(b) Let $f : R^n \to R$ be homogeneous of degree r and $g : R^n \to R$/be homogeneous of degree s. Then check the homogeneity of the functions $f + g, f \cdot g$ and $\frac{f}{g}$. 3

(c) Prove that $(9^n - 1)$ is divisible by 8 for every integer $n \geq 1$. 3

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ARTS/ECO/UG/SEC/11/101/2024

BACHELOR OF ARTS EXAMINATION, 2024

(First Year, First Semester)

ECONOMICS

(Mathematical Methods in Economics—1)

Time : *Two Hours* *Full Marks* : 30

Answer question number 1 and *any two* questions from the rest.

1. *(a)* State and explain whether the following statements are true or false : 5

(i) Let us consider the optimization problem :

Maximize $z = f(x)$ s.t. $g(x) = c$. The second order (sufficient) condition for identifying the maximum is $d^2 z < 0$.

(ii) An optimum solution will exist to the following problem :

Minimize $h(x_1, x_2, x_3)$ s.t. $g^1(x_1, x_2, x_3) = k_1$; $g^2(x_1, x_2, x_3) = k_2$; $g^3(x_1, x_2, x_3) = k_3$.

(iii) If (x_1^*, x_2^*, x_3^*) is the optimal solution to the problem. Maximize $f(x_1, x_2, x_3)$ and $(\overline{x_1}, \overline{x_2}, \overline{x_3})$ is the solution to the optimization problem :

Maximize $f(x_1, x_2, x_3)$, s.t. $h(x_1, x_2, x_3) = 115$, then $f(x_1^*, x_2^*, x_3^*) < f(\overline{x_1}, \overline{x_2}, \overline{x_3})$.

(iv) If a function is concave, then the domain of the function will always be a convex set.

(v) A linear function is a convex function but is not strictly convex.

(b) Given the function $f(x,y,z)=\left(\frac{x-y+z}{x+y-z}\right)^n$, use the properties of homogeneous function to prove : $x\frac{\partial f}{\partial x}+y\frac{\partial f}{\partial y}+z\frac{\partial f}{\partial z}=0$. 3

(c) Negate the following statements : 2

(i) $\forall x \exists y$, S.T. $x^2+y^2>1$

(ii) $\forall x \forall y,\ x^2+y^2>1$

(iii) $\exists x \forall y$, S.T. $x^2+y^2>1$

(iv) $\neg\exists x,\ \neg(x^2=1)$

(v) $\neg\exists x, (x^2=1)$

(vi) $\neg\forall x\ \neg(x^2=1)$

2. *(a)* Suppose $\exists f: A \to B$ such that *f* is bijective. Prove that $\exists f^{-1}$(inverse of f)$: B \to A$. 2

(b) Identify the extremum of the following function :

$F(x_1,x_2,x_3)=x_1^4+(x_1+x_2)^2+(x_1+x_3)^2$. Is it also a global optimum? Explain your answer. 3

(c) Write down the conditions for solving the following optimization problem :

Maximize : $f(x_1,x_2,x_3,l)$ subject to the constraints :

$a_1x_1+a_2x_2+a_3x_3+bl=k_1$; Interpret the Lagrange multiplier associated with the constraint with proof. 2.5+2.5

3. *(a)* For the function $f(x,y)=3x^2+2y^3-6xy$, identify and classify all the critical points. 3

(b) Solve the following constrained optimization problem :

Maximize $f(L,K)=L^{0.2}K^{0.8}$ subject to the constraint $2L + 8K = 50$. Give an interpretation to the first order condition assuming that *f* represents a production function and the constraint is a cost function. What is the speciality of the function *f*? What is the interpretation of Lagrange Multiplier in this problem? 4

(c) Prove, for every integer $n \geq 1$, that

$$\sum_{k=1}^{n}\frac{1}{4k^2-1}=\frac{n}{2n+1}$$ 3

4. *(a)* Let $f(x,y)=x^2y-2xy^2+3xy+4$. Identify and classify the critical points. Also if there is a maximum &/or minimum identify whether it's a local or global maximum/minimum. 2.5+1.5