1+\beta_1\right)\frac{r_{\pi 2}}{r_{\pi 1}}V_{\pi 1}$$

Substituting for $V_{\pi 2}$ in the expression for $v_o$ and simplifying;

$$v_o = -\left[\frac{\beta_1 + \left(1+\beta_1\right)\beta_2}{r_{\pi 1}}\right]R_C V_{\pi 1}$$

$$v_s = V_{\pi 1} + V_{\pi 2}$$

$$V_{\pi 2} = (1+\beta_1)\frac{r_{\pi 2}}{r_{\pi 1}} V_{\pi 1}$$

$$v_o = -\left[\frac{\beta_1 + (1+\beta_1)\beta_2}{r_{\pi 1}}\right] R_C V_{\pi 1}$$

Substituting for $V_{\pi 2}$;

$$v_s = V_{\pi 1} + (1+\beta_1)\frac{r_{\pi 2}}{r_{\pi 1}} V_{\pi 1}$$

$$A_v = \frac{v_o}{v_s} = \frac{-\left[\frac{\beta_1 + (1+\beta_1)\beta_2}{r_{\pi 1}}\right] R_C V_{\pi 1}}{V_{\pi 1} + (1+\beta_1)\frac{r_{\pi 2}}{r_{\pi 1}} V_{\pi 1}}$$

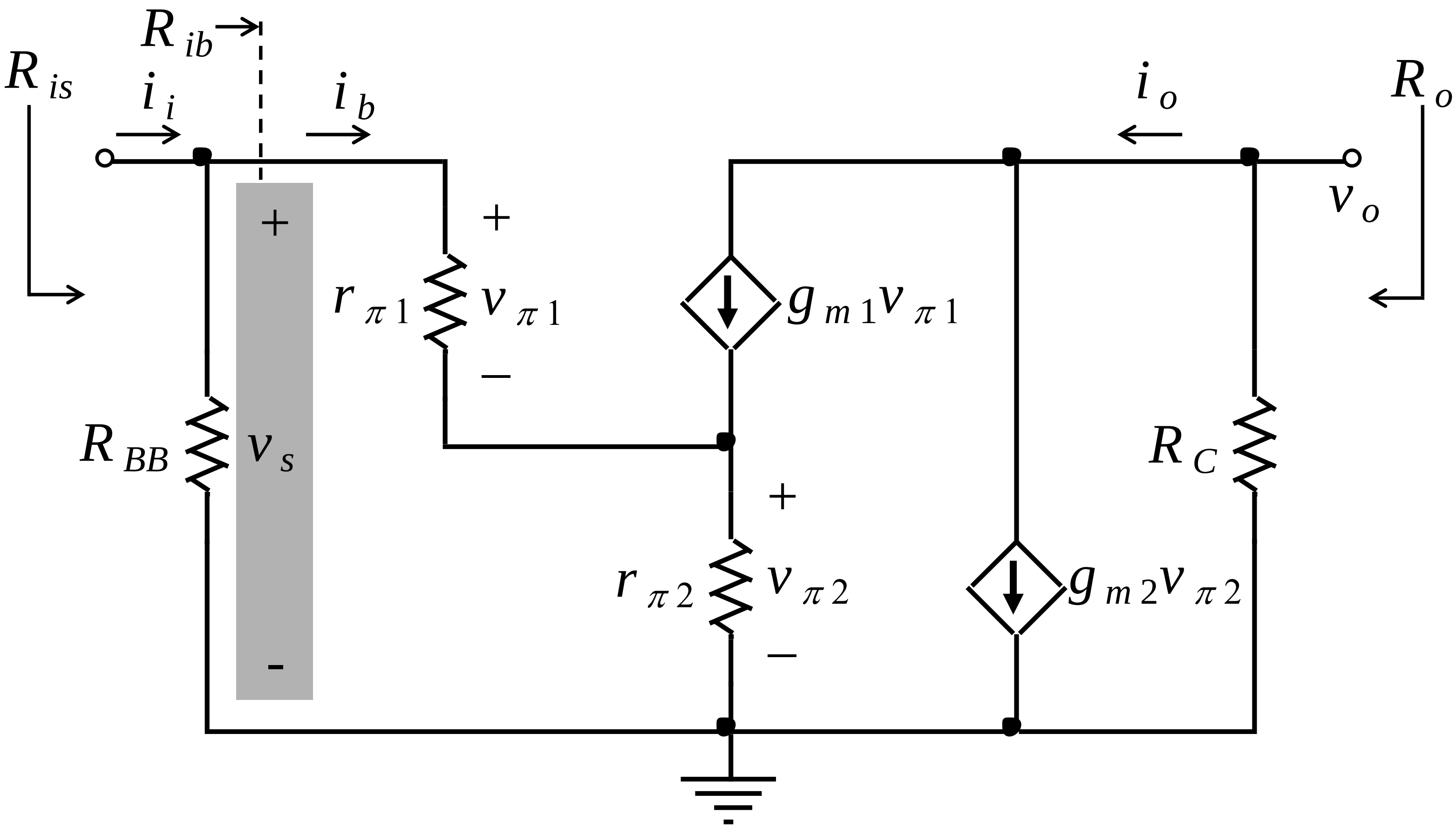
Simplifying;

$$A_v = \frac{v_o}{v_s} = -\frac{[\beta_1 + (1+\beta_1)\beta_2] R_C}{r_{\pi 1} + (1+\beta_1) r_{\pi 2}}$$

