Abstract

Title: A STUDY ON STRUCTURE SPACES OF SEMIRINGS, Γ-SEMIRINGS AND RELATED SEMIMODULES

Submitted by: Soumi Basu; Index No. 116/19/Maths./26

The present work is a study on the structure spaces of semirings, semimodules, Γ-semirings and ΓS-semimodules showing a nice interplay between the algebraic structures and the topological structures. At first, different kinds of congruences on a Γ-semiring, viz. cancellative congruence, left and right regular congruence, maximal congruence and prime congruence have been defined and several results on those congruences have been studied. Then in order to study a Γ-semiring via its operator semirings, one-to-one correpondences between the sets of different types congruences on a Γ-semiring with those on its operator semirings have been established. Next, the structure space of semirings, consisting of all prime congruences on a semiring has been defined and its topological properties have been studied. Likewise the structure space of Γ-semirings, topologizing the set of all prime congruences on a commutative Γ-semiring with strong unities equipped with the Hull Kernel topology, has been defined and investigation of the topological properties of that space via the left operator semiring has been done establishing the topological isomorphism between the structure space of a Γ-semiring and that of its left operator semiring. Next, it has been acknowledged that the set $C_-(X)$ of all non-positive valued continuous functions over a topological space X falls into the algebraic structure viz. Γ-semiring with pointwise addition and multiplication taking $\Gamma = C_-(X)$ and not only that it is found that the left operator semiring of $C_-(X)$ is isomorphic to the semiring $C_+(X)$ of all non-negative valued continuous functions over a topological space X. Then many results have been proved on the congruences (such as maximal regular, prime congruences) and ideals (such as z-ideals, z°-ideals) of the Γ-semiring $C_-(X)$ via the connection with those of the semiring $C_+(X)$. It has been established that the structure space and the real structure space of $C_-(X)$ are models of Stone-Čech compactification and Hewitt Realcompactification of a Tychonoff space X respectively. Also the Γ-semiring analogues of the 'Banach-Stone Theorem' and 'Hewitt Isomorphism Theorem' are obtained. In addition some results on z-ideals, z°-ideals of $C_-(X)$, characterizing the topological space X, have been studied. After that the notion

of prime ΓS-subsemimodules of an unitary ΓS-semimodule has been introduced and the structure space of a ΓS-semimodule is defined. Then a correspondence between the set of all prime ΓS-subsemimodules of a ΓS-semimodule and the set of all prime subsemimodules of the associated L-semimodule is obtained, where S is a Γ-semiring and L is its left operator semiring. After that several properties of the prime ΓS-subsemimodules over multiplication ΓS-semimodules, finitely generated ΓS-semimodules, k-finitely generated ΓS-semimodules, are studied via the associated L-semimodules. Thereafter the Hull Kernel topology is defined on the prime subsemimodules of a multiplication semimodule as well as on the set of all prime ΓS-subsemimodules of a multiplication ΓS-semimodule and those spaces have been named as the structure spaces of semimodules and ΓS-semimodules respectively. Then the topological properties of the structure space of semimodules have been examined and applying those the structure space of ΓS-semimodules has been studied via the homeomorphism of that space with the space of prime L-subsemimodules of the associated L-semimodule.

Soumi Baon 03.09.24

Candidate

03.09.24

Supervisor

Professor
DEPARTMENT OF MATHEMATICS
Jadavpur University
Kolkata - 700 032, West Bengal

Sarbani Mukherjee (Goswami)
03/09/2024

Co-superviser

SARBANI MUKHERJEE (GOSWAMI)
Assistant Professor (W.B.E.S)
Department of Mathematics
Lady Brabourne College