

Abstract

Title: Best coapproximation in Banach spaces: A Birkhoff-James orthogonality approach


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In this dissertation, we explore the best coapproximation problem, a notion complementary to the classical best approximation problem, within the framework of Banach space using the notion of Birkhoff-James orthogonality. We study the problem in the space of all $n \times n$ diagonal matrices from a computational perspective. We completely characterize the best coapproximation(s) to a given matrix T out of a given subspace \mathbb{Y} of the space of diagonal matrices \mathcal{D}_n . As a consequence of our study, we solve the problem computationally in the space ℓ_∞^n . We present a tractable approach to solve the best coapproximation problem in the space ℓ_1^n , leading to a complete characterization of coproximal and co-Chebyshev subspaces. We next introduce two types of subspaces which may be regarded as the least favorable situations from the existence of best coapproximations, named as anti-coproximal subspaces and strongly anti-coproximal subspaces. We obtain some necessary and sufficient conditions for strongly anti-coproximal subspaces in general Banach spaces, separately. In particular, we show that strictly convex and smooth Banach spaces does not contain any strongly anti-coproximal subspaces, and we provide a characterization of the anti-coproximal subspaces in smooth Banach spaces. The geometry of such subspaces is further explored in finite-dimensional real polyhedral Banach spaces, revealing intriguing structural properties. Extending beyond finite dimensions, we analyze the problem in the space of scalar-valued continuous functions, where anti-coproximal and strongly anti-coproximal subspaces coincide, and offer a full characterization. We also examine the stability of these notions in the setting of vector-valued continuous functions. Finally, we explore these subspaces in the space of all bounded linear operators, further broadening the scope of our study.


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