Substituting values;

$$A_v = -\frac{\left[100 + 100 + 100^2\right]2.2}{\left(203.25 + 2 + 100 \times 2\right)}$$

$$A_v = -55.4$$




$$R_{is} = R_{BB} // R_{ib}$$

$$R_{ib} = \frac{v_s}{i_b}$$

$$v_s = V_{\pi 1} + V_{\pi 2}$$

$$V_{\pi 1} = i_b r_{\pi 1}$$

$$V_{\pi 2} = \left(i_b + g_{m1} V_{\pi 1}\right) r_{\pi 2} = \left(i_b + g_{m1} r_{\pi 1} i_b\right) r_{\pi 2}$$
$$= \left(1 + \beta_1\right) r_{\pi 2} i_b$$

$$v_s = r_{\pi 1} i_b + \left(1 + \beta_1\right) r_{\pi 2} i_b$$

$$R_{ib} = \frac{v_s}{i_b} = r_{\pi 1} + \left(1 + \beta_1\right) r_{\pi 2}$$

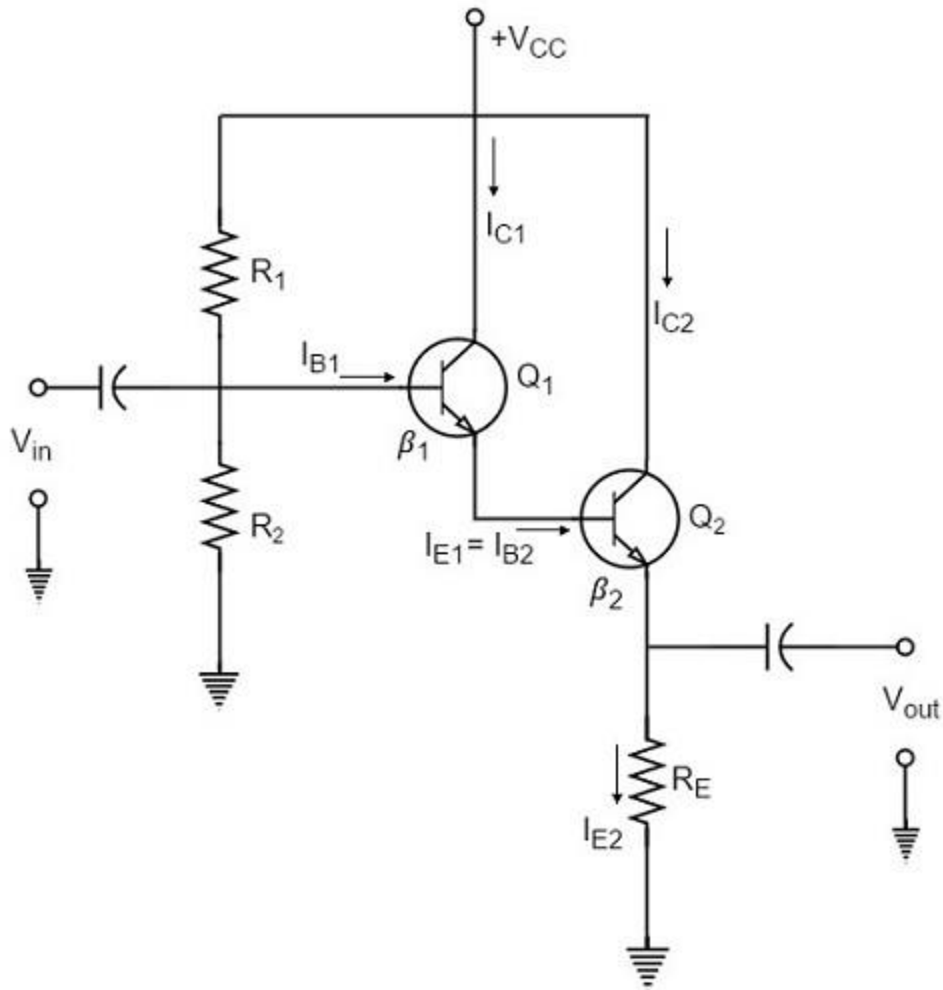
Substituting values;

$$R_{ib} = 203 + \left(1 + 100\right) 2 = 405\ \mathrm{k\Omega}$$

$$R_{is} = R_{BB} // R_{ib} = \frac{91 \times 405}{91 + 405}$$

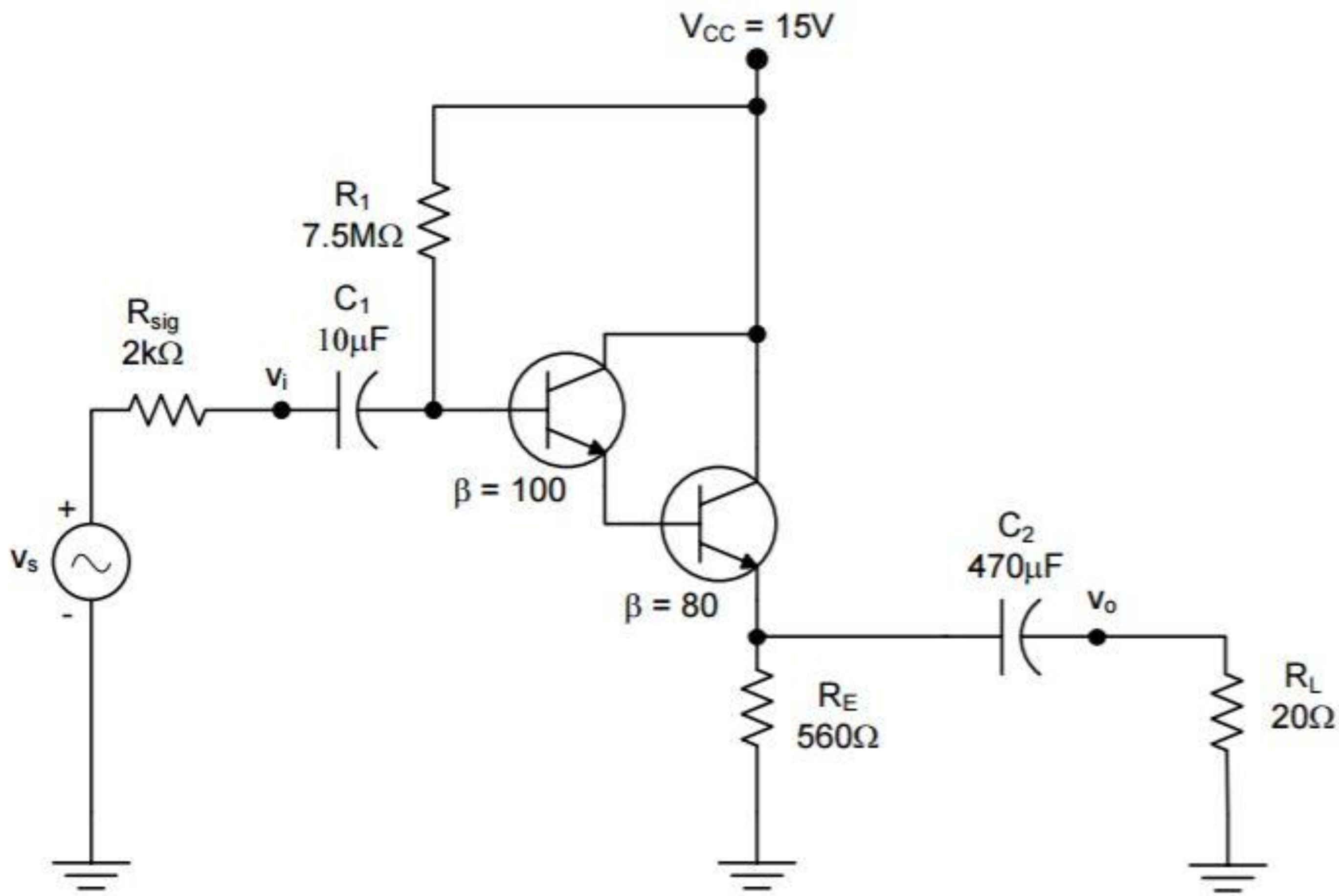
$$R_{is} = 73.6\,\text{k}\Omega$$

$$R_o = R_C = 2.2\,\text{k}\Omega$$

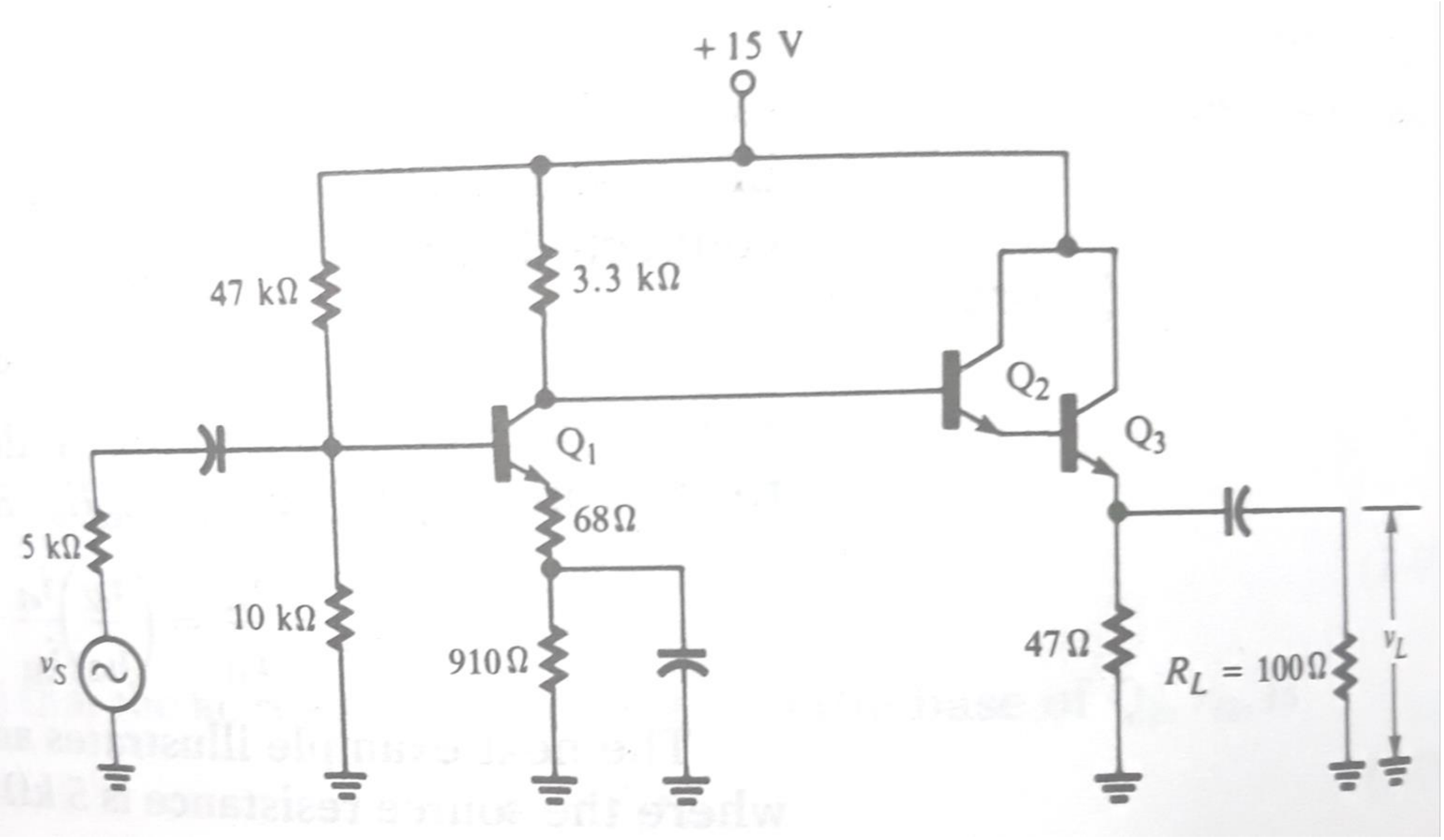


+VCC
IC1
IC2
R1
IB1
Q1
Vin
β1
R2
Q2
IE1= IB2
β2
Vout
RE
IE2

3. Consider the Darlington emitter follower pictured below.




a) Determine the input impedance, output impedance, the loaded voltage gain, and the current gain for the amplifier circuit.
b) Find the approximate lower cutoff frequency for the amplifier.




Bogart

# High Input Resistance circuits:

The ideal voltage amplifier should have infinite input impedance and zero output impedance.

The CC and CE with $R_E$ basic amplifiers have these properties.

- The Input impedance of these amplifiers is
- $Ri = hie+(1+hfe)R_E$ using the simplified model (assuming that $hoeR_E << 0.1$)
- As $R_E$ increases this equation suggests that Ri increases more

However, as $R_E \rightarrow \propto$ the assumption $hoeR_E >>1$ is no longer valid. And the more accurate equation is

$Ri = hie + (1+hfe)R_E / (1+hoeR_E)$ which $Ri \rightarrow hfe/hoe$ $\rightarrow$**theoretical limitation on Ri.**

**There are other practical limitations also.**

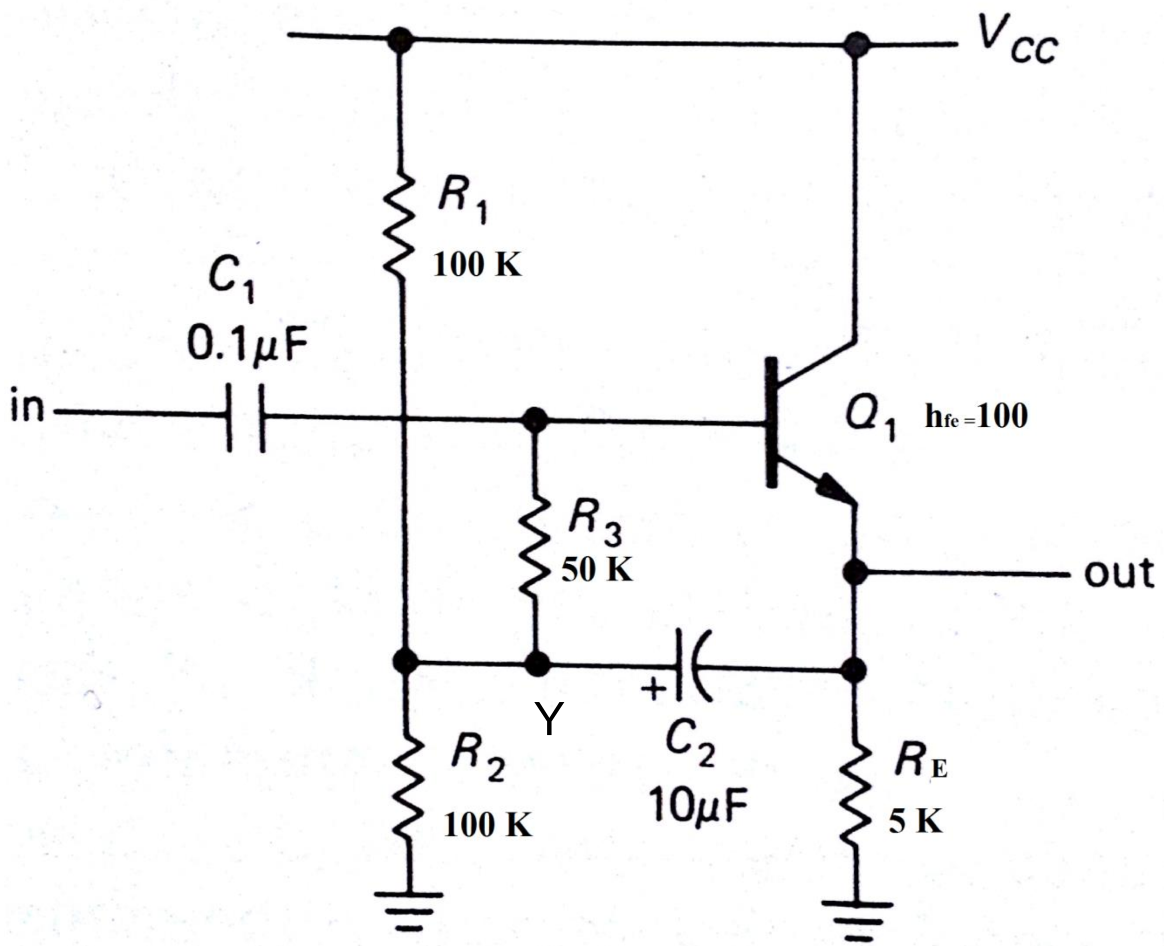
1. As $R_E$ increases the bias current causes a larger voltage drop across it. For middle of operating range $V_{CE} = V_{RE} = VCC/2$. We thus require larger power supply voltage
2. In integrated circuits $R_E$ occupies chip area. Larger the value, greater is the chip area occupied, leaving less for other components.
3. Bias resistance appear in parallel with the Ri and with typical values of a few 100K the parallel combination is now decided by the bias resistance, and is hence lower.

**Solutions:**

The DARLINGTON PAIR circuit increases the input resistance but biasing of $2^{nd}$ stage is difficult and should be a power transistor .

The bias resistance problem (3) may be solved by the **BOOTSTRAPPING technique.**

**Cancelling the effect of the bias resistance by Bootstrapping:**



- Here $R_1$ and $R_2$ are the bias resistances.
- Base bias through R3. $C_2$ short-circuits the output to point Y at the junction of the bias resistance.
- Thus signal voltage at Y = $V_{out}$ and at Base signal voltage is $V_{in}$.
- Then $i_{R3} = (Vin - Vout) / R_3$
- So effective resistance of the combination seen by the source is
  $R_{eff} = V_{in} / i_{R3} = V_{in} R_3 / (V_{in} - V_{out}) = R_3 / (1 – Av)$ where Av = voltage gain $V_{out}/V_{in}$

for the CC and Darlington circuits, Av< ≈1, 1 – Av and is a very small number. This means effective resistance of the bias combination is increased greatly.

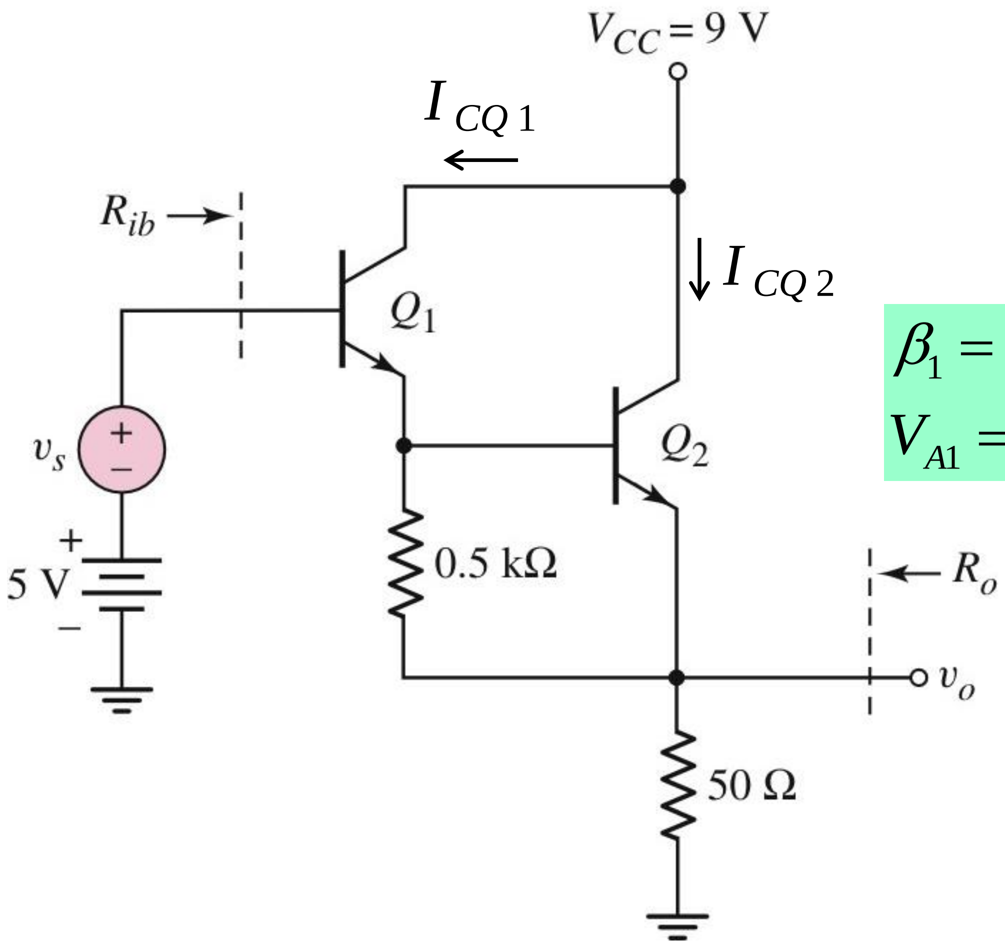
If Av is 0.95 and R3 is 50K , R1 = 100K and R2 = 100K then without bootstrapping, bias combination would give

$R_i$= (hie + $A_{I2}R_{i2}$) || (R1||R2) =$(1+hfe)^2$ $R_E$/(1+hoehfe$R_E$) || 50 K = 50 M ||50K=50K

$$(1+100)^2 \times 5K / (1+25\times10^{-6} \times100\times5K) \cong 50M$$

With Bootstrapping, the resistance is 50K / (1 – 0.95) = 50K *20 = 1000K = 1M.

Find; (a) $I_{CQ1}$ and $I_{CQ2}$ (b) $A_v = v_o/v_s$ (c) $R_{ib}$ and $R_o$




$$\beta_1 = \beta_2 = 100$$
$$V_{A1} = V_{A2} = \infty$$

Answers:
(a) 2.08 mA & 69.9 mA
(b) 0.99 V/V
(c) 480 kΩ & 0.469 Ω

Thus, the ratio of the transformer input and output resistances varies directly as the square of the transformer turn ratio:

$$\frac{R_{Lp}}{R_L} = \left(\frac{Np}{Ns}\right)^2 = n^2$$

Giving us the equation finding the reflected impedance,

$$R_{Lp} = \left(n^2\right) \times R_L$$

where

- n is the ratio of primary to secondary turns of the step-down transformer
- $R_{Lp}$ is the reflected impedance in the primary

**Advantages of Transformer Coupled Amplifier**

- The following are the advantages of a transformer coupled amplifier −
- An excellent impedance matching is provided.
- Gain achieved is higher.
- There will be no power loss in collector and base resistors.
- Efficient in operation.

**Disadvantages of Transformer Coupled Amplifier**

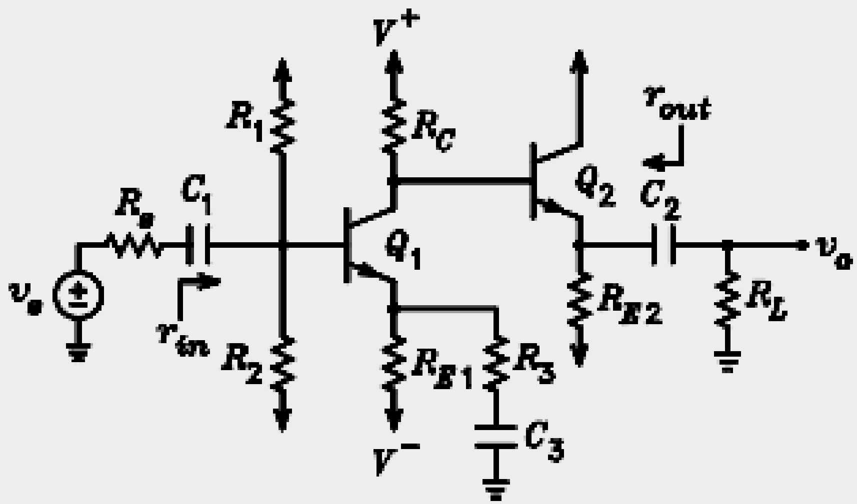
- The following are the disadvantages of a transformer coupled amplifier −
- Though the gain is high, it varies considerably with frequency. Hence a poor frequency response.
- Frequency distortion is higher.
- Transformers tend to produce hum noise.
- Transformers are bulky and costly.

**Applications**

✓The following are the applications of a transformer coupled amplifier −
✓Mostly used for impedance matching purposes.
✓Used for Power amplification.
✓Used in applications where maximum power transfer is needed.

# CE - CC Amplifier Example

For the circuits in the figure, it is given that $V^+ = 10\,\text{V}$, $V^- = -10\,\text{V}$, $R_s = 5\,\text{k}\Omega$, $R_1 = 100\,\text{k}\Omega$, $R_2 = 120\,\text{k}\Omega$, $R_{E1} = 2\,\text{k}\Omega$, $R_3 = 51\,\Omega$, $R_C = 2.4\,\text{k}\Omega$, $R_{E2} = 2\,\text{k}\Omega$, $R_L = 1\,\text{k}\Omega$, $V_{BE} = 0.65\,\text{V}$, $V_T = 0.025\,\text{V}$, $\alpha = 0.99$, $\beta = 99$, $r_x = 20\,\Omega$, and $r_0 = 50\,\text{k}\Omega$. The capacitors are ac short circuits and dc open circuits.




## DC Solution

The dc solution for $Q_1$ is the same as for the CE amplifier and is repeated. To solve for $I_{E1}$, replace the capacitors with open circuits. Look out the base and form a Thévenin equivalent circuit. We have

$$\begin{aligned}
V_{BB1} &= \frac{V^+R_2 + V^-R_1}{R_1 + R_2} = 10\frac{120\,\text{k}\Omega}{100\,\text{k}\Omega + 120\,\text{k}\Omega} - 10\frac{100\,\text{k}\Omega}{100\,\text{k}\Omega + 120\,\text{k}\Omega} = \frac{10}{11} \\
R_{BB1} &= R_1 \| R_2 = 100\,\text{k}\Omega \| 120\,\text{k}\Omega = 54.55\,\text{k}\Omega \\
V_{EE1} &= V^- = -10
\end{aligned}$$

$$R_{EE1} = R_E = 2\,\mathrm{k\Omega}$$

The emitter current in $Q_1$ is given by

$$I_{E1} = \frac{V_{BB1} - V_{BE1} - V_{EE1}}{R_{BB1}/(1+\beta) + R_{EE1}} = \frac{10/11 - 0.65 - (-10)}{54.55\,\mathrm{k\Omega}/(1+99) + 2\,\mathrm{k\Omega}} = 4.031\,\mathrm{mA}$$

The ac emitter intrinsic resistance of $Q_1$ is

$$r_{e1} = \frac{V_T}{I_{E1}} = \frac{25\,\mathrm{mV}}{4.031\,\mathrm{mA}} = 6.202\,\Omega$$

Look out of the base and emitter of $Q_2$ and form Thévenin equivalent circuits. We have

$$\begin{aligned}
V_{BB2} &= V^+ - \alpha I_{E1} R_{C1} = 10 - 0.99 \times 4.031\,\mathrm{mA} \times 2.4\,\mathrm{k\Omega} = 0.4223\,\mathrm{V} \\
R_{BB2} &= 2.4\,\mathrm{k\Omega} \\
V_{EE2} &= V^- = -10\,\mathrm{V} \\
R_{EE2} &= R_{E2} = 2\,\mathrm{k\Omega}
\end{aligned}$$

The emitter current in $Q_2$ is given by

$$I_{E2} = \frac{V_{BB2} - V_{BE2} - V_{EE2}}{R_{BB2}/(1+\beta) + R_{EE2}} = \frac{0.4223 - 0.65 - (-10)}{2.4\,\mathrm{k\Omega}/(1+99) + 2\,\mathrm{k\Omega}} = 4.828\,\mathrm{mA}$$

The ac emitter intrinsic resistance of $Q_2$ is

$$r_{e2} = \frac{V_T}{I_{E2}} = \frac{25\,\mathrm{mV}}{4.828\,\mathrm{mA}} = 5.178\,\Omega$$

**AC Solution - Method 1**

Zero the dc supplies and short the capacitors. Look out the base of $Q_1$ and make a Thévenin equivalent circuit. We have

$$\begin{aligned} v_{tb1} &= v_s \frac{R_1 \| R_2}{R_s + R_1 \| R_2} = v_s \frac{100\,\mathrm{k}\Omega \| 120\,\mathrm{k}\Omega}{5\,\mathrm{k}\Omega + 100\,\mathrm{k}\Omega \| 120\,\mathrm{k}\Omega} = \frac{v_s}{1.092} = 0.9160 v_s \\ R_{tb1} &= R_s \| R_1 \| R_2 = 5\,\mathrm{k}\Omega \| 100\,\mathrm{k}\Omega \| 120\,\mathrm{k}\Omega = 4.580\,\mathrm{k}\Omega \end{aligned}$$

The Thévenin equivalent circuit looking into the $i'_{e1}$ branch is $v_{tb1}$ in series with $r'_{e1}$, where

$$r'_{e1} = \frac{R_{tb1} + r_{x1}}{1 + \beta_1} + r_{e1} = \frac{4.580\,\mathrm{k}\Omega + 20}{1 + 99} + 6.202 = 52.20\,\Omega$$

The resistance looking out of the emitter of $Q_1$ is

$$R_{te1} = R_E \| R_3 = 2\,\mathrm{k}\Omega \| 51 = 49.73\,\Omega$$

The resistance looking into the collector of $Q_1$ is

$$r_{ic1} = \frac{r_{01} + r'_{e1} \| R_{te1}}{1 - \alpha_1 R_{te1} / \left(r'_{e1} + R_{te1}\right)} = \frac{50\,\mathrm{k}\Omega + 52.20 \| 49.73}{1 - 0.99 \times 49.73 / \left(49.73 + 52.20\right)} = 97.76\,\mathrm{k}\Omega$$

The short circuit collector output current from $Q_1$ is

$$\begin{aligned} i_{c1(sc)} &= G_{mb1} v_{tb1} = \frac{\alpha_1}{r'_{e1} + R_{te1}} \frac{r_{01} - R_{te1}/\beta_1}{r_{01} + r'_{e1} \| R_{te1}} v_{tb1} \\ &= \frac{0.99}{52.20 + 49.73} \frac{50\,\mathrm{k}\Omega - 49.73/99}{50\,\mathrm{k}\Omega + 52.20 \| 49.73} v_{tb1} = \frac{v_{tb1}}{103.0} = \frac{v_s}{112.4} \end{aligned}$$

Replace $Q_2$ with its simplified T model. Looking into the $r'_{e2}$ branch, we see $v_{tb2}$ in series with $r'_{e2}$ given by

$$r'_{e2} = \frac{R_{tb2} + r_{x2}}{1 + \beta_2} + r_{e2} = \frac{2.342\,\mathrm{k}\Omega + 20}{1 + 99} + 5.178 = 28.80$$

The resistance seen looking out of the emitter of $Q_2$ is

$$R_{te2} = R_{E2} \| R_L = 666.7\,\Omega$$

By voltage division, $v_o$ is given by

$$v_o = v_{tb2} \frac{r_{02} \| R_{te2}}{r'_{e2} + r_{02} \| R_{te2}} = -20.84 v_s \frac{50\,\mathrm{k}\Omega \| 666.7}{28.80 + 50\,\mathrm{k}\Omega \| 666.7} = -19.97 v_s$$

Thus the voltage gain is

$$\frac{v_o}{v_s} = -19.97$$

The output resistance is

$$r_{\text{out}} = R_{E2} \| r_{02} \| r'_{e2} = 2\,\text{k}\Omega \| 50\,\text{k}\Omega \| 28.80 = 28.38\ \Omega$$

To solve for the input resistance, we need $r_{ib1}$. To calculate this, we need $R_{tc1}$, which requires us to know $r_{ib2}$. For the latter, we have

$$\begin{aligned} r_{ib2} &= r_{x2} + (1+\beta_2)\, r_{e2} + R_{te2} \frac{(1+\beta_2)\, r_{02} + R_{tc2}}{r_{02} + R_{te2} + R_{tc2}} \\ &= 20 + (1+99)\, 5.178 + 666.7 \frac{(1+99)\, 50\,\text{k}\Omega}{50\,\text{k}\Omega + 666.7} \\ &= 66.33\,\text{k}\Omega \end{aligned}$$

Thus the resistance seen looking out of the collector of $Q_1$ is

$$R_{tc1} = R_C \| r_{ib2} = 2.4\,\text{k}\Omega \| 66.33\,\text{k}\Omega = 2.316\,\text{k}\Omega$$

The resistance looking into the base of $Q_1$ is

$$\begin{aligned} r_{ib1} &= r_{x1} + (1+\beta_1)\, r_{e1} + R_{te1} \frac{(1+\beta_1)\, r_{01} + R_{tc1}}{r_{01} + R_{te1} + R_{tc1}} \\ &= 5.391\,\text{k}\Omega \end{aligned}$$

The input resistance is

$$r_{in} = R_1 \| R_2 \| r_{ib1} = 100\,\text{k}\Omega \| 120\,\text{k}\Omega \| 5.613\,\text{k}\Omega = 5.089\,\text{k}\Omega$$

If $Q_2$ and $R_{E2}$ are omitted from the circuit and the left node of $C_2$ is connected to the collector of $Q_1$, we have a common-emitter amplifier. In this case, the output voltage is

$$v_o = -i_{c1(sc)} R_C \| r_{ic1} \| R_L = \frac{-v_s}{112.4} \times 2.4\,\text{k}\Omega \| 97.76\,\text{k}\Omega \| 1\,\text{k}\Omega = -6.235 v_o$$

This is lower than with the CC stage by a factor of 3.25 or by 10.2 dB. This illustrates how a stage that has a gain less than unity can increase the gain of a circuit when it is used to drive the load resistor.

## AC Solution - Method 2

For this solution, we use the $r_0$ approximations for $Q_1$. That is, we neglect the current through $r_{01}$ in calculating $i_{c1(\mathrm{sc})}$ but not in calculating $r_{ic1}$. The short circuit collector output current of $Q_1$ is

$$i_{c1(sc)} = G_{m1} v_{tb1} = \frac{\alpha}{r'_{e1} + R_{te1}} v_{tb1} = \frac{0.99 v_{tb1}}{52.20 + 49.73} = \frac{v_{tb1}}{103.0} = \frac{v_{s1}}{111.3}$$

Look out of the base of $Q_2$ and make a Thévenin equivalent circuit. We have

$$\begin{aligned} v_{tb2} &= -i_{c1(sc)} R_C \| r_{ic1} = \frac{-v_s}{111.3} \times 2.4\,\mathrm{k\Omega} \| 97.76\,\mathrm{k\Omega} = -21.05 v_s \\ R_{tb2} &= R_C \| r_{ic1} = 2.4\,\mathrm{k\Omega} \| 97.76\,\mathrm{k\Omega} = 2.342\,\mathrm{k\Omega} \end{aligned}$$

Replace $Q_2$ with its simplified T model. Looking into the $r'_{e2}$ branch, we see $v_{tb2}$ in series with $r'_{e2}$ given by

$$r'_{e2} = \frac{R_{tb2} + r_{x2}}{1 + \beta_2} + r_{e2} = \frac{2.342\,\mathrm{k\Omega} + 20}{1 + 99} + 5.178 = 28.80$$

By voltage division, $v_o$ is given by

$$v_o = v_{tb2} \frac{r_{02} \| R_{te2}}{r'_{e2} + r_{02} \| R_{te2}} = -21.05 v_s \frac{50\,\mathrm{k\Omega} \| 666.7}{28.80 + 50\,\mathrm{k\Omega} \| 666.7} = -20.17 v_s$$

Thus the voltage gain is

$$\frac{v_o}{v_s} = -20.17$$

This differs from the answer by Method 1 by 0.99%.

The solutions for $r_{\mathrm{out}}$ and $r_{\mathrm{in}}$ are the same as for Method 1.